UNCERTAINTY MODELING IN HEALTH RISK ASSESSMENT AND GROUNDWATER RESOURCES MANAGEMENT

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UNCERTAINTY MODELING
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To my parents, Azade and Fevzi Kentel,

and my sister, Secil Kentel Toros

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SUMMARY

Real-world problems, especially those that involve natural systems, are complex and composed of many non-deterministic components. Uncertainties associated with these non-deterministic components may originate from randomness or from imprecision due to lack of information. Until recently, uncertainty, regardless of its nature or source has been treated using probability theory concepts. However, uncertainties associated with real-world systems are not limited to randomness. Imprecise, vague, or incomplete information may better be represented by other mathematical tools, such as fuzzy set theory, possibility theory, belief functions, etc. New approaches which allow utilization of probability theory in combination with these new mathematical tools have found applications in various engineering fields. One such environmental engineering field is human health risk assessment.

In the first part of this thesis two new approaches which utilize both probability theory and fuzzy set theory concepts to treat parameter uncertainties in carcinogenic risk assessment are proposed. As a result of these approaches fuzzy health risks are generated. For the fuzzy risk to be useful for practical purposes its acceptability with respect to the compliance guideline has to be evaluated. A new fuzzy measure, the risk tolerance measure, is proposed for this purpose. The risk tolerance measure is a weighted average of the possibility and the necessity measures which are currently used for decision-making purposes.

In the second part of this thesis two decision-making frameworks are proposed to determine the best groundwater resources management strategy in the Savannah, GA region. Groundwater resources management problems, especially those in coastal areas, are complex and require treatment of various uncertain inputs.

The first decision-making framework proposed in this study is composed of a coupled simulation-optimization model followed by a fuzzy multi-objective decision-making approach. The deterministic results obtained from the coupled simulation-optimization model are used to evaluate the degree of satisfaction of each alternative management strategy with respect to conflicting fuzzy objectives. The individual satisfactions associated with each management strategy are aggregated into a single overall performance value by using various aggregator operators. The management alternative with the highest overall performance is selected as the best management strategy.

The second decision-making framework includes a groundwater flow model in which the parameters of the flow equation are characterized by fuzzy numbers and a decision-making approach which utilizes the risk tolerance measure proposed in the first part of this thesis. The groundwater flow in the Savannah region is simulated by the groundwater model operator method proposed by Dou et al. (1995) and fuzzy hydraulic heads are calculated within the model domain. The fuzzy hydraulic heads are used to calculate fuzzy drawdowns at critical locations in the region and acceptability of these drawdowns with respect to crisp constraints are evaluated by using the risk tolerance measure. Similar to the first decision-making framework, the risk tolerance measures associated with various constraints are aggregated into an overall performance value and the overall performance value is used to select the best management strategy.

1 INTRODUCTION

Success of a decision-making process depends on data collection and processing the knowledge extracted from this data. As our ability to collect data and information about the components of a system increases, the need for new mathematical tools to include all of the available information into the decision-making process increases. Conventional tools become insufficient as we wish to include all of the available information, to consider usually conflicting goals of all parties involved, to include decision-makers' preferences, to satisfy all quantitative and qualitative constraints, etc.

Over time various mathematical tools have been developed to effectively process information. Most information is associated with various types of uncertainties. Historically, probability theory has been used to represent uncertain information. Probability theory has been applied to many engineering problems and proved to be successful when the uncertainty is due to randomness, and when enough statistical data about the non-deterministic components of the problem exist.

For complex problems in which the dependencies between variables are not well defined, sufficient statistical data is not available, or expert knowledge and emotions play a significant role in characterizing the data, tools of probability theory are less effective in treating associated uncertainties. For such problems the knowledge of probabilistic variables is imprecise or incomplete. Thus, it becomes infeasible to determine the associated probabilities or probability density functions for the components of the system.

Another significant restriction associated with probability theory is that it does not provide means to treat uncertainties which are not characterized by randomness. For the situations in which the source of imprecision is a random variable, probability theory is the methodology of choice for dealing with uncertainty and imprecision. However, uncertainties associated with most of the large-scale real-world systems in which human judgment, perception, and emotions play an important role are not limited to randomness. Such humanistic systems are typified by socioeconomic systems, transportation systems, environmental control systems, food production systems, education systems, health-care delivery systems, criminal justice systems, information dissemination systems and the like (Zadeh 1981a). Such systems are too complex or too ill-defined to admit precise analysis. “Fuzzy thinking” may guide us in developing solutions to the problems which are much too complex for precise analysis.

As stated by Zadeh (1981b) “.. *in general, the uncertainty which is intrinsic in soft data is a mixture of probabilistic and possibilistic constituents and, as such, must be dealt with by a combination of probabilistic and possibilistic methods.”* Possibility theory which uses fuzzy set theory concepts and probability theory are complementary. They deal with two different kinds of uncertainties which almost certainly exist in most of the complex real-world systems. The concept of possibility is an abstraction of our intuitive perception of ease of attainment or degree of compatibility, whereas the concept of probability is rooted in the perception of likelihood, frequency, proportion or strength of belief. Furthermore, the rules governing the manipulation of possibilities are distinct from those which apply to probabilities (Zadeh 1981b). Thus, it is our responsibility to treat any kind of uncertainty present in the problem by the appropriate mathematical tools.

To summarize, most real-world decision-making problems are defined in environments in which the imprecision may stem from both randomness and fuzziness. Here by "fuzziness" we refer to imprecision due to lack of information. Depending on the available information, approaches using statistical tools, fuzzy arithmetic tools, or a combination of the two, or other theories may need to be considered in treating uncertainties. What is important when dealing with uncertainty in planning and engineering, is to identify the mathematical framework that is most suitable for the problem domain with respect to the nature of uncertainty, required levels of precision, and logical strength (Kikuchi and Pursula 1998).

In this study, we deal with two main research areas in environmental engineering: (i) human health risk assessment; and, (ii) groundwater resources management. Our main goal is to develop new methodologies to treat uncertainties that may exist in various components of these research areas, such as uncertainties in the parameters of analytical and numerical models, and uncertainties in the decision-making process. While doing this, we use respectively new mathematical tools such as fuzzy set theory and possibility theory together with conventional tools such as probability theory. A brief explanation of fuzzy set theory and two research areas we choose to study in this thesis are provided below.

1.1 FUZZY SET THEORY

Fuzzy set theory concepts are used to model a different form of uncertainty other than uncertainty due to randomness. This other form of uncertainty associated with complex real-world systems is due to objects without sharply defined boundaries in which the transition from membership to non-membership is gradual rather than abrupt. A fuzzy set is a generalization of an ordinary set. The elements of ordinary sets are deterministic, an object either belongs to the set or not. However, a fuzzy set allows degrees of membership for its members. For example, two fuzzy sets, one for "young" and one for "middle-aged" can be defined such that, a 35 year old man will belong to the fuzzy set of "young" with a membership value of 0.2, while at the same time he will belong to the fuzzy set of "middle-aged" with a membership value of 0.9. The membership values indicate how compatible the age of the man with the fuzzy concepts of "young" and "middle-aged." In dealing with systems of a high order of complexity, probability theory may need to be utilized in association with fuzzy set theory or other theories. Clearly, most of the classes of objects which are encountered in the real-world are fuzzy sets in the informal sense defined above (Zadeh 1979; Zadeh 1994). Possibility theory, a newly emerging measure of uncertainty, uses fuzzy set theory as the mathematical tool in incorporating uncertainties into the analysis.

Fuzzy set theory initiated by Zadeh in the early 1960s (Zadeh 1964) is a theory of classes without sharp boundaries. Based on its fuzzy rather than two-valued logic, fuzzy systems theory does not aim at the discovery of precise assertions about the behavior of complex systems. Rather it aims at an accommodation with the pervasive imprecision of real-world systems by abandoning the unattainable goals of classical systems theory and adopting instead a conceptual framework that is tolerant of imprecision and partial truths (Zadeh 1981a). Available fuzzy information about the system of concern has to be processed by tools of fuzzy set theory. Estimated values from incomplete information can be calculated using fuzzy set theory. Fuzzy set theory also allows us to represent linguistic variables such as *performance* whose linguistic values may be very "*good"*, "*satisfactory"*, "*acceptable"*, "*not-acceptable"*, *"bad"* defined by membership functions.

When the nature of uncertainty and available information about a specific component of the problem is appropriate to model it as a fuzzy set, a membership function is used to represent the evidence about that specific component. Each entity in the domain of the fuzzy component has a corresponding membership function value, μ in the interval [0,1]. The membership function of the fuzzy component assigns the grade of membership for each entity in the domain. The nearer the value of the membership function of an entity to unity, the higher the grade of membership of that entity in the fuzzy component. The membership function like probability density function is used to represent the available evidence. Two very important concepts of fuzzy sets are the support of a fuzzy set and the alpha-cut, $\alpha - cut$.

The support of a fuzzy set A in the universe of discourse U is a crisp set that contains all the elements of U that have nonzero membership values in A, that is (Wang 1997),

$$supp(A) = \left\{ x \in U \mid \mu_A > 0 \right\} \tag{1.1}$$

where *supp(A)* denotes the support o the fuzzy set A. The $\alpha - cut$ of a fuzzy set A is a crisp set A^{α_c} that contains all the elements in U that have membership values in A greater than or equal to α, that is (Wang 1997),

$$A^{\alpha_c} = \{x \in U \mid \mu_A(x) \geq \alpha\} \tag{1.2}$$

These two concepts are used extensively throughout this thesis.

1.2 HUMAN HEALTH RISK ASSESSMENT

A basic definition of risk assessment provided by the National Research Council (1983) is "Risk assessment is a process in which information is analyzed to determine if an environmental hazard might cause harm to exposed persons or ecosystems." Human health risk assessment deals with exposed persons and provides qualitative and quantitative characterization of the relationship between environmental exposures and effects observed in exposed individuals (U.S. EPA 2003). The health risk assessment process consists of four main steps: Source/release assessment, exposure assessment, dose-response assessment and risk characterization. In this study, we are only concerned with the risk characterization step.

The goal of risk assessment is to estimate the severity and likelihood of harm to human health from exposure to a substance or activity that under plausible circumstances can cause harm to human health. Risk assessment has been used to quantify human health impacts due to exposure to toxic substances via multiple exposure routes such as ingestion (drinking), inhalation (breathing volatilized contaminants during showering), and dermal contact (contact of contaminated water with skin, for example while showering). Quantitative risk characterization involves evaluating exposure estimates against a benchmark of toxicity, such as a cancer slope factor (or cancer potency factor). Risk is calculated by multiplying the cancer slope factor of the toxic substance by the dose an individual receives. In this study, we are only concerned with the cancer risk.

Cancer risk is calculated using an analytical equation. The cumulative risk equation and risks associated with various routes are explained in detail in Chapter 3. Uncertainties in the parameters of the risk equation (i.e., Equation (3.1)) can be propagated into the resulting human health risk using various combinations of probability theory and fuzzy set theory concepts. We will refer to the models which process both random and fuzzy information as hybrid models. The first part of this thesis includes two studies in which we proposed two hybrid models. In the first study, we proposed a new methodology to combine probability density functions of random variables and membership functions of fuzzy variables in calculating fuzzy risk estimates for individuals at certain fractiles of risk. The second study provides an alternative for 2-Dimensional Monte Carlo Analysis (2D MCA). The 2D MCA is one of the advanced modeling approaches that may be used in probabilistic risk assessment studies. In the 2D MCA, the variables of the risk equation together with the parameters of these variables are modeled as random variables. For example, exposure frequency may be represented by a normal distribution whose mean and standard deviation are also normal distributions. In the alternative method, 2D Fuzzy

Monte Carlo Analysis (2D FMCA), the parameters of the random variables are characterized as fuzzy numbers. These two studies are given in detail in Chapter 3.

When possibilistic or hybrid models are used in human health risk assessment, fuzzy risks are generated. In order to determine acceptability of the resulting fuzzy risk, it has to be compared with a compliance criterion. Evaluation of the acceptability of a fuzzy risk with respect to a compliance guideline is a recently developing area. One method to evaluate the acceptability of the resulting fuzzy risk is proposed by Guyonnet et al (2003; 1999). They used the possibility and the necessity measures. Other methods such as defuzzification techniques exist; however, it is our understanding that these existing methods yield a loss of some valuable information that might have further enhanced the decision-making process. Thus, in this thesis we proposed a new measure, the risk tolerance measure, which utilizes a combination of the possibility and the necessity measures for making decisions in fuzzy environments. We also investigated how various membership functions affect the decision-making process when one of the existing methods or the proposed risk tolerance measure is used. The fuzzy decision-making framework for human health risk assessment is provided in Chapter 3 as well.

1.3 GROUNDWATER RESOURCES MANAGEMENT

In the second part of this thesis, we study a large scale application which is associated with a groundwater resources management problem in Savannah, GA. Determining the best groundwater management strategy involves combined utilization of various models:

(i) groundwater simulation models to simulate groundwater flow and to determine piezometric head distribution throughout the model domain; (ii) optimization models to determine optimum additional pumping rates in the presence of certain constraints, and; (iii) decision-making models to satisfy various management goals. In developing solution methodologies for the groundwater resources management problem in the Savannah region we use these three types of models. Our main focus is to develop mathematical tools which allow inclusion of uncertain information into these models. While doing this we use fuzzy set theory and possibility theory concepts.

The groundwater resources management problem in the Savannah region is defined in the following section. Next, we provide brief information on various models (i.e., crisp simulation-optimization model, fuzzy multi-objective decision-making model, numerical simulation model with fuzzy parameters, and a decision-making framework which utilizes the risk tolerance measure) that we utilize in developing solution methodologies for the groundwater management problem in the Savannah region.

1.3.1 Problem Definition

The Upper Floridan Aquifer (UFA) is a primary source of drinking and industrial process water in the Savannah region. Pumping from this aquifer at various locations has lowered groundwater levels resulting in encroachment of seawater into the aquifer at the northern end of Hilton Head Island, S.C. (Clarke and Krause 2000; Clarke and Krause 2001; Garza and Krause 1996). This saltwater contamination has constrained further development of the UFA in the coastal area and created competing demands for the limited supply of water (Leeth et al. 2003). Nevertheless, the coastal area of Georgia continues to grow and the best water resources management strategy has to be identified in order to maintain a continuous source of reliable water supply in the region. In this thesis, evaluation of only the groundwater resources is considered.

1.3.2 Coupled Simulation-Optimization Model

The U.S. Geological Survey (USGS) has developed a groundwater simulation model, the Savannah Area Model (Garza and Krause 1996), which utilizes the MODFLOW simulation code (McDonald and Harbaugh 1988) for the region. To simulate groundwater flow, we use the Savannah Area Model and its database in a desktop computer platform utilizing Processing ModFlow (PMWIN) computational environment (Chiang and Kinzelbach 2000). We developed a coupled simulation-optimization model to estimate the additional groundwater withdrawal potential in the Savannah region. The coupled simulation-optimization model is solved using GAMS software (Brooke et al. 1998). The coupled simulation-optimization model is deterministic (i.e., all the variables are treated as crisp numbers) and it is used to achieve two goals: (i) determining the spatial distribution of additional groundwater withdrawal potential within the model domain; and, (ii) evaluating multiple groundwater withdrawal permit applications.

The main objective of the optimization model is to maximize the additional groundwater withdrawal potential in the Savannah without increasing the risk of saltwater intrusion at the northern end of Hilton Head Island. While doing this, we wanted the user to have some control on maintaining uniform groundwater withdrawal rates. Drawdown at Hilton Head Island has been used as the critical constraint in determining future additional groundwater potential in the Savannah region in previous studies conducted by the USGS (Clarke and Krause 2000; Garza and Krause 1996) and the Georgia Environmental Protection Division (EPD 1997; EPD 2001). We use the same constraint in order to be consistent with previous work. The definition of the groundwater management problem in the Savannah region, existing approaches to solve the problem and the coupled simulation-optimization model together with its results for various scenarios are provided in detail in Chapter 4 as well.

1.3.3 Processing Uncertain Information

Real-world problems of modern engineering are complex, and they involve many non-deterministic components. The non-deterministic nature of inputs into these complex problems results in non-deterministic or uncertain outputs. Uncertainties associated with the results of complex problems have various sources. In the second part of this thesis we investigated treatment of uncertainty in various stages of the groundwater resources management process: (i) fuzzy multi-objective decision-making, which utilizes the results of the coupled simulation-optimization model; and, (ii) simulation of groundwater flow by finite difference method with fuzzy parameters, followed by evaluation of the resulting fuzzy drawdown with respect to a crisp drawdown requirement at the northern end of Hilton Head Island.

1.3.3.1 Treatment of uncertain information in fuzzy multi-objective decision-making

Most of the time water resources management problems involve various conflicting objectives. Generally, it is more convenient to represent these objectives in linguistic terms: “good satisfaction of the demand,” “low drawdowns at critical locations,” “fair groundwater availability for all users,” etc. Thus, we proposed a multi-objective decision-making framework in which the objectives are represented as fuzzy sets. The results of the coupled simulation-optimization model for various scenarios are used within this decision-making framework to select the best management scenario. The fuzzy multi-objective decision-making framework involves aggregation of individual satisfactions of each management scenario with respect to the fuzzy objectives into a single overall performance by using various aggregation operators such as “and,” “or,” and “ordered weighted averaging (OWA).” Details of the fuzzy multi-objective decision-making process are provided in Chapter 4.

1.3.3.2 Treatment of uncertain information in groundwater flow simulation

Another source of uncertainty that exists in the groundwater flow modeling stage of the management process is transmissivity within the model domain. In order to include the uncertainty associated with transmissivities in the Savannah region an idealized two-dimensional (2D) finite difference model for only the UFA is developed.

For situations in which the transmissivities with in the model domain can best be represented by fuzzy numbers, the system of equations resulting from the numerical representation of the governing differential equation can be transformed into a system of interval equations. The system of interval equations may be solved by using various methods. Here, we used groundwater model operator method proposed by Dou et al. (1995) to solve the numerical model with fuzzy transmissivities to determine fuzzy drawdown at the northern end of Hilton Head Island. The groundwater model operator method uses a nonlinear optimization algorithm to solve the system of interval equations. The nonlinear optimization model is solved using GAMS software (Brooke et al. 1998). In order to decide if the resulting fuzzy drawdown is acceptable or not, the risk tolerance measure proposed in Chapter 3 may be used. The solution algorithm, results of the numerical model with fuzzy parameters, and decision-making process are given in Chapter 5.

As the thesis includes various problems from two different environmental research areas, each part has its own conclusions section. However, in Chapter 6 the overall conclusion of the uncertainty studies conducted in this thesis is provided.

1.4 CLOSURE

Real-world problems of modern engineering are complex and they involve many non-deterministic components. The non-deterministic nature of inputs into these complex problems results in non-deterministic or uncertain outputs. Uncertainties associated with complex problems have various sources. Until recently, uncertainty, without considering the nature or source of it, has been treated by probability theory concepts. However, a growing interest for non-probabilistic methods has emerged due to criticism on the credibility of probabilistic analysis when it is based on limited information (Moens and Vandepitte 2005). For example, when the only information provided by an expert about the value of a parameter is “the value of x lies in an interval A,” assuming a uniform probability distribution for x in the interval A introduces information that in fact is not available (Baudrit et al. 2004; Helton 2004). Uncertainty in model parameters may arise from randomness or from imprecision due to lack of information. Baudrit and Dubois (2005) state that in practice, while information regarding variability is best conveyed using probability distributions, information regarding imprecision is more faithfully conveyed using families of probability distributions encoded either by probability-boxes (upper and lower cumulative distribution functions (Ferson et al. To appear; Ferson et al. 2003)) or possibility distributions (also called fuzzy intervals) (Dubois et al. 2000), or by random intervals using belief functions of Shafer (Shafer 1976). Thus, depending on the source and nature of available information probabilistic approaches or non-probabilistic approaches such as interval analysis, possibility theory, fuzzy set theory, or other theories may become more appropriate to propagate parameter uncertainties in modeling natural systems. The non-probabilistic approaches should be regarded as complementary rather than competitive to the probabilistic approaches (Moens and Vandepitte 2005).

Research on a broader conception of uncertainty-based information, liberated from the confines of classical set theory and probability theory, began in early 1980s (Klir and

Wierman 1999). In constructing a model, our goal is to characterize the system using the best possible formulation. This formulation should attempt to capture all the available information by using any feasible mathematical framework. Substantial research has been conducted in this direction (see the Closure section of Chapter 2). In addition to classical set theory and probability theory, uncertainty-based information is now well understood in fuzzy set theory, possibility set theory, and evidence theory (Klir and Wierman 1999).

In this thesis we propose various models in which available information is processed by utilizing mathematical tools of probability theory, fuzzy set theory, and possibility theory, or a combination of these. We work on both hypothetical and real-world problems to demonstrate new methodologies in two environmental engineering research areas: human health risk assessment and groundwater resources management.

2 LITERATURE SURVEY

2.1 PROBABILITY THEORY AND FUZZY SET THEORY

Probability theory had been used as the sole tool for modeling uncertainty until the 1960s. When it was recognized that probability theory is capable of representing only one of the several distinct types of uncertainty, new theories for treating uncertainty emerged. One of the milestones in the evolution of these new uncertainty theories is the seminal paper by Lofti A. Zadeh (1965). He proposed a new mathematical tool in his paper and called this new mathematical tool "fuzzy sets." Although the American philosopher Max Black (1937) studied some ideas presented in Zadeh's paper about 30 years earlier, most of the fundamental concepts in fuzzy set theory were developed by Zadeh (1965; 1971; 1972a; 1974; 1979; 1981b). He proposed the concept of fuzzy algorithms in 1968 (Zadeh 1968a), and together with Bellman, proposed a new approach for decision-making in fuzzy environments in 1970 (Bellman and Zadeh 1970). In 1973, he published "Outline of a new approach to the analysis of complex systems and decision processes" (Zadeh 1973), in which he defined linguistic and fuzzy variables, fuzzy conditional statements, and fuzzy algorithms.

Fuzzy set theory and fuzzy modeling was initiated by L. A. Zadeh in 1965; and since then, many researchers have produced valuable references on this topic (Bezdek et al. 1999; Dubois et al. 1997; Kaufmann and Gupta 1985; Kaufmann and Gupta 1988; Klir and Yuan 1995; Nguyen and Sugeno 1998; Nguyen and Walker 1997; Wang 1982; Yager

1982; Yager and Filev 1994; Zimmermann 1985). A collection of recent literature on theoretical advances and applications of fuzzy set theory concepts in engineering, economics, medicine, ecology, etc. is provided by Dubois et al. (2001).

Despite opposition from mathematicians in statistics and probability who claimed that probability is sufficient to characterize uncertainty and any problem that fuzzy theory can solve can be solved equally well or better by probability theory, researchers began to work in this new field. Especially with the success of fuzzy controllers (Holmblad and Østergaard 1982; Mamdani and Assilian 1975; Sugeno et al. 1989; Yasunobu and Miyamoto 1985) many scientists and engineers began to look at fuzzy set theory seriously. Numerous studies on the discussion of probability versus possibility (fuzziness) are provided in the special issue of the IEEE Transactions on Fuzzy Systems (Vol. 2, No. 2, 1994). Following these developments, mathematicians came to the conclusion that "fuzzy set theory and probability theory are complementary, and they deal with different types of uncertainties" (Wang 1997).

Fundamental procedures to allow combined utilization of fuzzy set theory and probability theory to treat uncertainties have been proposed and developed since the emergence of fuzzy set theory. L. A. Zadeh provided the definition of "the probability of a fuzzy event" in 1968 (Zadeh 1968b). Comparisons of probability theory with possibility theory, what kind of uncertainties they treat, general definitions of probability theory, and fuzzy set theory concepts in the context of uncertainty modeling are provided in many references (Dubois and Prade 1980; Dubois and Prade 1993; Dubois and Prade 1994; Ferson and

Ginzburg 1996; Jumarie 1995; Kandel 1986; Kangas and Kangas 2004; Klir 1995; Moens and Vandepitte 2005; Nau 2002; Nguyen 1997; Pedrycz and Gomide 1998; Ross 1995; Sandri et al. 1995; Wang 1997; Xia 2001; Yager 1999b; Zadeh 1995). Taking it one step further, Ferson and Ginzburg (1995) proposed hybrid arithmetic to treat uncertain numbers which are represented by only fuzzily known probability functions. Similar mathematical tools have been proposed by various other researchers (Cooper et al. 1996; Ferson et al. 2002; Ferson and Kreinovich 2001; Kreinovich et al. 2001).

Once the theoretical background for treating uncertainties using fuzzy set theory was established, researchers began working on application of this new tool to real-world problems. A substantial amount of research using fuzzy set theory in environmental engineering problems has been conducted (Akter and Simonovic 2005; Bárdossy 2003; Bárdossy et al. 1990b; Bárdossy and Disse 1993; Coppola Jr et al. 2002; Dou et al. 1999; Dou et al. 1997b; Ghosh and Mujumdar 2006; Ravi and Reddy 1999; Schulz and Huwe 1997; Vamvakeridou-Lyroudia et al. 2005; Woldt et al. 1996). Several researchers used fuzzy arithmetic and various statistical tools to model uncertainties and compared the results they obtained by using these mathematical tools. For example, Buckley (1990) considered a linear programming problem with uncertain parameters that are modeled as either random variables or as fuzzy variables, and he provided a comparison of the results. Schulz and Huwe (1997) pointed out the main differences between fuzzy and stochastic concepts to account for uncertainties through a water flow modeling problem in the unsaturated zone. Chan and Govindaraju (2001) provided a comparison of three different methods (i.e., interval computing method, Monte Carlo simulations, and stochastic theories) for analyzing a field-scale solute transport. Coppola et al. (2002) developed a fuzzy rule-based methodology for estimating monthly groundwater recharge. They compared the accuracy of this method with that of ordinary linear regression.

Today's challenge is utilization of fuzzy set theory and probability theory simultaneously in treating uncertainties for real-world problem. Recently, researchers have begun working on applications of hybrid fuzzy-probability models, i.e., "hybrid models", to real-world problems. Different frameworks which make use of fuzzy set theory and probability theory in an integrated fashion have been proposed for problems in the environmental engineering field (Kikuchi and Pursula 1998; Liu et al. 2004b; Meghdadi and Akbarzadeh-T. 2003; Mujumdar and Sasikumar 2002; Sasikumar and Mujumdar 2000). Decision-making is one of the research areas in which simultaneous utilization of fuzzy and probability tools have recently been explored (Khadam and Kaluarachchi 2003; Liu et al. 2004b; Maqsood et al. 2005; Ravi and Reddy 1999; Yager 1999b; Yager 2000; Yager 2002).

2.2 HUMAN HEALTH RISK ASSESSMENT MODELS

One of the environmental engineering fields in which probabilistic uncertainty modeling has been widely applied is environmental health risk analysis. Upon development of relevant mathematical tools, fuzzy set theory has also been used for modeling parameter uncertainties in risk calculations. Simultaneous applications of fuzzy set theory and probability theory in health risk assessment studies are recent. This thesis includes two different cancer risk estimation methodologies, which use hybrid models.

The goal of risk assessment is to estimate the severity and likelihood of harm to human health from exposure to a substance or activity that under plausible circumstances can cause harm to human health. The Environmental Protection Agency (EPA) provides methods and procedures to conduct health risk assessments (U.S. EPA 1989). Details of risk assessment can also be found in other references (Cohrssen and Covello 1989; Glickman and Gough 1990; Kolluru et al. 1996; Louvar and Louvar 1998). Traditionally, statistical methods are used for quantitative treatment of variability and uncertainty in health risk assessment context (U.S. EPA 1997; U.S. EPA 1998).

Probabilistic risk assessment is a general term for risk assessment studies that use probability models to represent the likelihood of different risk levels in a population (i.e., variability) or to characterize uncertainty in risk estimates. The application of probabilistic analysis to human health is a relatively recent development that is used to obtain a probabilistic approximation to the solution of a mathematical equation or model (U.S. EPA 2001). Many authors studied different aspects of uncertainty modeling in risk assessment analysis. Most of the work involves utilization of probability theory in treating uncertainty and variability for multi-pathway exposure scenarios (Bennett et al. 1999; Bogen and Spear 1987; Critto et al. 2005; Cullen and Frey 1999; Finley et al. 1992; Glorennec 2006; Hamed 1997; Labieniec et al. 1996; Labieniec et al. 1997; Ma 2000; Ma

2002; Maxwell and Kastenberg 1999; Maxwell et al. 1998; McKone and Bogen 1991; Mohamed and Côté 1999; Valberg et al. 1996; Vose 1996; Wong and Yeh 2002).

After fuzzy set theory was recognized as a tool for treating uncertainties, researchers started using this tool for risk assessment modeling and risk management (Bogardi et al. 1990; Chen et al. 1998; Huang et al. 1999; Lee et al. 1994; Lee et al. 1995; McKone and Deshpande 2005; Ozbek and Pinder 1998; Rajkumar and Guesgen 1996). Guyonnet et al. (1999) provides a comparison of two methods – a possibilistic approach using fuzzy set theory and a probabilistic approach using Monte Carlo analysis – for addressing uncertainties in risk assessment. Recently, hybrid models which utilize both fuzzy set theory and probability theory simultaneously to treat uncertainties in risk assessment studies have been developed (Baudrit and Dubois 2005; Chen et al. 2003; European Commission 1995; Guyonnet et al. 2003; Kentel and Aral 2004; Kentel and Aral 2005; Liu et al. 2004a; McKone and Deshpande 2005).

The application of the fuzzy set theory in propagating uncertainties in health risk assessment is rather new, thus well-established procedures to evaluate acceptability of fuzzy risk with respect to a crisp guideline do not exist. One method proposed by Guyonnet et al. (2003; 1999) uses the possibility and the necessity measures for evaluating acceptability of a fuzzy risk. Another alternative approach to check the acceptability of the fuzzy risk with respect to a compliance criterion is defuzzifying (i.e., converting a fuzzy set into a crisp value) the fuzzy risk and then comparing it with the crisp standard. Various defuzzification methods have been proposed in the literature:

maximum method, center of gravity method, center of maxima method, mean of maxima method, etc. These methods lead to different results. Detailed information about different defuzification methods can be found in Yager and Filev (1994), Klir and Yuan (Klir and Yuan 1995), Wang (Wang 1997). Defuzzified results are not capable of representing the information that is delivered by the shape of the membership function.

2.3 GROUNDWATER RESOURCES MANAGEMENT IN COASTAL AREAS

Freshwater demand in coastal areas is increasing due to population growth. Intensive pumping of coastal aquifers causes saltwater intrusion and seawater encroachment. Thus, there is a need for determining the best groundwater management strategy which will guide future groundwater development. Commonly, decision-makers desire to select the best management strategy which maximizes the total pumping from the coastal aquifer, while protecting the wells from saltwater intrusion. Groundwater resources management in coastal areas have been studied by many researchers (Bauer et al. 2006; Don et al. 2005; H. A. Loáiciga 2000; Hallaji and Yazicigil 1996; Ji and Chang 2005; Kooi and Groen 2003; Liu et al. 2001; Melloul and Collin 2000; Rao et al. 2004).

2.3.1 COUPLED SIMULATION-OPTIMIZATION MODEL

Simulation of the groundwater behavior, together with fulfillment of the objectives of the optimization model is accomplished simultaneously by a coupled simulation-optimization model. Coupling the simulation model with the optimization model can be accomplished at the optimization stage by using “embedding method” or “response matrix approach.” In the embedding method (Aguado and Remson 1974), the finite difference or the finite element approximations of the governing groundwater flow equations are included in a linear or nonlinear programming model in the form of constraints. Numerous researchers have used the embedding method for the hydraulic management of aquifers (Das and Datta 1999a; Das and Datta 1999b; Das and Datta 2001; Wagner et al. 1992; Yazicigil and Rasheeduddin 1987). In the response matrix approach, the simulation model is externally used to develop a unit response matrix, and this response matrix is coupled with the optimization model.

The response matrix approach initially occurred in the petroleum engineering literature (Lee and Aronofsky 1958) and was soon utilized in the groundwater literature (Aral 1989; Colarullo et al. 1984; Daskin and Gorelick 1985; Deninger 1970; Gorelick and Remson 1982; Hallaji and Yazicigil 1996; Heidari 1982; Maddock III 1972; Maddock III and Lacher 1991; Mylopoulos et al. 1999; Ndambuki et al. 2000; Shen et al. 2004; Theodossiou 2004; Willis and Finney 1988; Yazicigil et al. 1987; Zhou et al. 2003). The response of the aquifer system (i.e., hydraulic head or drawdown) to pumping or recharge is generated and stored in a matrix, called the response matrix. In this study, the response matrix approach is used to couple the simulation model with the optimization model. The management models utilize various types of programming techniques such as linear, quadratic, dynamic, heuristic, etc. as the optimization algorithm while the hydraulics of groundwater is modeled by case-specific simulation models. Management models which couple an optimization algorithm with a groundwater hydraulics simulation approach were developed by many researchers (Emch and Yeh 1998; Finney et al. 1992; Gorelick et al. 1984; Kaunas and Haimes 1985; Kwanyuen and Fontane 1998; Lindner et al. 1988; Maddock III 1974; Mantoglou 2003; Mantoglou et al. 2004; McPhee and Yeh 2004; Rao et al. 2003; Shamir et al. 1984; Willis and Finney 1988). Papadopoulou (2002) provides a detailed review of the most recently developed groundwater management models.

2.3.2 Processing Uncertain Information

Recently, uncertainties associated with the groundwater flow system have been included into the management models via different mathematical tools. Sawyer and Lin (1998) developed a chance-constrained programming optimization method that accounts for uncertainties in the coefficients of the models and transforms a stochastic model into a deterministic equivalent which can be solved more simply. Aly and Peralta (1999) used a combined genetic algorithm-artificial neural network methodology, to account for uncertainty in hydraulic conductivity in a field-scale problem. Ndambuki et al. (2003) described how to reformulate a groundwater management problem where input parameters are uncertain as a second-order cone optimization problem. More recently, Feyen and Gorelick (2004) formulated a stochastic groundwater management model, aimed at preserving the hydroecological balance in wetland areas, that accounts for uncertainty in simulation predictions when planning regionally distributed groundwater production. Decision support systems have also been developed for various management problems in environmental engineering. For example, McPhee and Yeh (2004) constructed a decision support system for solving a groundwater management problem on a basin-wide scale, with explicit consideration of environmental objectives. They applied fuzzy set theory concepts to rank the alternatives and to assist decision-makers in selecting a suitable policy.

Treatment of uncertain information at two different stages is considered in developing solution methodologies for the groundwater resources management problem in the Savannah region: (i) Various groundwater management scenarios are evaluated with respect to multiple fuzzy objectives, and (ii) Uncertainties associated with the parameters of the groundwater flow are integrated into the management process. Background research related with these two different sources of uncertainty and how they are treated in management models are provided below.

2.3.2.1 TREATMENT OF UNCERTAIN INFORMATION IN FUZZY MULTI-OBJECTIVE DECISION-MAKING

The elements of the real-world decision-making problem - the parameters, feasible actions, objectives, constraints, states of nature, etc - often involve fuzziness together with randomness. Depending on the type of uncertainty, probability theory, fuzzy set theory, a combination of both, or another approach, may be selected as the appropriate tool to include uncertainties associated with various components of the system into the solution of the problem. Recognizing uncertainties and incorporating them into the analysis may lead to more informed decisions and may provide valuable information about the credibility of the results in representing the real-world situation. Thus, numerous approaches have been proposed to include uncertainties into decision-making.

Since we are mainly concerned with approaches utilizing fuzzy set theory, a brief history of application of fuzzy set theory concepts in decision-making is provided below.

After the concepts of fuzzy set theory were established in the mid-sixties, application of the fuzzy set approach to decision-making was suggested by Bellman and Zadeh (1970) and Chang (1971). Early work on decision-making in a fuzzy environment provide definitions and situations in which fuzzy set theory concepts are appropriate to model uncertainties (Jain 1976; Tanaka et al. 1974; Tanaka et al. 1975; Watson et al. 1979; Zadeh 1973; Zimmermann 1985).

Fuzzy set theory concepts have been used for various classes of decision-making problems, such as fuzzy preferences and choice, group decision-making, multi-criteria decision-making, multistage decision-making, optimization, mathematical programming, dynamic programming, etc. Various books that cover these topics have been published on decision-making in fuzzy environments (Bezdek et al. 1999; Carlsson and Fullér 2002; Kacprzyk and Fedrizzi 1990; Kacprzyk and Orlovski 1987; Lai and Hwang 1992; Lai and Hwang 1994; Slovinski 1998) together with many journal articles (Bender and Simonovic 2000; Bodjanova 1997; Carlsson and Fullér 1996; Ekel et al. 1998; Ekel 2001; Ekel 2002; Ramik and Vlach 2002; Ribeiro 1996; Roubens 1997; Sakawa et al. 1993; Stanciulescu et al. 2003; Wang and Huang 2002; Wang 2001; Yager 1995; Yager 2004a). Recently, much research involves utilization of possibility theory concepts in decision-making (Dubois et al. 1999; Guo and Tanaka 1996; Inuiguchi et al. 2003; Inuiguchi et al. 1989; Inuiguchi and Tanino 2002; Lee et al. 2000).

Real-world environmental engineering problems involve uncertainties which may be modeled using fuzzy set theory concepts. For example, most of the time, the goals, the constraints, or the evaluation criteria of practical environmental problems cannot be defined precisely. Thus, researchers recently started utilizing fuzzy set theory concepts for decision-making problems in environmental engineering fields (Bonano et al. 2000; Geng et al. 2001; Mohamed and Côté 1999; Mujumdar and Sasikumar 2002; Tilmant et al. 2002; Wang and Huang 2002; Wang and McTernan 2002; Yin et al. 1999).

For groundwater resources management problems, although an ultimate objective may exist, decision-makers usually want to satisfy some additional non-crisp/fuzzy objectives depending on the specific problem. These additional objectives are characterized by fuzzy sets in this thesis. Then a solution methodology which utilizes a fuzzy multi-objective decision-making framework is proposed for the groundwater management problem in the Savannah region. Various methodologies have been proposed for solving multi-objective decision problems which involve uncertainties and many researchers have contributed to this topic (Akter and Simonovic 2005; Bender and Simonovic 2000; Despic and Simonovic 2000; Inuiguchi et al. 1987; Kacprzyk and Nurmi 1998; Lai et al. 1994; Murata et al. 1996; Padet et al. 1995; Sakawa et al. 1999; Yager 1977; Yager 1984).

Various approaches for calculating an overall performance of the multiple objectives have been proposed. Bellman and Zadeh (1970) formulated a decision function which assigns equal importance to each objective. In their formulation they used the intersection operator (i.e., conjunctive operator) to aggregate individual performances into a single value. Yager (1978) formulated a fuzzy decision-making procedure for problems with multiple objectives with unequal importance for the intersection operator. Ravi and Reddy (1999) used Yager's approach to rank Indian coals. Another similar decision-making application is provided by McPhee and Yeh (2004); they used Yager's (1988) ordered weighted averaging method to rank groundwater management alternatives. In this study, the determination of the best management strategy is formulated as a fuzzy multi-objective decision-making problem, and a conjunctive (i.e., and), a disjunctive (i.e., or), and an averaging (i.e., OWA) operator is used to evaluate overall performance values. In addition, Yager's (1978) approach is used to evaluate unequal fuzzy objectives to determine the best management strategy with the intersection and OWA operators. Detailed explanations of conjunctive, disjunctive, and averaging aggregation operators are provided in Slovinski (1998), Yager and Filev (1994), Nguyen and Walker (1997), Ross (1995), Wang (1997), and Klir and Yuan (1995). As an example of the averaging operators we used the ordered weighted averaging (OWA) operator. OWA operators are introduced by Yager (1988). Yager provided the mathematical background and various applications of OWA operators in his studies (Calvo et al. 2004; Filev and Yager 1994; Yager 1993; Yager 1996a; Yager 1996b; Yager 1998a; Yager 1998b; Yager 2004b; Yager 2004c; Yager and Filev 1999). For water resources management in the Savannah region examples of all three aggregation operators, conjunctive (i.e., intersection operator), disjunctive (i.e., union operator), and averaging operators (i.e., OWA) are used and the details of the results are presented in Chapter 4.

2.3.2.2 Treatment of uncertain information in groundwater flow simulation

Generally the hydrogeological parameters of the groundwater flow equation are measured at specific locations in space and time. Since it is not feasible to measure these parameters at every point in the model domain, some estimated values are used at locations with no observations. There are several options for representing these parameters throughout the model domain. They might be represented by crisp numbers, random numbers, or fuzzy numbers. When the parameters are represented by crisp numbers the groundwater flow model becomes a deterministic model and classical types of numerical methods such as Finite Difference, Finite Element, and Finite Volume Methods are used to solve the governing differential equations. These methods have been extensively used in groundwater flow modeling. Detailed information on numerical modeling of groundwater flow can be found in Remson et al. (1971), Bear (1972), Huyakorn and Pinder (1983), and Frind (2003).

As in most engineering applications, model parameters cannot be determined precisely. Traditionally scientists modeled these parameters as random variables. Introducing these random variables into the flow equation results in a stochastic partial differential equation, whose solution consists of not only the mean value but also the statistics of the head at every location (Shafike 1994). The details of the solution of the stochastic partial differential equation can be found in de Marsily (1986), Orr (1993), Harter (1994) and Shafike (1994). More recently, a collection of short articles on stochastic hydrogeology was published in the Stochastic Environmental Research and Risk Assessment Journal:

Zhang and Zhang (2004), Dagan (2004), Neuman (2004), Winter (2004), Christakos (2004), Molz (2004), Rubin (2004), Grin (2004), Sudicky (2004), and Freeze (2004). A stochastic partial differential equation is used to represent the groundwater flow when model parameters, boundary or initial conditions, or the source/sink terms are described by random variables. Probability density functions are used to represent the inputs which are random variables. Then the output also becomes a random variable. Perturbation techniques, spectral analysis, and Monte Carlo simulations are some of the methods used to solve stochastic partial differential equations (Shafike 1994). Various applications of these techniques can be found in Neuman and Yakowitze (1979), Dagan (1982), Gelhar and Axness (1983), Dagan (1986), Gelhar (1986), Wagner and Gorelick (1989), Georgakakos and Vlatsa (1991), Sun and Yeh (1992), Zhang (2002).

Utilizing fuzzy numbers for the parameters of the numerical model is another way of treating parameter uncertainty. When it is best to represent the parameter uncertainty in terms of membership functions, fuzzy set theory can be used to propagate uncertainties to the results of groundwater flow simulation. Imprecise parameters which originate from incomplete information, indirect measurements, subjective interpretations, and expert judgment of available information can be incorporated into the groundwater flow models via various techniques. Groundwater flow simulation using numerical groundwater flow models with fuzzy parameters is a new research area.

Shafike (1994) proposed a new approach to propagate parameter uncertainty in terms of imprecise information into the dependent variable, the head, of the flow equation. In their approach they considered the model parameters to be fuzzy numbers and represented them by membership functions. More specifically, they formulated the groundwater flow problem numerically using fuzzy set theory coupled with interval analysis. Their technique allows characterizing the model parameter, boundary conditions, initial conditions, and pumping rates as fuzzy numbers. They demonstrated this technique by a numerical example of a two-dimensional transient flow problem. Transmissivity is allowed to be a fuzzy number and a finite element technique is used to solve the groundwater flow problem. At each alpha-cut level, the solution of the flow equation with the finite element technique resulted in an interval system of linear equations. The interval system of linear equations are solved by an iterative algorithm proposed by Moore (1979). As a result, the dependent variable (i.e., the hydraulic head) at each node is calculated in terms of fuzzy numbers. Shafike (1994) concluded that the new approach they proposed provides a more realistic methodology for handling the problem of incomplete and imprecise data.

Dou et al. (1995) proposed another method, the groundwater model operator method, to incorporate imprecise parameters into steady state groundwater flow models. They used fuzzy numbers to represent imprecise parameters. Dou et al. (1995) state that the iterative method used by Shafike (1994) provides an enclosure of the real solution set for a linear interval system of equations, but it is normally wider than the real solution set. Another issue pointed out by Dou et al. (1995) associated with the solution methods of linear interval equations is that the coefficient matrix and the right-hand vector are assumed to vary independently of each other in their given intervals. However, in numerical methods of groundwater flow, since flow equations are derived based on finite difference or finite element techniques the coefficients and the right-hand vector are related to each other. Thus, Dou et al (1995) concluded that neglecting this dependence results in much wider hydraulic head membership functions, or an overestimate of the uncertainty in head values. The alternative method, the groundwater model operator method, proposed by Dou et al (1995) is based on a constrained nonlinear optimization algorithm to solve the linear interval equations at different alpha-cut levels. Dependence in hydraulic head coefficients is considered in this method, thus overestimation is prevented. Dou et al (1995) concluded that the hydraulic head imprecision is quite sensitive to the dependence of elements in the coefficient matrix and the right-hand vector in the groundwater flow model.

Later, Woldt et al. (1996) used the constrained nonlinear solution formulation to approximate the unsteady groundwater flow in a heterogeneous, isotropic, and confined aquifer, in which aquifer parameters such as transmissivity are considered to be fuzzy numbers. A method to solve the solute transport problem when the loading concentration is fuzzy is also presented in the same study (Woldt et al. 1996). Another application of a transient fuzzy groundwater flow model is provided in Dou et al (1997a). As a result of this study, Dou et al (1997a) concluded that the fuzzy modeling technique can handle imprecise information directly without generating a large number of realizations. They also observed that the hydraulic head is not guaranteed to be a monotonic function of transmissivity at all nodes in a group in a spatially variable transmissivity field. The same group of researchers extended their research into numerical solute transport simulation using a fuzzy sets approach in a paper published the same year (Dou et al. 1997b).

More recently, various researchers presented solution methodologies for numerical methods with fuzzy parameters, especially fuzzy finite element methods. Akpan et al. (2001a) provided a practical approach for analyzing the response of structures with fuzzy parameters. The methodology they presented involves integrated finite element modeling, response surface analysis, and implicit fuzzy analysis procedures. In another study Akpan et al. (2001b) used a fuzzy finite element based approach for modeling smart structures with vague and imprecise uncertainties. In their paper, they used fuzzy sets to represent the uncertainties present in the piezometric, mechanical, thermal, and physical properties of the smart structure.

Treating uncertainties as interval values for various engineering problems became popular in recent years. Köylüoğlu et al. (1995) developed an interval approach which uses the finite element method (FEM) to deal with pattern loading and structural uncertainties. Rao and Sawyer (1995), Rao and Berke (1997), and Rao and Chen (1998) developed different versions of interval based FEM to treat uncertainties in engineering problems. Muhanna and Mullen (1995; 1999; 2001) used finite element analysis procedures which utilize the concepts of fuzzy sets through interval analysis for mechanics problems. A survey of non-probabilistic uncertainty treatment in finite element analysis is provided by Moens and Vanderpitte (2005).

2.4 CLOSURE

Integrating all the available information into humanistic systems such as socioeconomic systems, transportation systems, environmental control systems, food production systems, education systems, health-care delivery systems, criminal justice systems, information dissemination systems as indicated by Zadeh (1981a) is very important. Most of the available data and information that exist are associated with some type of uncertainty. Thus, appropriate modeling of such systems requires utilization of mathematical tools which allow processing of all the available information, including uncertain information.

Recently, researchers have been working on developing mathematical tools that may help processing uncertain information in engineering problems. Results obtained from Science Citation Index Expanded (i.e., SCI-EXPANDED) when we searched for the words "uncertain or uncertainty" on March 14, 2006 indicates this fact. The results of our search are summarized in Table 2.1.

Table 2.1 Search results for "uncertain or uncertainty" in SCI-EXPANDED database

Year range	Number of results found
1960-1975	1,647
1976-1990	5,515
1991-2006	89,749

As can be seen from Table 2.1 number of publications in which the word "uncertain or uncertainty" occurs increased tremendously in the last 15 years. It has been recognized that processing the available uncertain information is necessary for better representation and modeling of complex real-world systems. Traditionally, this is accomplished by using the probability theory. Then, mathematical tools such as fuzzy set theory and possibility theory emerged and being used for processing uncertain information. We conducted another research in SCI-EXPANDED database on how utilization of probability theory, fuzzy set theory or possibility theory, and a combination of these two progressed during years. The results of our search are provided in Table 2.2. Fuzzy set theory was first proposed by Zadeh in 1965 (Zadeh 1965) and as can be seen from Table 2.2 utilization of fuzzy set theory or possibility theory continuously increased since then.

Table 2.2 Search results for “probability”, “fuzzy or possibility”, and “probability and fuzzy or possibility” in SCI-EXPANDED database

Year range	Number of results found when searched for		
	“probability”	“fuzzy or possibility”	“probability and fuzzy or possibility”
1960-1975	4,657	2,747	129
1976-1990	7,145	7,457	2,461
1991-1995	30,448	46,810	5,498
1995-2000	41,640	63,115	9,415
2000-2006	56,548	77,770	14,147

This thesis includes various studies conducted in two main research areas in environmental engineering: human health risk assessment and groundwater resources management. Our goal in all of these studies was to propose methodologies which allow characterization of uncertain information by using fuzzy set theory and possibility theory concepts. We are aware of the fact that there are problems which may require combined utilization of probability theory and fuzzy set theory to process all the available information. Recently, research which includes a combination of probability theory and fuzzy set theory or possibility theory concepts have increased, as indicated by Table 2.2. Hence, we also proposed methods which allow integrated utilization of mathematical tools from both probability theory and fuzzy set theory. We think that it is the engineer's responsibility to select the best available tool to treat uncertainties in modeling complex systems and the nature of uncertainties should guide the engineer in making this decision.

3 UNCERTAINTY PROPAGATION IN HUMAN HEALTH RISK ASSESSMENT

Even at its best, risk assessment does not estimate risk with absolute certainty (U.S. EPA 2004). The National Research Council (1994) states why characterizing uncertainty and variability in risk assessment is important as follows: "The very heart of risk assessment is the responsibility to use whatever information is at hand or can be generated to produce an estimate, a range, a probability distribution — whatever best expresses the present state of knowledge about the effects of some hazard in some specific setting. To ignore the uncertainty in any process is almost sure to leave critical parts of the process incompletely examined and hence to increase the probability of generating a risk estimate that is incorrect, incomplete, or misleading." Hence it is important that human health risk assessment process treats uncertainty and variability in a scientific and robust manner.

First, we investigate uncertainty propagation in human health risk assessment models. Then, we provide two hybrid approaches which allow combined utilization of probability theory and fuzzy set theory tools for integrating uncertainty and variability into the human health risk assessment models: (i) Probabilistic-fuzzy health risk modeling; (ii) 2D Monte Carlo versus 2D Fuzzy Monte Carlo health risk assessment. As a result of possibilistic or hybrid risk assessment studies, fuzzy risks may be generated. In order to evaluate the acceptability of the resulting fuzzy risk, it has to be compared with the compliance guideline set by the regulatory agency. In Section 3.5 we proposed a decision-making procedure to achieve this goal.

3.1 Processing Uncertain Information in Human Health Risk Assessment Studies

Humans are exposed to a mixture of pollutants from multiple sources. Thus, recent health risk assessment studies often consider aggregate exposures and cumulative risk calculations. Aggregate exposures and cumulative risk calculations involve many complicated phenomena and consequently require characterization and treatment of uncertain and variable information at various stages of the risk analysis. A better understanding of uncertainty and variability in the factors that contribute to exposure and risk assessment is required to be able to accurately represent the problem.

One of the major research areas in human health risk assessment studies is the development of approaches which allow integration of uncertainty and variability into the analysis. Health risk is related to an individual's location, activity, and behavior or preference, as well as pollutant emission rate, and physical, chemical, and biological processes involved in the fate and transport of the pollutant to humans. Intrinsic variability and extensive uncertainties exist in this spectrum of variables and processes. One such uncertainty is associated with the relevant parameters of the health risk assessment model since many of these parameters, in real-world problems, may not be known with certainty (Kelly and Campbell 2000; Liu 2004b). Probabilistic risk assessment (PRA), especially Monte Carlo Analysis, which uses statistical tools, is currently the most commonly utilized method for evaluating uncertainty and variability in health risk assessment (Cullen and Frey 1999; Schuhmacher et al. 2001; Vose 1996).

Statistical analysis provides robust, well accepted, and widely applied techniques to treat uncertainty and variability in risk assessment studies when and if sufficient data is available. However, most of the time, available data and information, and the accuracy and reliability of such data are not sufficient enough to conduct PRA. In addition, all uncertainties in data or model parameters are not due to randomness. Thus, other theories and computational methods are necessary to propagate uncertainty and variability in risk assessment studies. One such theory is possibility theory, which uses fuzzy set theory. Possibility theory will allow utilization of incomplete information (incomplete information includes vague, imprecise information which is not sufficient to generate probability distributions) together with expert judgment to treat uncertainties. Fuzzy set theory, which utilizes a membership function to describe a variable, can be used to represent uncertainty associated with exposure variables. In both of the hybrid models we proposed to treat parameter uncertainties that may exist in the health risk assessment process, we used Monte Carlo Analysis in combination with fuzzy arithmetic. Environmental decision-making will benefit from integrated probabilistic-fuzzy approaches by providing flexibility in treating uncertainty and variability associated with aggregate exposure and risk assessment studies.

3.2 EVALUATING THE CANCER RISK

Currently, health risk assessment studies concentrate on aggregate exposures and cumulative risk calculations since humans are exposed to a mixture of pollutants from multiple sources. Exposure assessment techniques estimate or directly measure the quantities of concentration of risk agents received by individuals, populations, or ecosystems. In this study, we carry out risk assessment analysis only from the individual risk perspective and not the population or ecosystem risk assessment, and we are only concerned with cancer risk. The risk is defined to be the increased individual chance of developing cancer during a specified time interval as a result of such an exposure.

In this study we assume that the source is contaminated tap water. Possible routes of exposure are ingestion, inhalation, and dermal contact. The cumulative risk equation and risk equations for these specific routes are given as follows (U.S. EPA 1989):

$$\text{Risk} = \sum_r Risk_r = \sum_r CDI_r \times CSF_r \quad r = ingestion, inhalation, dermal\,contact \tag{3.1}$$

where CDI is the chronic daily intake averaged over 70 years (mg/kg-day) and CSF is the cancer slope factor $(\text{mg/kg-day})^{-1}$.

Risk due to ingestion of chemicals in drinking water:

$$Risk_{ingestion} = \frac{CW \times IR \times EF \times ED}{BW \times AT} \times CSF_{ing} \tag{3.2}$$

where CW is chemical concentration in tap water (mg/L), IR is the ingestion rate (L/day), EF is the exposure frequency (days/year), ED is exposure duration (years), BW is the body weight (kg), AT is averaging time (equal to 70 years x 365 days/year), and CSF_{ing} is the cancer slope or potency factor associated with ingestion $(mg/kg\text{-}day)^{-1}$.

Risk due to inhalation of airborne chemicals:

$$Risk_{inhalation} = \frac{CA \times IH \times ET \times EF \times ED}{BW \times AT} \times CSF_{inh} \tag{3.3}$$

where CA is chemical concentration in air (mg/m^3), IH is the inhalation rate (m^3/hr) and CSF_{inh} is the cancer slope factor associated with inhalation $(mg/kg\text{-}day)^{-1}$.

A modified risk equation due to indoor inhalation of vaporized chemicals is given by McKone and Bogen (1991). The concentration of vaporized chemicals is estimated assuming a linear transfer function between tap water and air in various locations in the home. The contaminant vapor concentration in indoor air can be estimated from:

$$AC_i = CW \times \frac{W_i \times TE_i}{VR_i} \quad ; \quad i = s, b, h \tag{3.4}$$

where AC_i is the contaminant concentration in indoor air (mg/m^3), W_i is the tap water use rate (L/hr), TE_i is the transfer efficiency from tap water to air (unitless), VR_i is the air exchange rate (m^3/hr), and i takes the subscripts s, b, and h, for the shower, bathroom and house, respectively. Thus the modified risk equation due to inhalation of airborne chemicals is:

$$Risk_{inhalation} = [(AC_s \times ET_s) + (AC_b \times ET_b) + (AC_h \times ET_h)] \times \frac{IH \times EF \times ED}{BW \times AT} \times CSF_{inh} \qquad (3.5)$$

Risk due to dermal contact with chemicals in water (modified according to McKone and Bogen (1991)):

$$Risk_{dermal} = \frac{CW \times SA \times F \times PC \times ET \times EF \times ED \times CF}{BW \times AT} \times CSF_{der} \qquad (3.6)$$

where SA is skin surface area available (cm^2), F is the fraction of skin in contact with water, PC is chemical-specific dermal permeability constant (cm/hr), CF is the volumetric conversion factor for water (1 lt/1,000 cm^3) and CSF_{der} is the cancer slope factor associated with dermal contact $(mg/kg\text{-}day)^{-1}$.

Because of data gaps, as well as uncertainty and variability in the available data, risk cannot be known or calculated with absolute certainty (U.S. EPA 2004). While propagating uncertainty and variability associated with the parameters of the risk equation into the resulting risk, terms used in Equations (3.2), (3.5), and (3.6) can be characterized as crisp numbers, random numbers, fuzzy numbers, or random numbers with fuzzy parameters (i.e., mean and standard deviation of a normal variable is represented by fuzzy numbers). How each parameter can best be represented depends on the form of the available information about that parameter. Uncertainty and variability in a human health risk assessment context is explained below.

Uncertainty and Variability: Uncertainty and variability exist in all risk assessment studies (U.S. EPA 2004). As defined in U.S. EPA (2001), variability is the true heterogeneity or diversity that characterizes an exposure variable or response in a population. Further study (e.g., increasing sample size) will not reduce variability, but it can provide greater confidence, that is lower uncertainty, in quantitative characterization of variability. Variability may be associated with parameters related to characteristics of individuals, individual behavior patterns, and individual variations in dose-response sensitivity to risk agents, and characteristics of the environment. For example, different individuals in a population have different body weights and no matter how carefully we measure them the variability in the body weight will not reduce. On the other hand, uncertainty is somewhat different than variability and is defined as a lack of knowledge about specific variables, parameters, models, or other factors (U.S. EPA 2001). Examples include limited data regarding the concentration of a contaminant in an environmental medium, or lack of information on cancer potency factors. The uncertainty associated with the contaminant concentration in an environmental medium can be reduced by performing more measurement with more accurate measurement techniques. But most of the time it is not possible to remove uncertainty completely.

Parameter uncertainty refers to the lack of knowledge about the values of model parameters. Common sources of parameter uncertainty include random measurement errors, systematic measurement errors, use of surrogate data instead of direct measurements, misclassification of exposure status, random sampling errors, and use of an unrepresentative sample (U.S. EPA 2005). On the other hand some of the parameters of the risk equation involve uncertainty due to human and environmental variations. Human variation refers to person-to-person differences in biological susceptibility or in exposure (U.S. EPA 2005) while environmental variations refer to differences in the environmental conditions.

Most of the variables that occur in risk equations due to specific routes (i.e., Equations (3.2), (3.5), and (3.6)) are uncertain and/or variable. Thus, using single-value estimates for these variables may result in significant errors. In recent years, the probabilistic risk assessment (PRA) studies have become popular in analyzing uncertainty and variability associated with the parameters of the risk equations. PRA is the general term for risk assessment that uses probability theory and probability models to represent the likelihood of different risk levels in a population (i.e., variability) or to characterize uncertainty in risk estimates (Ma 2000; Ma 2002; Ma et al. 2002; Maxwell and Kastenberg 1999; Maxwell et al. 1998). Assuming all uncertainty is due to randomness, PRA uses probability distributions for one or more variables in a risk equation in order to quantitatively characterize variability and/or uncertainty in the outcome. The output of a PRA is a probability distribution of risk that reflects the combination of the input probability distributions. Using this approach, if the input distributions represent variability in a probabilistic sense, then the output risk distribution may provide information on the variability in risk in the population of concern. If the input distributions reflect uncertainty in a probabilistic sense, then the output risk distribution may provide information about uncertainty in the risk estimate (U.S. EPA 2001).

With appropriate data and expert judgment, formal approaches to PRA can be applied to provide insight into the overall extent and dominant sources of human variation and uncertainty (U.S. EPA 2005). However, in all scientific and engineering fields, data, time, and economical restrictions limit the application of established methods. Moreover, if uncertainty is not due to randomness or if the available information is in the form of expert and subjective interpretations of system parameters, or imprecisely defined boundaries of parameters than probabilistic analysis may not be effective in representing the true nature of uncertainty. Advances in uncertainty analysis, especially in the direction of incorporating incomplete information and expert knowledge into the health risk assessment processes are emerging (Bárdossy et al. 1991; Guyonnet et al. 2003; Guyonnet et al. 1999; Kentel and Aral 2004; Kentel and Aral 2005; McKone and Deshpande 2005). For example, possibility theory and fuzzy set theory provide mathematical tools for integrating incomplete information and expert judgment into the analysis. Development of hybrid models which allow simultaneous utilization of random and fuzzy variables in human health risk assessment is an effort to improve quantitative uncertainty analysis and to bring flexibility to it. The two hybrid health risk assessment models we proposed are provided in Sections 3.3 and 3.4.

3.3 PROBABILISTIC-FUZZY HEALTH RISK MODELING

An improved quantitative treatment of uncertainty and variability in human health risk assessment studies can produce richer and more robust information which might be used in decision-making. Thus, propagation of variable and uncertain information into the resulting risk is crucial in health risk analysis. The main objective of this first study is to propose a systematic health risk assessment approach which allows modeling variables of the risk equations due to various routes (Equations (3.2), (3.5), and (3.6)) as crisp, random, or fuzzy variables depending on the form of the available information.

Our approach for calculating risk involves a multi-pathway exposure, an individual health risk model, and a framework for evaluating the membership function of health risk as a function of variability and uncertainty.

3.3.1 MULTIPLE EXPOSURE PATHWAYS

We characterized most of the parameters occurring in daily intake formulas for ingestion, inhalation, and dermal contact routes (Equations (3.2), (3.5), and (3.6)) as variable parameters and we used probability density functions (pdfs) instead of single value estimates to model them. However, tap water concentration and cancer slope or potency factors associated with ingestion, inhalation, and dermal contact may not exhibit only variability as defined previously. Depending on the form of the available information, it may be better to interpret them as uncertain parameters. Then one has to use a method other than the probabilistic approach when incorporating them into the mechanistic models. Here, we model these two parameters as fuzzy variables. Why we chose to model tap water concentration and cancer potency factors as fuzzy variables and the rest of the parameters as crisp and random variables is explained in the following section.

3.3.2 Fuzzy and Random Variables

All the parameters in the risk equations associated with ingestion, inhalation, and dermal contact routes as given in Equations (3.2), (3.5), and (3.6), respectively, may contain uncertainty and variability. Depending on the available data, the risk analyst may choose to model these parameters as crisp, random, or fuzzy variables. Which parameters of the risk equations can better be represented by what kind of variable (i.e., crisp, random, or fuzzy) is a case-specific decision which is directly related to the amount and type of available information about these parameters, expert knowledge, and judgment. Our goal in this study is to demonstrate a new hybrid approach for a hypothetical case and we do not have problem-specific data. Thus, we decided to represent the tap water contaminant concentration, and the cancer potency factors as fuzzy variables due to the knowledge we gained from literature. The following paragraphs explain why we think these two parameters might better be represented as uncertain variables.

Exposure analysis is used to estimate the daily intake, while epidemiological studies are used to determine the dose-response relationship. To estimate the responses of populations exposed to a given dose of contaminant, risk analysts conduct mathematical extrapolations. Determining how much of a hazard the absorbed substance poses is a complicated problem. Clear-cut relations are rare since epidemiological studies are not very sensitive in detecting health effects from relatively low levels of exposure (Hattis and Kennedy 1986). As explained in detail in Hattis and Kennedy (1986), the problem is that the rates of specific illnesses from a given hazard often must be several times above average before one can conclude that they aren't simply random fluctuations. Moreover, in great majority of the cases where the epidemiological evidence is incomplete or ambiguous, using animal studies to make projections is more reasonable. However, such studies suffer from their own serious uncertainties. In order to ensure that the toxic effects will appear at statistically significant levels, the animals are generally exposed to high concentrations of chemical. A dose-response curve is fitted to the resulting data obtained from these experimental animals. The dose-response curve is used to assess the probability of humans developing cancer to this chemical, but most of the time, at much lower levels. Using such high doses on experimental animals can complicate the interpretation of results in various ways. Thus, the different mathematical models produce widely varying results. Other serious difficulty is in the process of interpreting animal studies and extrapolating them to humans. Because animals and humans metabolize substances differently, the level of the test chemical that reaches various parts of the animal and the human body can vary widely. Hence, humans and animals may suffer from different health effects. As the National Research Council, which often advises the federal government on scientific issues, points out in a congressionally commissioned report that correcting for these difficulties are not easy because researchers often lack enough information about human and animal systems (National Research Council 1983). Thus assessing toxicity values - which defines the quantitative relationship between dose and response - of risk agents on humans is full of uncertainties, and the information available is not enough to provide clear-cut values. Based on this observation, cancer potency factors may be treated more appropriately as non-random but uncertain variables and using single value estimates or probabilistic estimates for this parameter may not be the proper interpretation. In an application, the analyst must gather all the available information about cancer potency factors and include this information – even if it is not complete – into risk assessment studies. Thus, as an initial approximation, modeling these parameters as fuzzy variables may reduce the associated interpretation errors and may lead to more informed decisions.

Contaminant concentration in the tap water is considered as the toxicant source in this study. Calculation of tap water concentration requires detailed studies as well, which has its own uncertainty structure. Contaminant concentration within the medium which supplies tap water must be determined through detailed environmental modeling studies or measurements. Depending on the source of tap water (i.e., groundwater, surface water, or combination of both) groundwater flow modeling, surface water flow modeling, or combined modeling of groundwater flow and surface water flow together with contaminant transport modeling may be required. It is not our intention here to provide groundwater or surface water modeling studies to estimate tap water concentration. However, we are aware of the fact that contaminant concentration in the tap water is another uncertain parameter and must be treated as such in its own uncertainty structure. Thus, in this study, the contaminant concentration in tap water is also treated as a fuzzy variable. Measured contaminant concentrations in tap water or expert knowledge may help the analyst to assign a membership function for this fuzzy variable.

The rest of the parameters of the risk equations for ingestion, inhalation, and dermal contact routes (Equations (3.2), (3.5), and (3.6)) are modeled as random or crisp variables. We assumed that the average lifetime of an individual is 70 years and the individual is exposed to the toxic substance everyday for a period of 30 years. Consequently, averaging time, AT is used as 365x70=25550 days. Thus, these three variables (i.e., exposure frequency, EF, exposure duration, ED, and averaging time AT) are modeled as constant values. This will allow us to present how the new hybrid model we are proposing treats crisp, random, and fuzzy variables simultaneously. The effect of human and environmental variations in the rest of the parameters is incorporated into the risk equations by characterizing them as random variables. Parameters such as ingestion rate, inhalation rate, body weight, and water use rate, exposure time in shower, bathroom, and house are modeled as random variables due to "human-variation." By "human-variation" we mean both personal characteristics of the individual like body weight and personal preferences like water use rate. Other parameters like air exchange rate in shower, bathroom, and house, or transfer efficiency from water to air are modeled as random variables due to variability in the environmental characteristics such as temperature.

To summarize, in this study we considered only the cancer potency factors (indicators of toxicological response of the individual to a specific risk agent) associated with different exposure pathways and tap water contaminant concentration as non-random but uncertain parameters. We represented these uncertain parameters as fuzzy variables. Other than exposure frequency, exposure duration, and averaging time, which are treated as crisp values, the rest of the parameters of the risk equations are treated as random variables and the associated probability distributions of these variables are used. In Table 3.1, a list of all of the parameters used in risk calculations are shown, along with the type of variability/uncertainty associated with them and how they are treated (random or fuzzy) in this study. The corresponding distributions used to represent the variability/uncertainty (pdfs for random variables and membership functions for fuzzy variables) are given in the last column of Table 3.1. The membership functions of the fuzzy variables are given in Figure 3.1.

For ease of calculation, triangular membership functions are used for membership functions of the fuzzy variables. The PFRA method is not restricted to this membership function choice. Also we should emphasize that the proposed approach is not limited to the specific selection of the parameters of the risk equations as crisp, random, and fuzzy variables as given in Table 3.1. As will become apparent, any combination of crisp, random, and fuzzy variables can be analyzed using the hybrid approach described in the following section.

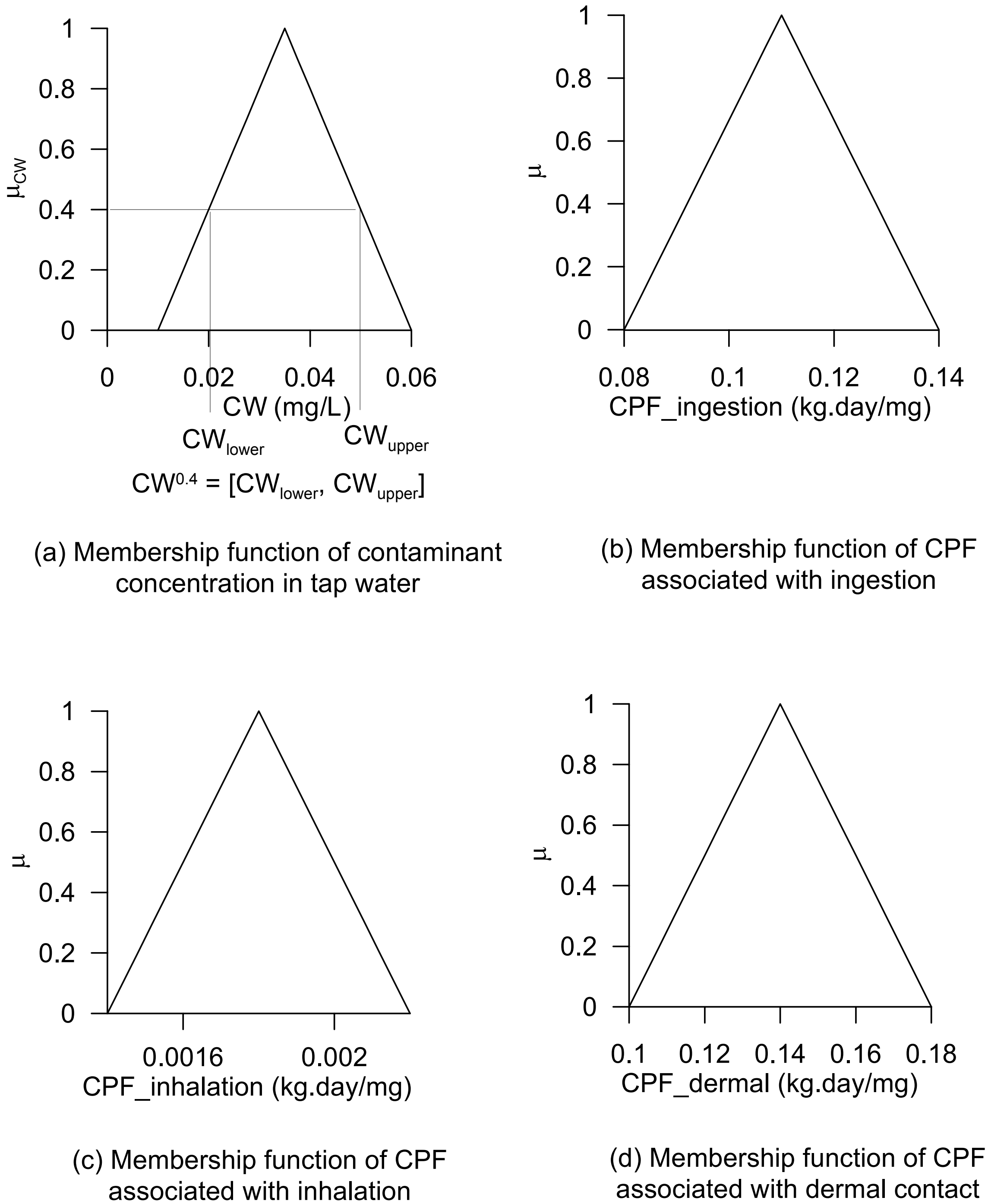
Table 3.1 Parameters of Risk Calculations

Parameter	Symbol	Units	Type of Uncertainty	Variable Type	Value/Distribution*
Averaging time	AT	days	none	constant	25550 (70 years)
Exposure duration	ED	years	none	constant	30
Exposure frequency	EF	d/year	none	constant	350
Ingestion rate per body weight	IR/BW	L/d/(kg body wt)	variable	random	lognormal ~ (3.3e-2, 1.3e-2)
Water use rate	W_i	L/hr			
Shower	W_s	L/hr	variable	random	lognormal ~ (480,160)
Bathroom	W_b	L/hr	variable	random	lognormal ~ (40,15)
House	W_h	L/hr	variable	random	lognormal ~ (40,15)
Transfer efficiency from water to air					
Shower	TE_i	-	variable	random	
Bathroom	TE_s	-	variable	random	triangular ~ (0.1, 0.5, 0.9)
House	TE_b	-	variable	random	triangular ~ (0.1, 0.43, 0.9)
	TE_h	-	variable	random	triangular ~ (0.1, 0.43, 0.9)
Air exchange rate	VR_i	m^3/hr	variable	random	
Shower	VR_s	m^3/hr	variable	random	uniform ~ (4-20)
Bathroom	VR_b	m^3/hr	variable	random	uniform ~ (10-100)
House	VR_h	m^3/hr	variable	random	uniform ~ (300-1200)
Exposure time	ET_i	hr/d	variable	random	
Shower	ET_s	hr/d	variable	random	lognormal ~ (0.13, 0.09)
Bathroom	ET_b	hr/d	variable	random	lognormal ~ (0.32, 0.21)
House	ET_h	hr/d	variable	random	uniform ~ (8-20)
Inhalation rate per unit body weight	IH/BW	m^3/d/(kg body wt)	variable	random	lognormal ~ (0.39, 0.5)
Skin surface area available per body weight	SA/BW	m^2/kg	variable	random	lognormal ~ (0.39, 0.5)
Chemical-specific dermal permeability constant	PC	m/hr	variable	random	uniform ~ (0.004, 0.01)
Fraction of skin in contact with water	F	-	variable	random	unirorm ~ (0.4-0.9)
Cancer potency factor	CPF	(kg.day)/mg	uncertain	fuzzy	
Ingestion	$CPF_{ingestion}$	(kg.day)/mg	uncertain	fuzzy	triangular: (0.08, 0.11, 0.14)
Inhalation	$CPF_{inhalation}$	(kg.day)/mg	uncertain	fuzzy	triangular: (0.0014, 0.0018, 0.0022)
Dermal	CPF_{dermal}	(kg.day)/mg	uncertain	fuzzy	triangular: (0.1, 0.14, 0.18)
Contaminant (PCE) concentration in tap water	CW	mg/L	uncertain	fuzzy	triangular: (0.012, 0.015, 0.018)

All parameters other than fuzzy variables are from McKone and Bogen (1991)

*For lognormal and normal distributions, the values given in parentheses represent the arithmetic mean and standard deviation, respectively. For uniform distributions the numbers in parentheses represent the minimum and maximum values in the distribution. For triangular distributions (probability density functions), the numbers in parentheses represent the minimum, likeliest and maximum values, respectively.

For triangular membership functions the numbers in parentheses represent the zero membership, one membership and zero membership values of the triangular distribution. Refer to text for detailed explanation.



(a) Membership function of contaminant concentration in tap water

(b) Membership function of CPF associated with ingestion

(c) Membership function of CPF associated with inhalation

(d) Membership function of CPF associated with dermal contact

Figure 3.1 Membership functions of contaminant concentration in tap water and cancer potency factors (CPF) for ingestion, inhalation, and dermal contact

3.3.3 PROBABILISTIC-FUZZY RISK ASSESSMENT ANALYSIS

The initial step of PFRA is data collection and evaluation. Once all available information is gathered and evaluated, appropriate probability density functions and membership functions can be specified for variable and uncertain parameters, respectively. However, since our goal is to present the hybrid approach, this initial step is not conducted in this study. All the parameters of the risk equations (Equations (3.2), (3.5), and (3.6)) are characterized as crisp, random, or fuzzy (see Table 3.1) variables mainly by using the information provided in McKone and Bogen (1991).

Crisp variables do not contain any uncertainty, thus they are represented by a single value. Uncertainty associated with fuzzy variables is represented by membership functions while variability associated with random variables are represented by probability density functions. In this study, we use Monte Carlo Analysis (MCA) to propagate information supplied by probability density functions of the random variables. On the other hand, fuzzy arithmetic and interval analysis are used to integrate uncertainty associated with the fuzzy variables, namely contaminant concentration in tap water, and cancer potency factors for ingestion, inhalation, and dermal contact routes. Probability-fuzzy risk assessment method we are proposing is explained below.

The risk model we propose is as follows (Kentel and Aral 2004):

$$Risk = \underset{\text{fuzzy}}{\underset{\downarrow}{CW}} \times \left[\underset{\text{random}}{\underset{\downarrow}{f(V_{1_{ing}},...,V_{n_{ing}})}} \times \underset{\text{fuzzy}}{\underset{\downarrow}{CPF_{ing}}} + \underset{\text{random}}{\underset{\downarrow}{g(V_{1_{inh}},...,V_{m_{inh}})}} \times \underset{\text{fuzzy}}{\underset{\downarrow}{CPF_{inh}}} + \underset{\text{random}}{\underset{\downarrow}{q(V_{1_{der}},...,V_{k_{der}})}} \times \underset{\text{fuzzy}}{\underset{\downarrow}{CPF_{der}}} \right] \tag{3.7}$$

where $f(V_{1_{ing}},...,V_{n_{ing}})$ represents the function involving random variables occurring in the risk formula associated with ingestion route, Equation (3.2), n is the number of random variables occurring in Equation (3.2), $g(V_{1_{inh}},...,V_{m_{inh}})$ represents the function involving random variables occurring in the risk formula associated with inhalation route, Equation (3.5), m is the number of random variables occurring in Equation (3.5), and $q(V_{1_{der}},...,V_{k_{der}})$ represents the function which involves random variables occurring in the risk formula associated with dermal contact route, Equation (3.6), k is the number of random variables occurring in Equation (3.6). In this study, as indicated in Table 3.1, some of the variables occurring in functions f, g, and q are defined as constants. However, the solution methodology will not alter if they are chosen as random variables. Here, $\left(V_{i_{ing}}, V_{j_{inh}}, V_{k_{der}}\right)$ are vectors that contain the sampled values of these parameters from the probability distributions selected for these parameters, respectively (Table 3.1).

As can be seen in Equation (3.7), the risk equation is composed of various fuzzy and random variables. In this study, we carried out the PFRA analysis for two different scenarios. In the first scenario, it is assumed that only the contaminant concentration in the tap water is treated as a fuzzy variable and cancer potency factors associated with ingestion, inhalation, and dermal contact are chosen as constant values. This would represent a simpler case, where only the environmental modeling outcome is assumed to be uncertain. In the second scenario, cancer potency factors associated with all three routes together with the contaminant concentration in the tap water are treated as uncertain variables, and thus represented with membership functions. For both cases, most of the terms in functions f, g, and q are modeled as random variables while some are defined as constants.

Scenario 1: In this scenario, as a simpler case of Equation (3.7), only the contaminant concentration in the tap water is treated as a fuzzy variable which has a triangular membership function, Figure 3.1a. The total risk equation for this case can be given as:

$$Risk = \underbrace{CW}_{\textbf{fuzzy}} \times \left[f(\underbrace{V_{1_{ing}},...,V_{n_{ing}}}_{\textbf{random}}) \times \underbrace{CPF_{ing}}_{\textbf{constant}} + g(\underbrace{V_{1_{inh}},...,V_{m_{inh}}}_{\textbf{random}}) \times \underbrace{CPF_{inh}}_{\textbf{constant}} + q(\underbrace{V_{1_{der}},...,V_{k_{der}}}_{\textbf{random}}) \times \underbrace{CPF_{der}}_{\textbf{constant}} \right] \quad (3.8)$$

Monte Carlo Analysis is used together with interval analysis to generate cumulative distribution functions of risk for upper and lower limits of each alpha-cut level. PFRA study is conducted with 5,000 and 10,000 Monte Carlo simulations. Number of Monte Carlo simulations in the order of thousands are chosen in recent risk assessment studies (Barbeau 2000; Clewell et al. 1999; Daniels et al. 2000; Maxwell and Kastenberg 1999; Smith 1994). Thus we initially tried 5,000 and 10,000 simulations. Since the results obtained are very similar, it is decided to proceed with 5,000 Monte Carlo simulations for the rest of the study. The procedure to conduct PFRA is given below.

The term inside the square parenthesis in Equation (3.8) (this term will be referred as $\boldsymbol{P}$ from here on, as given in Equation (3.9)) involves only random variables and constants. 5,000 Monte Carlo simulations are conducted to generate sets of random variables by sampling from the probability distribution functions of the associated variables provided in Table 3.1. Then, the cumulative distribution function of $\boldsymbol{P}$, $\left[P\right]_{cdf}$, is generated.

$$\boldsymbol{P} = f\,(V_{1_{ing}},...,V_{n_{ing}}) \times CPF_{ing} + g(V_{1_{inh}},...,V_{m_{inh}}) \times CPF_{inh} + q(V_{1_{der}},...,V_{k_{der}}) \times CPF_{der} \qquad (3.9)$$

The next step is to include the uncertainty due to fuzziness associated with the contaminant concentration in the tap water, CW.

The total risk equation, Equation (3.8), being a monotonic function, will allow us to use interval analysis to carry out fuzzy calculations. Interval analysis involves discretizing the membership domain of the fuzzy variable into a specified number of alpha-cut, α_c, levels. First, an alpha-cut interval, in this case 0.1, is selected (i.e.: 0.0:0.1:1.0). The lower and upper bounds of the intervals for each α_c for fuzzy variable CW (this concept is depicted in Figure 3.1a) can be given as:

$$CW^{\alpha_c} = [CW^{\alpha_c}_{lower},\ CW^{\alpha_c}_{upper}] \qquad \forall \quad \alpha_c = 0:0.1:1.0 \qquad (3.10)$$

Fuzzy arithmetic is carried out for each α_c level in accordance with interval analysis, and this procedure is summarized below. Detailed explanations of interval analysis can be found in Moore (1979).

The lower and upper values of CW (i.e., $CW_{lower}^{\alpha_c}$ *and* $CW_{upper}^{\alpha_c}$) are used together with the previously calculated cumulative distribution function (cdf), $[P]_{cdf}$, to calculate two cdfs of risk corresponding to each α_c level (i.e., a total of 21 cdfs for risk are calculated).

$$\begin{aligned}\left[Risk_{lower}^{\alpha_c}\right]_{cdf} &= [P]_{cdf} \times CW_{lower}^{\alpha_c} \\ \left[Risk_{upper}^{\alpha_c}\right]_{cdf} &= [P]_{cdf} \times CW_{upper}^{\alpha_c}\end{aligned} \qquad \forall \quad \alpha_c = 0:0.1:1.0 \qquad (3.11)$$

Multiplying $[P]_{cdf}$ with a constant, either $CW_{lower}^{\alpha_c}$ *or* $CW_{upper}^{\alpha_c}$, results in shifting the cumulative distribution function $CW_{lower}^{\alpha_c}$ *or* $CW_{upper}^{\alpha_c}$ units, and these new cumulative distribution functions are referred to as $\left[Risk_{lower}^{\alpha_c}\right]_{cdf}$ and $\left[Risk_{upper}^{\alpha_c}\right]_{cdf}$ in Equation (3.11) respectively. Thus, the multiplication operation used in Equation (3.11) represents multiplication of a cdf with a constant. The collection of all of these risk cdfs (i.e., 21 cdfs corresponding to lower and upper limits of each α_c) will be referred to as $[Risk]_{cdf}$.

The cdfs corresponding to lower and upper limits of all alpha-cut levels are generated. However, to provide a clear and simple presentation, cdfs for the lower and upper limits of only 0.0, 0.5 and 1.0 alpha-cut levels are given in Figure 3.2.

Using these cumulative distribution functions of risk, $[Risk]_{cdf}$, the membership function of the total risk to a specific fractile of individuals at risk can now be generated. This concept is explained in Figure 3.2. A horizontal line cutting through $[Risk]_{cdf}$ curves are drawn and risk values corresponding to upper and lower limits of each α_c are read.

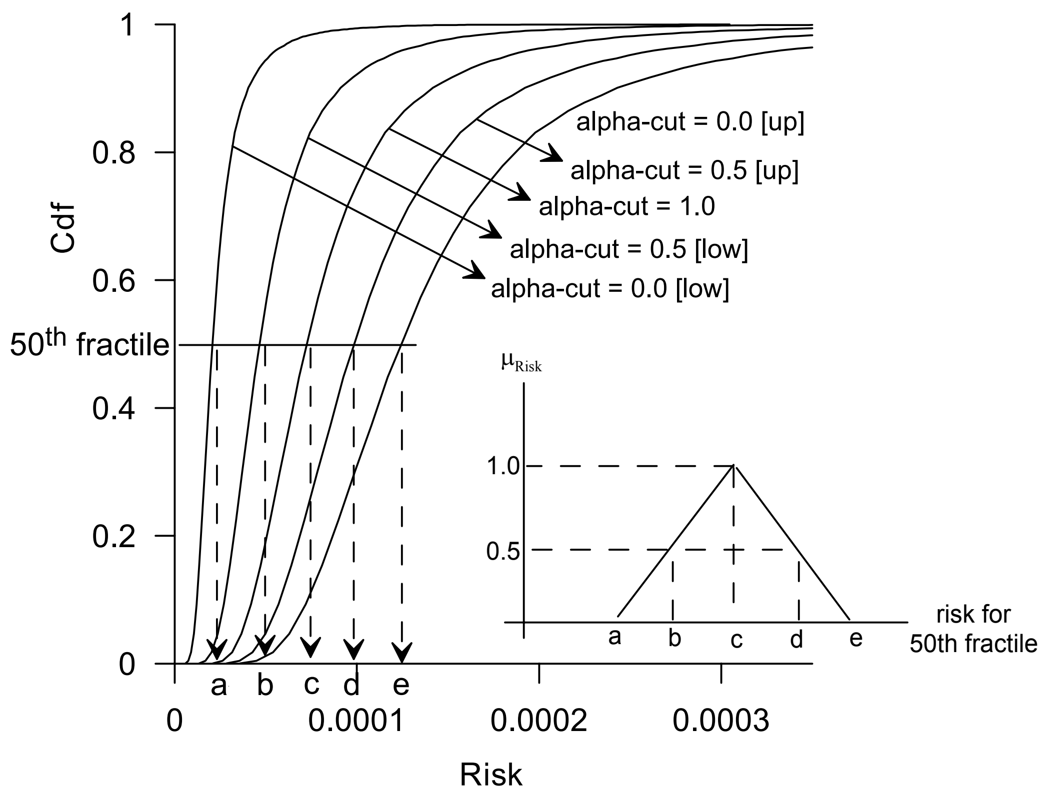


Figure 3.2 The cdfs of risk for 0, 0.5, and 1.0 alpha-cut levels for Scenario 1

For example, a and e are risk values having membership values of zero for 50th fractile. Similarly, all of the other risk values and associated membership values are read from the graph and used to plot the fuzzy risk membership function corresponding to 50th fractile. Figure 3.3 depicts the membership function of total risk to individuals at 30th, 60th, and 90th fractiles of risk.

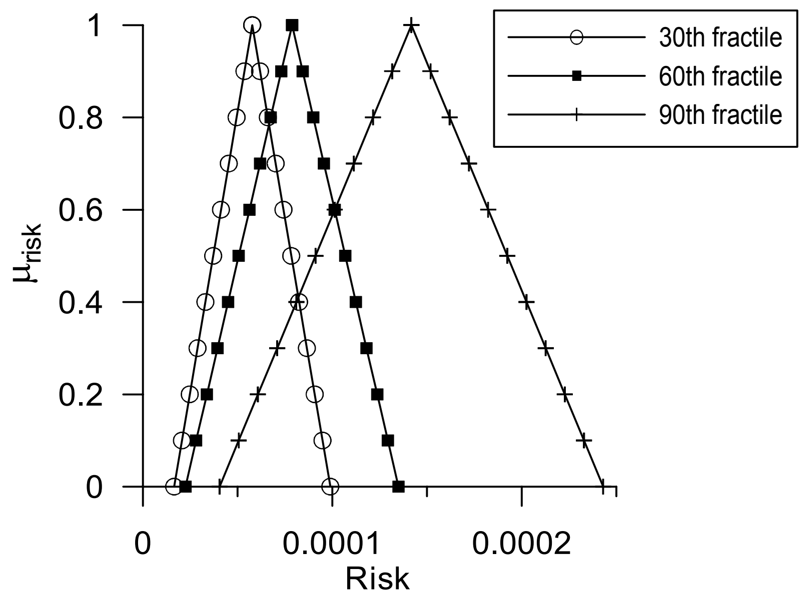


Figure 3.3 The membership functions of risk to individuals at 30th, 60th, and 90th fractiles of risk for Scenario 1

In PFRA, all variable parameters are modeled as random variables (in this specific case study, some are taken as constants but the proposed method allows all of these parameters to be modeled as random variables). Here the assumption is that none of the variables are correlated with one another and an independent distribution over all individuals at risk is determined for each of these variable parameters; which we think is an acceptable assumption. A set of random variables is sampled from these distributions to construct an individual. This individual is not associated with any particular individual but is representative of a particular risk fractile in the population. Detailed explanation of the particular risk fractile concept can be found in Maxwell et al. (1998).

Scenario 2: In this case all the variable parameters are characterized as random variables as before and all the uncertain parameters (i.e., the contaminant concentration and cancer potency factors) are characterized as fuzzy variables. The risk equation associated with this case is given in Equation (3.7). A similar procedure to the one used in Scenario 1 is followed for this case. Lower and upper limits of each fuzzy variable for each α_c level are determined. Again 5,000 Monte Carlo simulations are conducted to generate random variables by sampling from the probability distribution functions of associated variables. Since the functions associated with ingestion, $f(V_{1_{ing}},...,V_{n_{ing}})$, inhalation, $g(V_{1_{inh}},...,V_{m_{inh}})$, and dermal contact, $q(V_{1_{der}},...,V_{k_{der}})$, are functions of random variables and constants, 5,000 different values for these three functions are generated:

$$
\begin{aligned}
\left[f\right] &= f\left(V_{i_{ing}}\right) \qquad & i &= 1,2,3,...,n \\
\left[g\right] &= g\left(V_{j_{inh}}\right) \qquad & j &= 1,2,3,...,m \\
\left[q\right] &= q\left(V_{l_{der}}\right) \qquad & l &= 1,2,3,...,k
\end{aligned}
\tag{3.12}
$$

As stated before, the total risk equation, Equation (3.7), is a monotonic function. This will allow us to use interval analysis to carry out fuzzy calculations. The lower and upper values of contaminant concentration, $CW_{lower}^{\alpha_c}$ and $CW_{upper}^{\alpha_c}$ **,** and cancer potency factors associated with ingestion, $\left(CPF_{ing}\right)_{lower}^{\alpha_c}$ and $\left(CPF_{ing}\right)_{upper}^{\alpha_c}$, inhalation, $\left(CPF_{inh}\right)_{lower}^{\alpha_c}$ and $\left(CPF_{inh}\right)_{upper}^{\alpha_c}$, and dermal contact, $\left(CPF_{der}\right)_{lower}^{\alpha_c}$ and $\left(CPF_{der}\right)_{upper}^{\alpha_c}$, corresponding to each α_c are used together with the previously calculated $[f]$, $[g]$ and $[q]$ results to compute two risk values for each of the 5,000 Monte Carlo simulation results (i.e., one for the lower and one for the upper limit of each alpha-cut of the membership functions of the contaminant concentration and cancer potency factors associated with ingestion, inhalation, and dermal contact):

$$\begin{aligned}
\left[Risk_{lower}^{\alpha_c}\right] &= CW_{lower}^{\alpha_c} \times \left([f]\times\left(CPF_{ing}\right)_{lower}^{\alpha_c} + [g]\times\left(CPF_{inh}\right)_{lower}^{\alpha_c} + [q]\times\left(CPF_{der}\right)_{lower}^{\alpha_c}\right) \\
\left[Risk_{upper}^{\alpha_c}\right] &= CW_{upper}^{\alpha_c} \times \left([f]\times\left(CPF_{ing}\right)_{upper}^{\alpha_c} + [g]\times\left(CPF_{inh}\right)_{upper}^{\alpha_c} + [q]\times\left(CPF_{der}\right)_{upper}^{\alpha_c}\right) \\
&\forall\, \alpha_c = 0:0.1:1.0
\end{aligned} \tag{3.13}$$

Then for upper and lower limits of each α_c level, $\left[Risk_{lower}^{\alpha_c}\right]$ and $\left[Risk_{upper}^{\alpha_c}\right]$ are used to generate cumulative distribution functions of risk, $\left[Risk_{lower}^{\alpha_c}\right]_{cdf}$ and $\left[Risk_{upper}^{\alpha_c}\right]_{cdf}$, respectively. Thus, a total of 21 cdfs for risk are calculated. The cdfs corresponding to lower and upper limits of 0.0, 0.5 and 1.0 α_c levels are given in Figure 3.4.

The cumulative distribution functions corresponding to lower and upper limits of each alpha-cut level are used to generate membership functions of total risk to individuals at 30th, 60th, and 90th fractiles of risk. A similar procedure to the one described for Scenario 1 is used for these calculations. The membership functions of total risk to individuals at 30th, 60th, and 90th fractiles of risk are given in Figure 3.5 together with the results of Scenario 1 for comparison purposes.

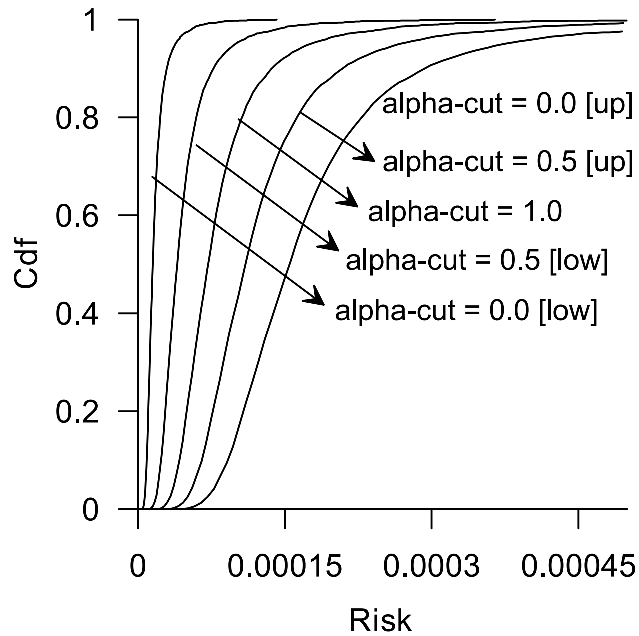


Figure 3.4 The cdfs of risk for 0.0, 0.5, and 1.0 alpha-cut levels for Scenario 2

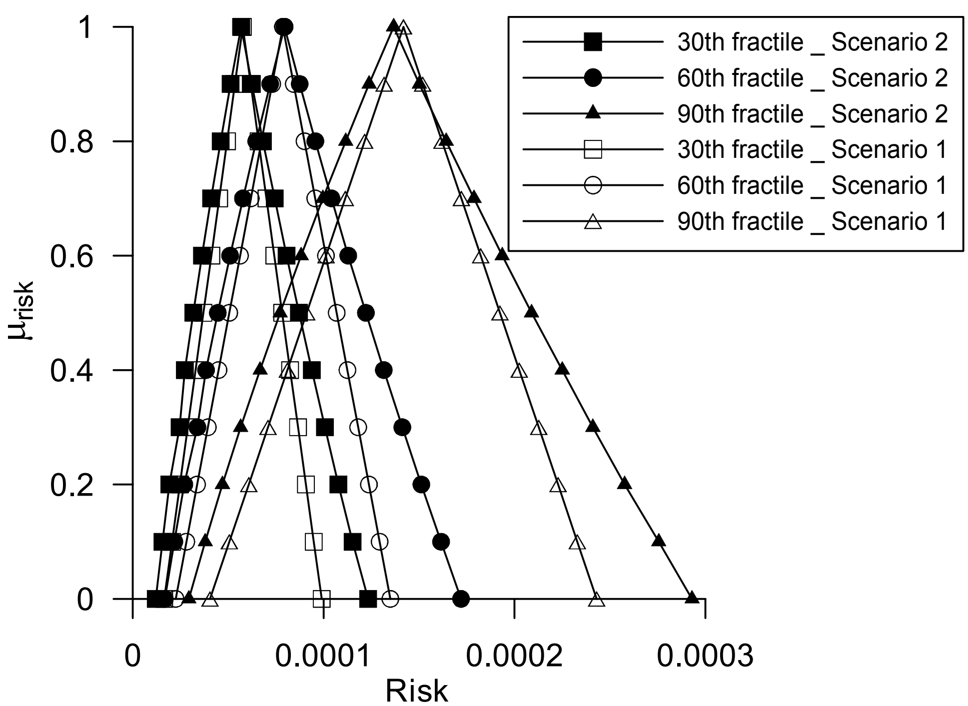


Figure 3.5 Comparison of membership functions of risk to individuals at certain fractiles for Scenario 1 and Scenario 2

3.3.4 Results For Probabilistic-Fuzzy Health Risk Modeling

In this study we used the probability density functions provided by McKone and Bogen (1991) for the variable parameters (see Table 3.1). For fuzzy variables, we used triangular membership functions which are given in Figure 3.1. The values for cancer potency factors associated with ingestion, inhalation, and dermal contact are provided in McKone and Bogen (1991) as constants. We used these values as the peak values (i.e., value which has a membership value of one) then we assigned triangular distributions for these fuzzy variables according to our interpretation of uncertainty in these parameters. Since our goal here is to present the framework of PFRA analysis, the particular selection of the support of an uncertain variable is not important and can be changed as one interprets the data. Again, since we are working with a hypothetical case, we assigned an arbitrary triangular membership function for the contaminant concentration in the tap water. Any kind of membership function is possible for fuzzy variables but for the sake of simplicity we preferred to work with triangular distributions. While conducting toxicant and site specific PFRA studies the analyst should collect all the available information about these uncertain parameters and assign the most appropriate membership functions to them as they interpret the uncertainty.

The proposed risk model is provided in Equation (3.7). First, we conducted simulations for a simple case, Scenario 1, in which only the contaminant concentration in the tap water is treated as a fuzzy variable, Equation (3.8). In this case, cancer potency factors associated with ingestion, inhalation, and dermal contact are modeled as constants. Variable parameters (see Table 3.1) are treated as random variables. Probabilistic-fuzzy risk assessment study is conducted as explained in Section 3.3.3 to generate cumulative density functions for lower and upper limits of each α_c. Then, these cumulative density functions are used to form membership functions of risk to individuals at certain fractiles of risk.

Interpretation of Results with Fuzzy Set Theory: Scenario 1 uses contaminant concentration in tap water as a fuzzy variable so the resultant risk is also a fuzzy variable. The membership functions of risk to individuals at 30th, 60th, and 90th fractiles of risk are given in Figure 3.3. As can be seen from Figure 3.3, the membership functions of risk to individuals at different fractiles of risk have triangular distributions. Triangular membership functions of risk can be interpreted as risk to individuals at a certain fractile of risk being around the peak value (i.e., value corresponding to a membership function of 1.0). A membership value of one reflects the most likely value for the variable. If the fuzzy set corresponding to the risk to the 90th most highly exposed person is called $\widetilde{R_{90}}$, then $\widetilde{R_{90}}$ can be defined as "around $1.5x10^{-4}$" (see Figure 3.3). The support of the membership function provides the possible range of the risk for individuals at the corresponding fractile. As can be seen from Figure 3.3, the possible range of risk to individuals at 90th fractile is $4.05x10^{-5}$ to $2.43x10^{-4}$ and the most likely outcome is $1.5x10^{-4}$. Risk values outside $4.05x10^{-5}$ - $2.43x10^{-4}$ range have zero membership values (i.e., no membership) and $1.5x10^{-4}$ has a membership value of one (i.e., full membership). Since the membership values indicate the degree to which a risk value is a member of the fuzzy set $\widetilde{R_{90}}$, values outside $4.05x10^{-5}$ - $2.43x10^{-4}$ range are not members of $\widetilde{R_{90}}$ and

1.5×10^{-4} is a full member of $\widetilde{R_{90}}$. As can be seen from Figure 3.3, uncertainty in added risk to individuals at 90th fractile is larger than that of 30th fractile (i.e., support of the membership function being wide implies that uncertainty is larger). This is due to the shapes of the cumulative distribution functions used to generate membership functions. As can be seen from Figure 3.2, cumulative distribution functions have higher slopes up to around 0.8 and then they level off. Thus, the rate of change of risk for lower fractiles is smaller than those of higher fractiles.

The triangular membership function obtained for risk for a certain fractile (Figure 3.3) is symmetric for Scenario 1. This is due to the fact that only the contaminant concentration is modeled as a fuzzy variable whose membership function is taken as a symmetric triangular distribution. The rest of the risk equation (i.e., $\boldsymbol{P}$ term given in Equation (3.9)) results in a single number for each of the 5,000 Monte Carlo simulations and then these results are used to form $[P]_{cdf}$. For each fractile one risk value can be obtained from this cdf. Thus, the fuzzy arithmetic carried out to generate the membership function of risk for a certain fractile involves multiplying the lower and upper bounds of a fuzzy number at a certain α_c level with a constant (i.e., the risk value for that certain fractile). This yields a risk membership function similar to the one used for contaminant concentration in tap water.

In the second scenario, PFRA is applied to generate membership function of risk to individuals at certain fractiles of risk using Equation (3.7). In this scenario, the contaminant concentration in tap water and cancer potency factors associated with ingestion, inhalation, and dermal contact are treated as fuzzy variables and the rest of the parameters other than exposure frequency, exposure duration, and averaging time are treated as random variables (see Table 3.1). The corresponding cumulative distribution curves for Scenario 2 are given in Figure 3.4 and comparison of membership functions of risk for certain fractiles for Scenario 1 and Scenario 2 is given in Figure 3.5. As can be seen from Figure 3.5, when cancer potency factors associated with ingestion, inhalation, and dermal contact are also treated as fuzzy variables, uncertainty in risk increases (i.e., compare support of triangular distributions corresponding to the same fractile for two different scenarios). The support of risk for individuals at 90th fractile of risk for Scenario 1 is 4.05×10^{-5} to 2.43×10^{-4} while for Scenario 2 it is 2.94×10^{-5} to 2.93×10^{-4}. For the latter case, uncertainty in the resulting risk is higher due to the fact that uncertainties in more parameters (i.e., cancer potency factors) are included into the study.

Another observation from Figure 3.5 is that the peak values corresponding to the same fractile of risk for both scenarios are almost the same. Ideally, they should be the same; however 5,000 Monte Carlo simulations are used in the calculation of these peak values. For two different scenarios two different sets of 5,000 Monte Carlo simulations are generated and they produce slightly different values for the most likely added risk values. The shapes of the membership functions of risk for a certain fractile for Scenario 2 are not similar to the shapes of input fuzzy variables (i.e., contaminant concentration in tap water and cancer potency factors associated with ingestion, inhalation, and dermal contact). This is due to the fact that, in Scenario 2, several of the variables are represented as fuzzy variables and the risk equation involves multiplication and addition of these fuzzy variables. Thus, the shape of the membership function for risk at a certain fractile is not symmetric but skewed for this case.

The selection of membership functions for the fuzzy variables and the probability distribution functions of the random variables will impact the shape of the membership function of risk obtained for a certain fractile. The shape of the membership function may also represent valuable information to the analyst.

There are several methods to defuzzify fuzzy variables (i.e., convert a fuzzy variable into a point estimate). One of the most popular methods is the center of gravity method (Ross 1995; Wang 1997). For example, if the shape of the membership function of risk is skewed to the left or right of the peak and if the fuzzy risk is deffuzified using the center of gravity method, a crisp risk value which is higher or lower depending on the orientation of the skewness of the triangle will be obtained. Thus, the deffuzified value of risk will be either smaller or larger than the most likely risk value (i.e., value corresponding to a membership value of one) of the fuzzy risk. This would indicate that a lower (or higher) risk value is possible when compared to the risk corresponding to the membership value of one, which also corresponds to the risk one would obtain from PRA approach for that fractile.

Membership functions can also be used to interpret the results from a possibility point of view. Interpretation of the results using possibility theory concepts is provided in the following section.

Interpretation of Results with Possibility Theory: A brief explanation of the possibility theory in relation to the fuzzy set theory is provided in Appendix A. The reader may refer to Zadeh (1978), Wang (1982), Dubois and Prade (1988), Klir and Yuan (1995), and Dubois and Prade (2003) for detailed explanations and applications of the possibility theory.

For individuals at 90^{th} fractile risk for Scenario 2, let F be the fuzzy subset defined as "around $1.4x10^{-4}$" (see Figure 3.5) and variable X be the risk for individuals at 90^{th} fractile risk. Then the proposition "risk for individuals at 90^{th} fractile is around $1.4x10^{-4}$" translates into:

$$R(risk\ for\ individuals\ at\ 90^{th}\ fractile\ of\ risk) = "around\ 1.4x10^{-4}" \tag{3.14}$$

which associates a possibility distribution, Π_{Risk}, with risk which is postulated to be equal to $R(Risk)$, i.e., $\Pi_{Risk} = R(Risk)$. Thus for our example, the possibility distribution associated with risk is denoted by π_{Risk} and is equal to the membership function of F, as given in Figure 3.5.

From Figure 3.5, we can conclude that for Scenario 2, for individuals at 90^{th} fractile risk, the possibility that risk is between $7.7x10^{-5}$ or $2.09x10^{-4}$, given that "risk to individuals at 90^{th} fractile risk is around $1.4x10^{-4}$" is at least 0.5. Another significant result that can be derived using possibility theory is that, the possibility that risk will be smaller than $2.94x10^{-5}$ and greater than $2.93x10^{-4}$ (this is the support of the membership function corresponding to Scenario 2, for the individual at 90^{th} fractile, see Figure 3.5) is zero, thus the probability is also zero. Similarly, we can assign possibility values for all the risk values covered by the domain of fuzzy restriction "around 1.4×10^{-4}." The results obtained from possibility theory application may be used in approximate reasoning studies. Membership function of the fuzzy risk may be used to evaluate compliance of the resulting fuzzy risk with respect to a crisp guideline. Decision-making in human health risk assessment context is studied further in Section 3.5.

3.3.5 Conclusions for Probabilistic-Fuzzy Health Risk Modeling

While conducting health risk assessment studies the first step is to collect all information that is available. Depending on the type of collected information, variables of the risk equation may be characterized as crisp, random, or fuzzy variables. The proposed probabilistic-fuzzy approach allows the analyst to include uncertainties in model parameters in the desired fashion (i.e., allows combining crisp, random, and fuzzy variables into the risk equations).

In this study, we proposed an approach to include two different kinds of uncertainties into risk assessment models using probability theory and fuzzy set theory simultaneously. Treating cancer potency factors associated with ingestion, inhalation, and dermal contact together with the contaminant concentration in tap water as fuzzy variables allowed us to include uncertainties due to reasons other than randomness into the risk assessment model. Fuzzy set calculations resulted in membership values of risk for individuals at a certain fractile of risk. Instead of a single probability distribution of risk as provided by PRA, PFRA approach provides the probability distributions of risk for various α_c levels. If the appropriate information is available, utilization of PFRA may provide results which may help the decision-maker to make more informed decisions.

Representation of uncertainty using fuzzy or random variables will impact the form of the uncertainty in the calculated risk. The membership function of risk for a certain fractile may provide significant information for the analyst. For example, the possibility of occurrence of risk values having zero membership values for a specific fractile are zero, while the risk value with a membership value of one is the most likely risk. The shape and the support of the risk provide extra information about the resulting uncertainty which is a combined effect of the random and fuzzy input variables. For example, uncertainty associated with a risk that has a small support is respectively smaller than that of a risk which has a larger support. For simplicity purposes triangular membership functions are used in this study; however, membership functions for fuzzy variables do not need to be triangular. If other membership functions are used for the input variables, the procedure to conduct PFRA will not change but the shape of resulting fuzzy risk may change.

3.4 2D Monte Carlo versus 2D Fuzzy Monte Carlo Health Risk Assessment

In this second application of hybrid models for human health risk assessment, we treat uncertainty and variability associated with one or more parameters of the risk equation for ingestion route (i.e., Equation (3.2)) by representing these parameter(s) by random variable(s) which have fuzzy parameters. For example exposure duration is characterized by a normal distribution where mean and standard deviation of this normal distribution are characterized by fuzzy variables. We only consider the ingestion route in this study, thus we will refer to Equation (3.2) as the risk equation from hereon in this section.

As explained earlier, a PRA can provide information about the probability of exceeding a risk level of concern, given the estimated variability or uncertainty in the elements of the risk equation. One of the advanced modeling approaches that may be used to conduct PRA studies is two-dimensional Monte Carlo Analysis (2D MCA). In 2D MCA, the parameters of the random variables of the health risk equation are characterized by probability density functions as well. 2D MCA procedure is briefly summarized in the following section since the method we are proposing is an extension and an alternative to this approach. Details of conducting a 2D MCA can be found in RAGS, Volume 3 (U.S. EPA 2001). In this study, we propose a new hybrid method identified as 2D Fuzzy Monte Carlo Analysis (2D FMCA) which can be utilized in health risk assessment as an alternative to 2D MCA. The parameters of the random variables are treated as fuzzy numbers instead of random numbers in this new method.

3.4.1 2D Monte Carlo Analysis

2D MCA is one of the advanced modeling approaches that may be used to conduct PRA (U.S. EPA 2001). The 2D MCA is a procedure that allows characterization of both uncertainty and variability in one or more of the input variables. All probability density functions used to describe the variability in a PRA model have a certain degree of uncertainty. Thus, the terms of the risk equation that are represented by random variables have parameters which are also represented by random variables (i.e., the mean and the standard deviation of a normal random variable are referred as parameters of the random variable). Such variables are called "second order random variables." For example, variability in the exposure frequency can be represented using a normal pdf with a mean, m and a standard deviation, s where m and s are treated as random variables to account for the uncertainty associated with them.

In a 2D MCA, probability density functions representing variability and uncertainty are sampled using nested computational loops. The inner loop simulates variability by repeatedly sampling values for each variable from a specific probability density function, the parameters of which are selected in the outer loop. Figure 3.6 shows 2D MCA procedure as provided in RAGS, Volume 3 (U.S. EPA 2001).

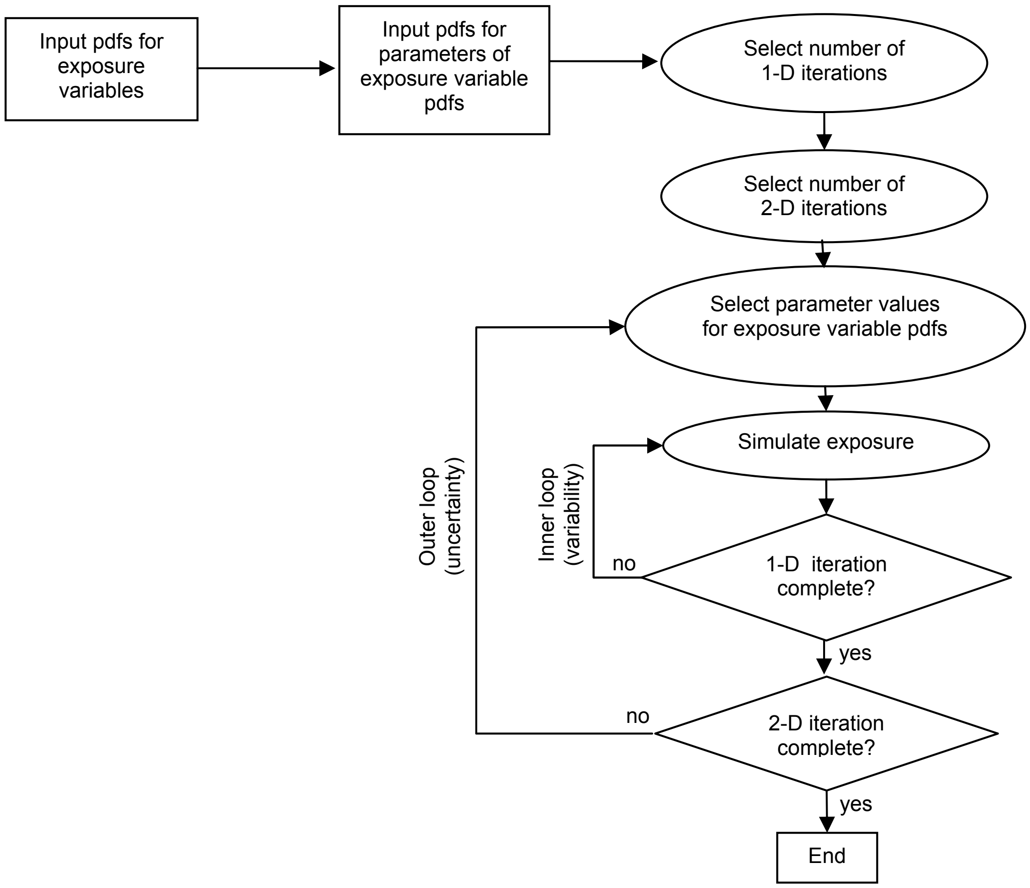


Figure 3.6 2D Monte Carlo procedure (U.S. EPA 2001)

In real-world situations, it is very hard to gather enough data to develop probability density functions for the parameters of the random variables. Moreover, there are situations in which available data and information is more appropriate to represent the parameters of the random variable by fuzzy numbers. Thus, a methodology, alternative to 2D MCA, is proposed in this second study to model uncertainty and variability associated with the variables of the risk equation. The new method is called 2D Fuzzy Monte Carlo Analysis (2D FMCA). The parameters of the random variables are modeled as fuzzy numbers, and fuzzy risks corresponding to different percentiles are generated as the result of 2D FMCA.

3.4.2 Characterization of Variability and Uncertainty

In this study, to simplify the calculations, we assumed that the individual is exposed to the contaminant through ingestion of the drinking water pathway only. The risk equation for the ingestion route is given in Equation (3.2). In more complex exposure applications the methodology discussed for this pathway can be extended to other pathways as well.

As a demonstration, we have applied 2D MCA and 2D FMCA analyses to two different scenarios. In the first scenario, only one of the variables of the risk Equation (3.2) (i.e., exposure frequency, EF) is treated as a random variable which is associated with uncertainty and variability – this scenario will be referred to as Case 1 from here on. In the second scenario, two variables of the risk Equation (3.2) (i.e., exposure frequency, EF and exposure duration, ED) are treated as random variables involving uncertainty and variability – this scenario will be referred to as Case 2 from here on. The analysis is carried out for a hypothetical case. The crisp input data selected in this analysis is as follows: the cancer slope factor is 0.11 (mg/kg-day)$^{-1}$, the concentration of the chemical in drinking water is 0.015 mg/L, the ingestion rate is 2 L/day, the exposure frequency is 320 days/year, the exposure duration is 30 years, the body weight is 70 kg, and the averaging time is 25,550 days.

For Case 1, only the exposure frequency is modeled as a second order random variable. The equations of the triangular pdfs used for the parameters of the random variable are provided in Equation (3.15). In this study, in order to be able to make comparisons with the proposed fuzzy Monte Carlo approach, we used triangular pdfs to define the mean and standard deviation of the random variables.

$$\begin{aligned}
&\text{exposure frequency} \sim N(m_{\mathrm{EF}}, s_{\mathrm{EF}}) \\
&\qquad m_{EF} \sim triangular,\ f(x) \\
&\qquad\qquad f(x) = \begin{cases} \dfrac{0.05}{20}(x-300) & 300 \le x \le 320 \\ 0.05 - \dfrac{0.05}{20}(x-320) & 320 < x \le 340 \end{cases} \\
&\qquad s_{EF} \sim triangular,\ g(x) \\
&\qquad\qquad g(x) = \begin{cases} (x-5) & 5 \le x \le 6 \\ 1-(x-6) & 6 < x \le 7 \end{cases}
\end{aligned} \tag{3.15}$$

where m_{EF} is the mean of the exposure frequency, s_{EF} is the standard deviation of the exposure frequency, and $\sim N(m_{EF}, s_{EF})$ assigns a normal pdf to exposure frequency.

For Case 2, the exposure frequency and the exposure duration are modeled as second order random variables. Triangular pdfs are also used for the parameters of exposure duration as given in Equation (3.16).

$$
\begin{aligned}
&\text{exposure duration} \sim N(m_{\mathrm{ED}}, s_{\mathrm{ED}}) \\
&\qquad m_{ED} \sim triangular,\ p(x) \\
&\qquad\qquad p(x) = \begin{cases} \dfrac{0.2}{5}(x-25) & 25 \le x \le 30 \\ 0.2 - \dfrac{0.2}{5}(x-30) & 30 < x \le 35 \end{cases} \\
&\qquad s_{ED} \sim triangular,\ q(x) \\
&\qquad\qquad q(x) = \begin{cases} (x-3) & 3 \le x \le 4 \\ 1-(x-4) & 4 < x \le 5 \end{cases}
\end{aligned}
\tag{3.16}
$$

where m_{ED} is the mean of the exposure duration, s_{ED} is the standard deviation of the exposure duration, and $\sim N(m_{ED}, s_{ED})$ assigns a normal pdf to exposure duration.

Normalized pdfs, $f_N(x)$, $g_N(x)$, $p_N(x)$, and $q_N(x)$ of $f(x)$, $g(x)$, $p(x)$, and $q(x)$ are given in Figure 3.7. Probability density functions are normalized such that the highest probability becomes one.

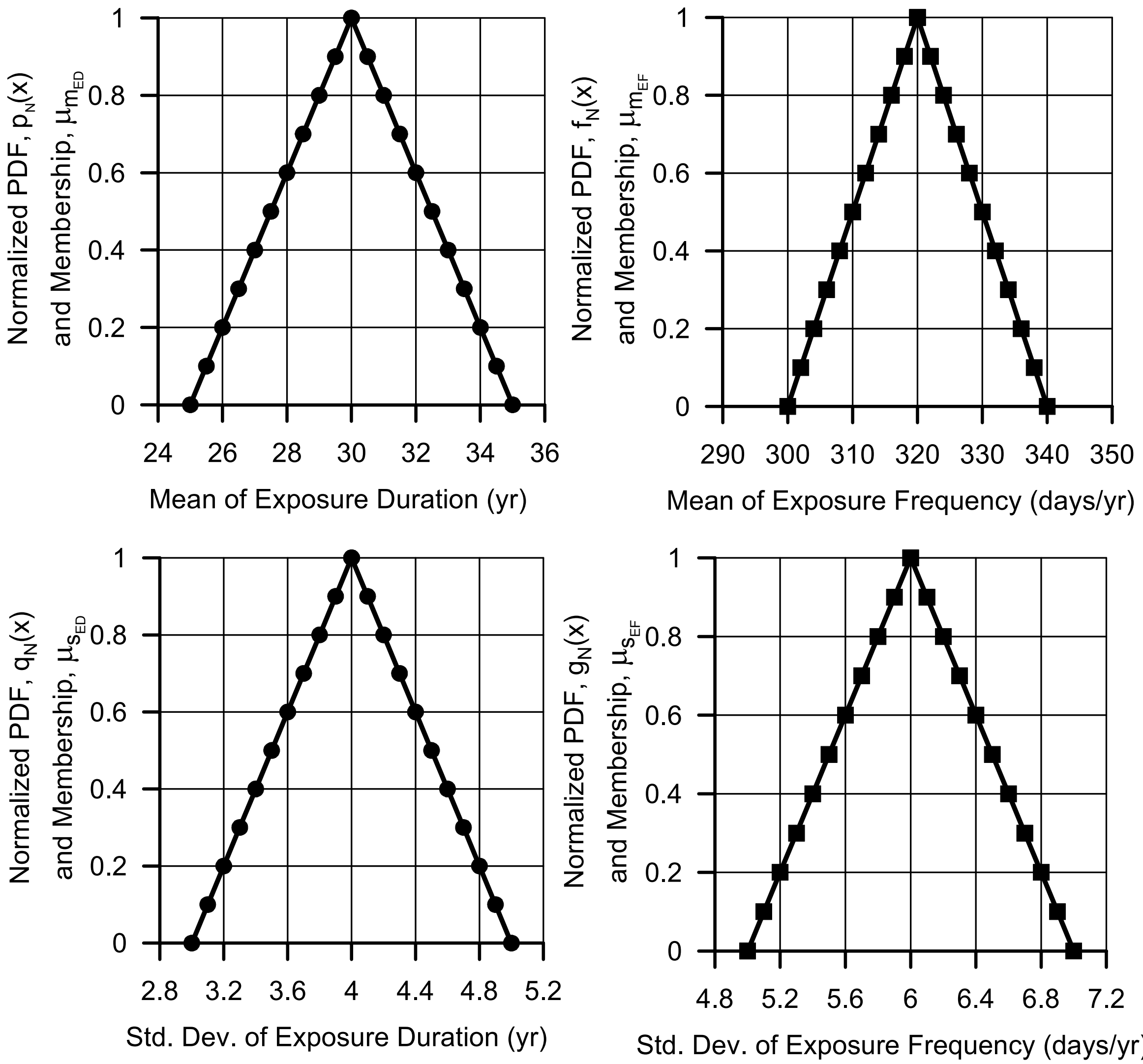


Figure 3.7 Normalized probability density functions and membership functions of means and standard deviations of EF and ED

In this study, the outer loop (i.e., uncertainty loop) iterates 1,000 times while the inner loop (i.e., variability loop) iterates 10,000 times. The total number of simulations required is equal to the number of outer loop iterations times the number of inner loop iterations. Thus, in our case a total of 1,000 cumulative distribution functions are generated and it takes a total of 10,000x1,000 iterations to generate these cdfs. In general, 2D MCA is computationally intensive since the two nested loops with large sampling within each loop are used. Another computational difficulty comes from sorting the risk values calculated in each one of the inner loops. For each cdf, 10,000 risk values are calculated and these have to be sorted to determine the risk value corresponding to each percentile (i.e., to form the cdf). Another method to form the cdf, is to discretize the risk domain and count the number of risk values in each interval, but risk values corresponding to specific percentiles can not be directly determined from this analysis.

3.4.3 2D Fuzzy Monte Carlo Analysis

2D FMCA, the new approach proposed in this study, uses a combination of probability and possibility theory to include imprecise and probabilistic information in the risk analysis model. Similar to 2D MCA, the variability in the random variables of the risk equation (i.e., exposure frequency and exposure duration) is characterized using normal pdfs and the uncertainty associated with them is characterized by using fuzzy numbers for the parameters of these random variables. That is, the means and the standard deviations of these pdfs are modeled as fuzzy numbers. We will refer to these variables as second order uncertain variables. Since it is more general and covers the one variable case as well, below we provide the application of the 2D FMCA procedure for Case 2 only.

The exposure frequency is assumed to be normally distributed with a fuzzy mean of "around 320.0" and a fuzzy standard deviation of "around 6.0" while exposure duration is assumed to be normally distributed with a fuzzy mean of "around 30.0" and a fuzzy standard deviation of "around 4.0." The membership functions of means and standard deviations of the exposure frequency ($\mu_{m_{EF}}$, $\mu_{s_{EF}}$) and the exposure duration ($\mu_{m_{ED}}$, $\mu_{s_{ED}}$) are given in Figure 3.7. The selected shapes of the membership functions and normalized triangular pdfs used for 2D MCA are identical. One important difference between triangular membership function and triangular pdf is that, the area below the pdf is equal to unity. Here, we should emphasize that it is not our intention to convert pdfs to membership functions. Our purpose is to provide an alternative approach to 2D MCA for treating uncertainties in the parameters of the pdfs using fuzzy set theory. If sufficient information to generate pdfs of the parameters of the random variables is not available, but expert knowledge or scarce data exists to represent the parameters of the pdf as fuzzy numbers then fuzzy set theory can be used to treat the uncertainties in these parameters. If the form of the available information is appropriate, another alternative might be using subjective probabilities which require probabilistic methods to carry out the PRA. Dubois and Parade (1992) provide a critical discussion of the Bayesian approach to the modeling of expert opinions in risk assessments. The reader may also refer to Guyonnet et al. (2003) for a brief discussion of the same topic. In this study, our focus is on the representation of uncertainty by fuzzy numbers. For cases where necessity of an advanced PRA has been justified, but due to lack of data, only a one-dimensional MCA can be conducted; it may be beneficial for the analyst to conduct a 2D FMCA which utilizes the available incomplete information or expert judgment.

The support of the membership function provides all the possible values for the variable and any number outside the support is not possible according to fuzzy set definition. The base of the probability density function covers all the values which have positive probabilities. Since, our goal is not to convert probability density functions into membership functions or vice versa or to utilize one in place of the other, no direct numerical comparisons for the calculated risk estimates are provided, nor one should attempt to provide such a comparison due to inherent differences in the definition, meaning, and treatment of uncertainty as utilized in each method. Here we provide computational framework for the 2D FMCA and the interpretation of the information generated from the proposed method.

In 2D FMCA, first, the fuzzy cumulative distributions of exposure frequency and exposure duration for each alpha cut (α_c) are calculated. The cdf, $F(x;\, m,\, s^2)$ defined as the probability of event, E(x) = {x | -∞ < X < x} is,

$$F(x;m,s^2) \;=\; \Pr\{E(x)\} \;=\; \int_{-\infty}^{\frac{x-m}{s}} g(t;0,1^2)\;\; dt$$

$$\Pr\{E(x)\} \;=\; \Pr(-\infty \le X \le x) \;=\; \Pr(X \le x) \;=\; \Phi\left(\frac{x-m}{s}\right) \;-\; \Phi\left(\frac{-\infty-m}{s}\right) \qquad (3.17)$$

$$F(x;\mu,s^2) \;= \Pr(X \le x) \;=\; \Phi\left(\frac{x-m}{s}\right)$$

where $g(t; m, s^2)$ is the probability density function, $\Phi(x)$ is the cumulative distribution function of a standard normal distribution. When the mean, m, and the standard deviation, s, are fuzzy numbers, the argument of Φ (i.e., $((x-m)/s)$) is also a fuzzy number.

A method to calculate probability density function when the mean and the standard deviation are fuzzy numbers is proposed by Kato et al. (1999). Using the heuristics of this method together with interval analysis and vertex method, fuzzy cumulative distribution functions for random variables can be calculated. This procedure, for a random variable X which has a normal distribution with fuzzy mean, m, and fuzzy standard deviation, s, is summarized below:

First, an alpha-cut interval, in this case 0.1, is selected (i.e.: $\alpha_c = 0.0:0.1:1.0$). The lower and the upper bounds of the intervals for each α_c for m and s, can be given as

$$\begin{Bmatrix} m^{\alpha_c} = [m_{lower}^{\alpha_c},\ m_{upper}^{\alpha_c}] \\ s^{\alpha_c} = [s_{lower}^{\alpha_c},\ s_{upper}^{\alpha_c}] \end{Bmatrix} \qquad \forall\, \alpha_c = 0.0:0.1:1.0 \tag{3.18}$$

Next, the random variables domain is discretized as follows:

$$x = x_i\ ; \qquad i = 0,1,2,\ldots,n \tag{3.19}$$

where x_n corresponds to $F(x_n) = 1$. For each alpha-cut level at x_i, the vertexes of $\Phi(x)$ can be calculated as

$$\left\{\begin{array}{lcl} vertex_1 & = & \Phi\left(\dfrac{x_i - m_{lower}^{\alpha_c}}{s_{lower}^{\alpha_c}}\right) \\ vertex_2 & = & \Phi\left(\dfrac{x_i - m_{upper}^{\alpha_c}}{s_{lower}^{\alpha_c}}\right) \\ vertex_3 & = & \Phi\left(\dfrac{x_i - m_{lower}^{\alpha_c}}{s_{upper}^{\alpha_c}}\right) \\ vertex_4 & = & \Phi\left(\dfrac{x_i - m_{upper}^{\alpha_c}}{s_{upper}^{\alpha_c}}\right) \end{array}\right\} \quad \forall\, \alpha_c = 0:0.1:1.0; \quad i = 0,1,2,...,n \tag{3.20}$$

The upper and the lower bounds of intervals of fuzzy cdf function for each alpha-cut level at x_i can now be calculated from

$$\left\{\begin{array}{lcl} F(x_i)_{lower}^{\alpha_c} & = & \min\limits_{j=1\ to\ 4}(vertex_j) \\ F(x_i)_{upper}^{\alpha_c} & = & \max\limits_{j=1\ to\ 4}(vertex_j) \end{array}\right\} \quad \forall\, \alpha_c = 0:0.1:1.0; \quad i = 0,1,2,...,n \tag{3.21}$$

Following the procedure summarized above, fuzzy cdfs for the exposure frequency, $F^{\alpha_c}\underset{EF}{|}$ and the exposure duration, $F^{\alpha_c}\underset{ED}{|}$ are generated. We refer to each cumulative distribution curve corresponding to the lower and the upper bounds of a certain alpha-cut as $F_{lower}^{\alpha_c}$ *and* $F_{upper}^{\alpha_c}$ (i.e., two curves for each alpha-cut, α_c) respectively, and we refer to the corresponding fuzzy cdfs as F^{α_c} (i.e., two curves for each alpha-cut for 11 alpha-cut levels, [0.0:0.1:1.0]; thus a total of 22 curves). For lower bounds of the exposure frequency, the exposure duration, and the risk we use $F_{lower}^{\alpha_c}\underset{EF}{|}$, $F_{lower}^{\alpha_c}\underset{ED}{|}$, and $F_{lower}^{\alpha_c}\underset{R}{|}$ respectively. As the next step, Monte Carlo analysis is used to calculate fuzzy risk cdfs and associated membership functions are assigned using the heuristics of the extension principle and interval analysis.

The process of performing addition, subtraction, multiplication, etc. with fuzzy numbers is identified as fuzzy arithmetic. The extension principle, which is one of the most important concepts of fuzzy set theory, is used to conduct arithmetic operations on fuzzy numbers. In general, it enables us to extend any point operations to operations involving fuzzy sets. A generalized version of the extension principle can be found in Appendix B.

In our case, $F^{\alpha_c}\big|_{EF}$ and $F^{\alpha_c}\big|_{ED}$ are fuzzy subsets, and the Cartesian product of $F^{\alpha_c}\big|_{EF}$ x $F^{\alpha_c}\big|_{ED}$ is mapped into the fuzzy subset risk, $F^{\alpha_c}\big|_{R}$. Monte Carlo analysis is used to generate the risk cdf (i.e., sample 10,000 times from the associated cdfs of 2-tuple, $F^{\alpha_c}\big|_{EF}$ and $F^{\alpha_c}\big|_{ED}$) and using the heuristics of the extension principle we assign $\min(\alpha_c$ of $F^{\alpha_c}\big|_{EF}$, α_c of $F^{\alpha_c}\big|_{ED})$ as the corresponding membership function for that cdf. Provided that equal alpha-cut intervals are used to discretize the membership domain, the number of cdfs which will have a membership value of α_c can be calculated from

$$2\left(2(n+1)-2\frac{\alpha_c}{\Delta\alpha_c}\right) + 2\left(2(n+1)-\left(2\frac{\alpha_c}{\Delta\alpha_c}-2\right)\right) \tag{3.22}$$

where $\Delta\alpha_c$ is the alpha-cut interval (0.1 for this study) and $n = 1/\Delta\alpha_c$.

Finally, the heuristics of interval analysis is used to calculate upper bound, $F^{\alpha_c}_{upper}\Big|_R$ and lower bound, $F^{\alpha_c}_{lower}\Big|_R$ of risk cdfs for each alpha-cut level. The algorithm to calculate $F^{\alpha_c}_{upper}\Big|_R$ and $F^{\alpha_c}_{lower}\Big|_R$ is as follows:

First cumulative probability axis is discretized as,

$$percentile = p_i \quad ; \quad i = 0,1,2,...,n \tag{3.23}$$

where n is the total number of intervals used (i.e., in this study 0.5 intervals is used for the percentile; so cumulative probability axis is divided into a total of 200 equal intervals). At each p_i the lower and the upper bounds of risk cdfs for each α_c are calculated as follows:

$$\left\{ \begin{aligned} F^{\alpha_c}_{lower}\Big|_R(p_i) &= \min \left(F^{\alpha_c}\Big|_R(p_i) \right) \\ F^{\alpha_c}_{upper}\Big|_R(p_i) &= \max \left(F^{\alpha_c}\Big|_R(p_i) \right) \end{aligned} \right\} \forall \alpha_c = 0.0:0.1:1.0; \quad i = 0,1,...,n \tag{3.24}$$

where $F^{\alpha_c}_{lower}\Big|_R (p_i)$ represents the lower bound of the risk cdf at the percentile corresponding to p_i for a membership value of α_c. Applying this methodology, frequency plots of risks for upper and lower limits of each alpha-cut are generated. These results provide the range of possible frequency plots corresponding to each alpha-cut level.

For 2D FMCA procedure again two nested loops are used. For Case 1, the outer loop only iterates 2x1.0/alpha-cut times. In this study we used an alpha-cut of 0.1 so the outer loop iterates 22 times. Similar to 2D MCA case, the inner loop iterates 10,000 times. Thus, for Case 1, total number of simulations required by 2D FMCA (i.e., 22x10,000) is much less than that of 2D MCA case when an alpha-cut of 0.1 is used which we think is an acceptable selection. For Case 2, the computational requirement of 2D FMCA increases as well. Initially 22 cdfs are generated for exposure frequency and 22 cdfs are generated for exposure duration. Then these 44 cdfs are paired with each other, resulting in a total of $22^2 = 484$ cdfs. Each of these 484 cdfs have a membership value determined using the heuristics of the extension principle. Thus, for Case 2, the outer loop iterates 484 times, so the total number of simulations required is 484x10,000. This is still computationally less intensive than 2D MCA assuming that the outer loop of 2D MCA is bigger than 484 which usually is the case.

3.4.4 Results for 2D Monte Carlo versus 2D Fuzzy Monte Carlo Health Risk Assessment

In this work we have introduced another methodology which utilizes fuzzy set theory together with probabilistic risk assessment. Currently, the most popular method to carry out the PRA is Monte Carlo analysis. For the cases where the parameters of the distributions of random variables are not well known (i.e., can not be represented as crisp values) a 2D MCA can be conducted. However, typically the data required to conduct 2D MCA is not readily available or it is too costly to collect the required data. Even in recently conducted risk assessment studies, the need for 2D MCA is usually stated, but due to data limitations this analysis is not conducted. Currently, instead of 2D MCA, various other methods such as multiple 1D MCA are used as alternative approaches.

In this study, we proposed an alternative approach in which the parameters of the random variables are characterized by fuzzy variables. Data required for this approach may be easier to obtain. The membership functions of the parameters of the random variables can be formed using imprecise, vague information or, expert judgment. Thus, application of the 2D FMCA approach to risk assessment problems instead of various 1D MCA approaches may be more realistic for some cases and may provide the analyst sufficient information for decision-making.

Results of 2D Monte Carlo Analysis: The output of a 2D MCA analysis is a collection of cdfs for each simulation of the outer loop. For Case 1, the exposure frequency is modeled as a normally distributed random variable with various mean and standard deviation combinations selected from triangular pdfs. In this study 1,000 different mean and standard deviation combinations (i.e., determined by the outer loop) are generated for the exposure frequency. For each mean and standard deviation combination 10,000 Monte Carlo simulations are conducted to generate one cdf. Thus, 2D MCA for Case 1 produced 1,000 cdfs for risk. For Case 2, again 1,000 cdfs for risk are calculated and for each cdf, 10,000 Monte Carlo simulations are conducted. The only difference between Case 1 and Case 2 is the exposure duration in addition to the exposure frequency is also modeled as a normally distributed random variable in Case 2. Thus, for each simulation of the inner loop, one combination of mean and standard deviation for the exposure frequency and one combination of mean and standard deviation for the exposure duration is generated and these distributions are used to sample values for the exposure frequency and the exposure duration.

Various forms of information can be extracted from the results of 2D MCA. For example, the 90% confidence interval for the median for selected percentiles of the risk distribution can be calculated. The 90% confidence interval for the median is the envelope covering the area between cdfs corresponding to the 5^{th} and the 95^{th} percentiles as defined in RAGS, Volume 3 (U.S. EPA 2001). 1,000 cdfs generated through 2D MCA are used to plot the 90% confidence interval for the median and the results are given in Figure 3.8.

Confidence limits for the 5^{th}, the 25^{th}, the 50^{th}, the 75^{th}, the 95^{th}, and the 99^{th} percentiles of risk for Case 1 are given in Figure 3.9. As can be seen from Figure 3.9, the simulation suggests that there is a 95% probability that the 75^{th} percentile risk estimate is below 1.87×10^{-5}.

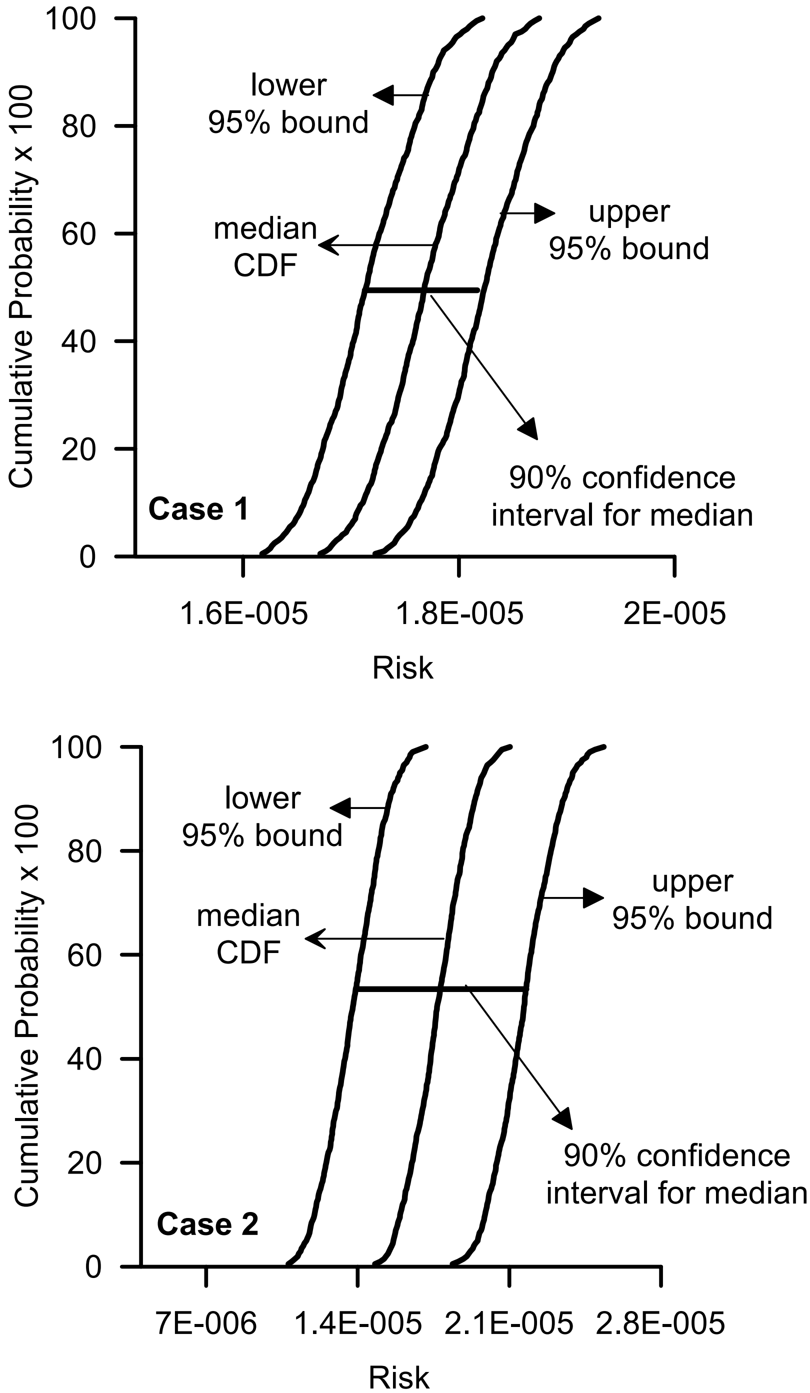


Figure 3.8 90% Confidence intervals for median for Case 1 and Case 2

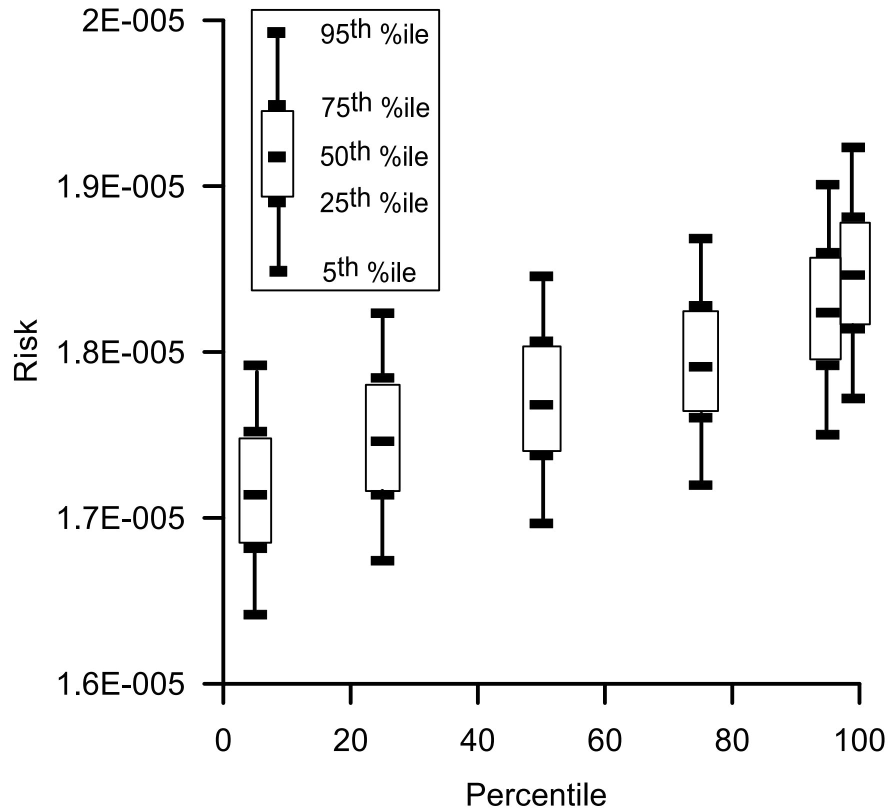


Figure 3.9 Variability statistics for Case 1

Results of 2D Fuzzy Monte Carlo Analysis: As a result of the 2D FMCA analysis, again a set of risk cdfs are generated. The 2D FMCA produces two cdfs (i.e., one for upper and one for lower bound) for each alpha-cut level except for alpha-cut 1.0 since the lower and the upper bound at alpha-cut 1.0 is the same. Thus, a total of 22 risk cdfs are generated. Five of these cdfs corresponding to 0.0, 0.5 and 1.0 alpha-cuts for Case 1 and Case 2 are given in Figure 3.10.

These 22 risk cdfs can be used to generate fuzzy risks corresponding to each percentile. For example, the membership function of fuzzy risks corresponding to the 5^{th}, 25^{th}, 50^{th}, 75th, 95^{th}, and 99^{th} percentiles for Case 1 and Case 2 are given in Figure 3.11.

The membership function for the 75^{th} percentile gives the possible values for each alpha-cut. For example, the support of the membership function for Case 1 is $1.68x10^{-5}$ to $1.91x10^{-5}$. This range gives all the possible risk values that can result with the specific pdfs used for the elements of the risk equation (i.e., exposure frequency) and the membership functions used for the parameters of these elements (i.e., triangular distributions defined in Equations (3.15) and (3.16)). The outputs of the fuzzy approach are all the possible risks and likeliness of occurrence of each risk value. The risk value corresponding to a membership value of 1.0 is the most possible/likely risk. For example, for 75^{th} percentile, the most likely risks are $1.79x10^{-5}$ and $1.93x10^{-5}$ for Case 1 and Case 2, respectively.

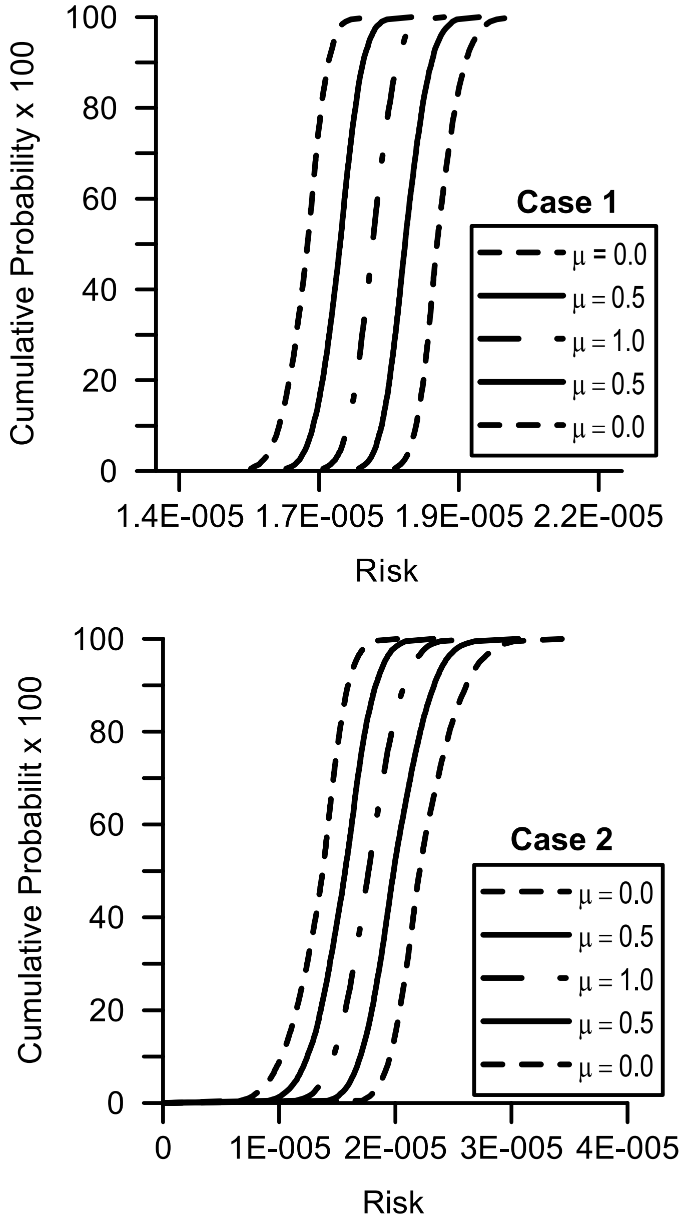


Figure 3.10 Cdfs for 0, 0.5, and 1.0 alpha-cut levels for Case 1 and Case 2

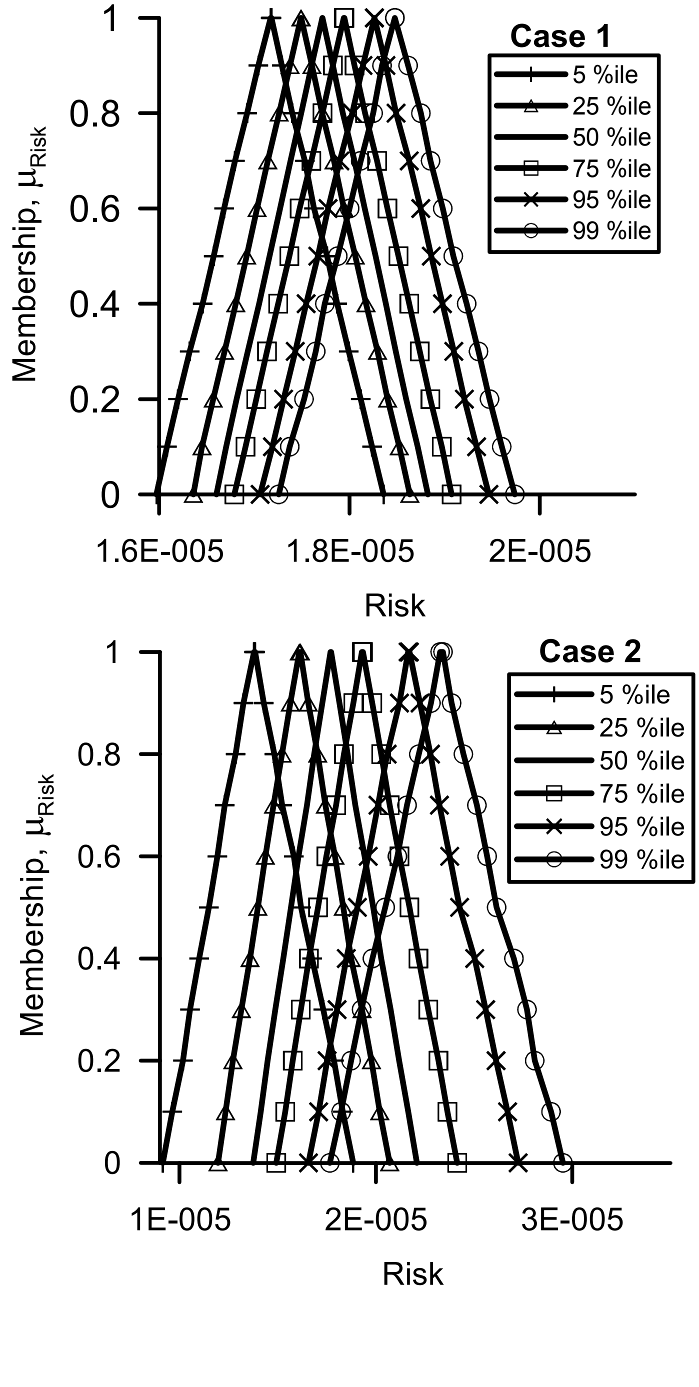


Figure 3.11 Membership functions of fuzzy risk for 5^{th}, 25^{th}, 50^{th}, 75^{th}, 95^{th}, and 99^{th} percentiles for Case 1 and Case 2

For Case 1 only the exposure frequency is characterized as a second order uncertain variable while for Case 2, both exposure frequency and exposure duration are characterized as second order uncertain variables. Thus, Case 2 involves higher uncertainty and variability. This can be seen from the supports of the membership functions of fuzzy risks generated for various percentiles. For example the support of fuzzy risk for 75th percentile for Case 1 ranges from 1.68×10^{-5} to 1.91×10^{-5} while the support ranges from 1.49×10^{-5} to 2.41×10^{-5} for Case 2. Since Case 2 allows more uncertain information to be included into the model with respect to Case 1, the resulting fuzzy risk has a larger range of possibilities (i.e., the support of the membership function is larger). Comparison of the results of 2D FMCA and 2D MCA is provided in the next section.

2D FMCA vs 2D MCA Analysis: To provide the results of 2D MCA and 2D FMCA analysis in relation to each other, pdfs and membership functions of 75 percentile risks are provided in Figure 3.12 for Case 1 and Case 2. The pdfs are multiplied by a factor such that the maximum relative frequencies equal one (i.e., normalized frequency). The bar chart on Figure 3.12 is the normalized frequency for risk obtained from 2D MCA and the solid line is the membership function of the risk obtained from 2D FMCA. Since both 2D MCA and 2D FMCA approaches provide cumulative distribution functions for any desired percentile, similar comparison plots can be generated for other percentiles such as 50th, 90th, 99th, etc.

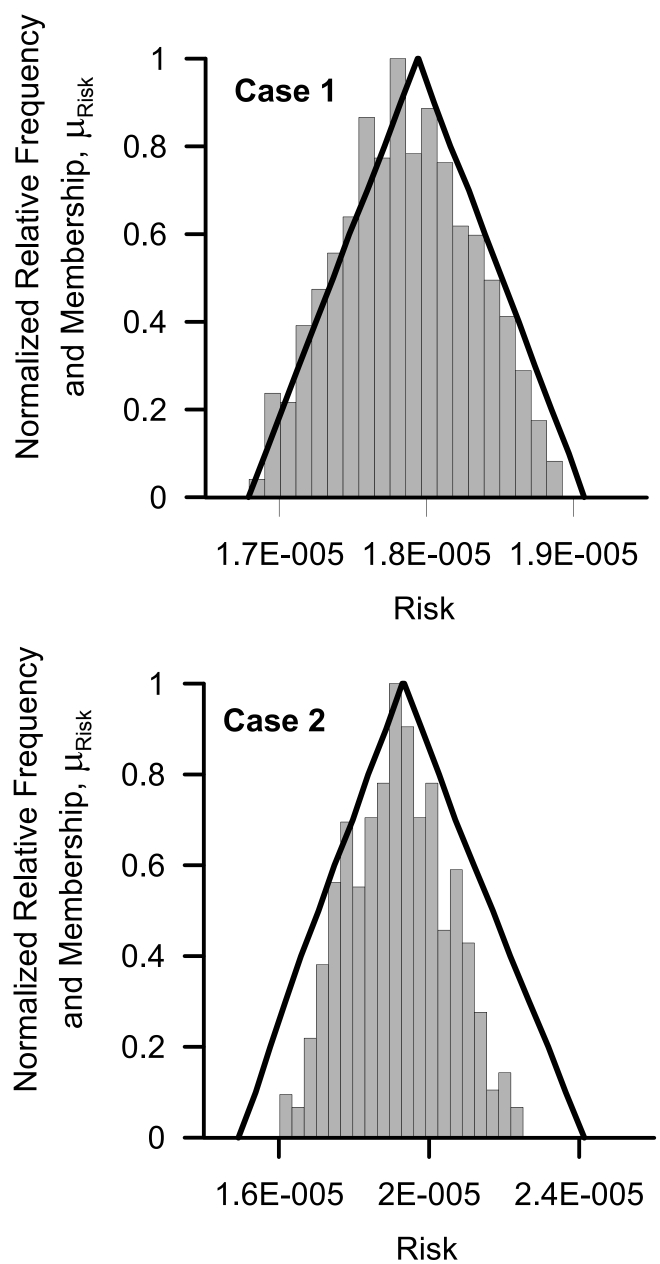


Figure 3.12 Comparison of 2D MCA and 2D FMCA for 75th percentile risk

As can be seen from the results given in Figure 3.12, shapes of pdf and membership function of risk are similar to each other for both cases, however, the spread of membership functions are greater than those of pdfs. This is due to the fact that fuzzy calculations take into consideration all possible combinations of parameter values rather than random sampling. For 75th percentile, the highest normalized frequencies correspond to risk values of $1.78x10^{-5}$ and $1.91x10^{-5}$ (result of 2D MCA) while the most likely risks are $1.79x10^{-5}$ and $1.93x10^{-5}$ (result of 2D FMCA) for Case 1 and Case 2, respectively. Acceptability of a fuzzy risk with respect to a crisp compliance guideline is investigated in Section 3.5. Existing approaches and a new decision-making tool is proposed in that section.

3.4.5 CONCLUSIONS FOR 2D MONTE CARLO VERSUS 2D FUZZY MONTE CARLO FOR HEALTH RISK ASSESSMENT

2D MCA is identified as one the advance PRA techniques in RAGS, Volume 3 (U.S. EPA 2001). In 2D MCA, both variability and uncertainty in a variable is treated by characterizing the variable as a second order random variable. For example, exposure frequency is characterized by a normal probability density function and the mean and the standard deviations of the normal distribution are characterized by normal probability density functions as well. One of the major limitations of 2D MCA is lack of sufficient data to characterize the variable as second order random variable. In real-world applications often the necessity of 2D MCA is justified however, since necessary information to conduct this analysis does not exit only a 1D MCA or multiple 1D MCA are conducted. For such situations the proposed hybrid method can be used as an alternative to 1D MCA or multiple 1D MCA. The analyst must decide which approach to use depending on the form of data/information available about the problem under consideration.

In risk assessment studies where decisions directly impact human life, it is necessary to consider all the possibilities and make decisions in the light of this information. Depending on the context of the study, the decision-maker can adjust the conservative nature of 2D FMCA using lower percentiles of risk, or utilizing various measures (i.e., the possibility, the necessity, or the risk tolerance measure as defined in Section 3.5) in making decisions.

3.5 Risk Tolerance Measure for Decision-Making in Fuzzy Health Risk Assessment

In the last part of this chapter, we deal with decision-making in fuzzy health risk assessment studies. As a result of the possibilistic or hybrid approaches as provided in the previous sections, fuzzy risks are generated. Since possibilistic and hybrid health risk assessment studies are relatively new, decision-making in human health risk assessment studies which result in fuzzy risks is a recently developing research area. In this section, we provide a review of several available approaches which may be used by the decision-makers in comparing the acceptability of the fuzzy health risk with respect to a crisp compliance guideline. The existing approaches involve defuzzification techniques, the possibility and the necessity measures. Here, we also propose a new measure, the ***risk tolerance measure***, which is a combination of the possibility and the necessity measures. Various hypothetical fuzzy risks which have different membership functions are evaluated with respect to the possibility, the necessity, and the risk tolerance measures and the results are discussed comparatively. First, we briefly explain the possibility and the necessity measures, and then define the risk tolerance measure that we propose as an alternative to these two measures.

3.5.1 THE POSSIBILITY MEASURE AND THE NECESSITY MEASURE

One method to compare the acceptability of the resulting fuzzy risk with respect to a compliance criterion is proposed by Guyonnet et al. (1999) and Guyonnet et al (2003). They used the possibility, $Poss$, and the necessity, Nec, measures for determining the acceptability of a fuzzy risk, $\widetilde{R}$ with respect to a compliance guideline, C_{comp}.

In order to measure the validity of the proposition, $\boldsymbol{P}$: "the fuzzy risk $\widetilde{R}$ is smaller than or equal to the compliance guideline C_{comp}" Guyonnet et al. (1999) and Guyonnet et al (2003) used the following definitions (Dubois and Prade 1988):

$$Poss(\boldsymbol{P}) = Poss\left(\widetilde{R} \le \widetilde{C}\right) = \sup_x \min\left[\mu_{\widetilde{R}}(x), \mu_{\widetilde{C}}(x)\right] \tag{3.25}$$

$$Nec(\boldsymbol{P}) = Nec\left(\widetilde{R} \le \widetilde{C}\right) = \inf_x \max\left[1-\mu_{\widetilde{R}}(x), \mu_{\widetilde{C}}(x)\right] \tag{3.26}$$

where $\mu_{\tilde{R}}(x) =$ membership function of $\tilde{R}$ for any value of x ; $\mu_{\tilde{C}}(x) =$ membership function of compliance criteria for any value x (note: $\mu_{\tilde{C}}(x) = 1 \quad if\ x \le C_{comp}$ and $\mu_{\tilde{C}}(x) = 0 \quad if\ x > C_{comp}$). The values of the possibility and the necessity measures range between [0, 1]. The reader may refer to Kikuchi and Pursula (1998) for basic explanations of the possibility and the necessity concepts. The fundamental properties of the possibility and the necessity measures are:

$$Nec(\boldsymbol{P}) = 1 - Poss(not\ \boldsymbol{P}) \tag{3.27}$$

The necessity measure takes the conservative view because it counts only the evidence that supports the impossibility of "*not* $\boldsymbol{P}$ ". Because of the way that evidence is accounted for:

$$\begin{aligned} Nec(\boldsymbol{P}) + Nec(not\ \boldsymbol{P}) \le 1 \\ Poss(\boldsymbol{P}) + Poss(not\ \boldsymbol{P}) \ge 1 \end{aligned} \tag{3.28}$$

Thus, the theory of possibility uses the possibility measure and the necessity measure to check the validity of a proposition. Unlike in the probability theory, neither the sum of the necessity measure and its compliment nor the sum or the possibility measure and its complement adds up to one. As can be seen from Equations (3.25) and (3.26), the necessity measure of the proposition is always smaller than its possibility measure.

As suggested by Comé et al. (1997), as a conservative approach, using the necessity measure instead of the possibility measure seems more reasonable in making comparisons with compliance guidelines for health risk assessment purposes. However, the possibility and the necessity measures provide two different forms of information about the validity of the proposition. Thus, a measure which combines the possibility and the necessity measures may provide a more complete representation of the available information. In the next section, we explain how these two measures can be combined into a single measure, identified as the risk tolerance measure, which can be used to verify the validity of the proposition.

3.5.2 The Risk Tolerance Measure

Consider the fuzzy risk, $\widetilde{R}$ and the compliance guideline, C_{comp} given in Figure 3.13. The evidence of risk being within a certain range (i.e., [a, b]) can be approximately represented by a horizontal strip with a representative membership value. We will associate the membership value of each strip with the mid value of the α-cuts associated with that strip. For example, for α-cut intervals of 0.1, the bottom strip will be associated with a membership value of 0.05. We will use this concept in explaining the risk tolerance measure.

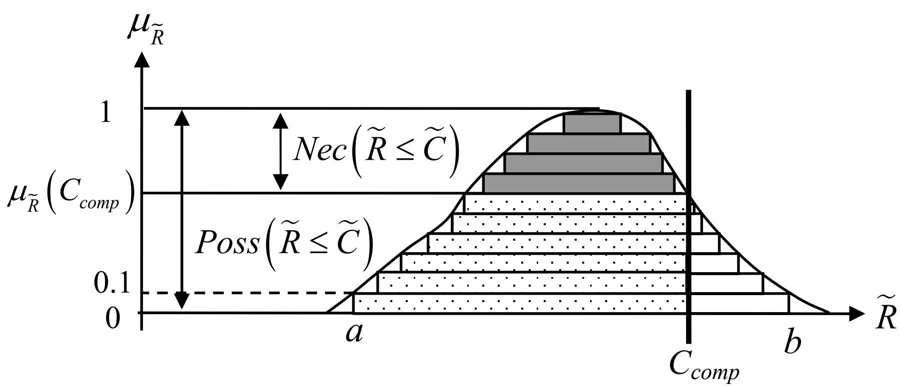


Figure 3.13 Fuzzy risk

The possibility and the necessity measures reveal two different types of information. In decision-making, using a risk tolerance measure which utilize information available in both the possibility and the necessity measures may be more coherent compared to using only the possibility measure or the necessity measure. The risk tolerance measure we propose here yields a unique criterion to evaluate the acceptance or rejection of the proposition and includes information from both measures as described below.

In Figure 3.13, the dotted sections of the solid white strips and the shaded strips provide evidence for the proposition $\boldsymbol{P}$ (i.e., $\widetilde{R} \le \widetilde{C}$). Since there is evidence which supports $\boldsymbol{P}$ throughout the entire membership domain [0,1], $Poss(\boldsymbol{P}) = 1$. However, in the membership range of [0, $\mu_{\widetilde{R}}(C_{comp})$], there is also evidence which supports ***not P*** (i.e., $\widetilde{R} > \widetilde{C}$). The membership function enveloping the solid white sections of the bottom strips to the right of compliance criteria represent evidence for ***not P***. On the other hand, as can be seen from Figure 3.13, in the membership range [$\mu_{\tilde{R}}(C_{comp})$, 1] there is no evidence which supports conflicting propositions. The necessity measure is determined considering only the evidence that supports the impossibility of ***not P*** (i.e., the shaded strips). In other words, the necessity measure is determined using the evidence that exist in the [$\mu_{\tilde{R}}(C_{comp})$, 1] membership range which supports only impossibility of ***not P*** .

The risk tolerance measure is a weighted average of the possibility and the necessity measures (see Equation (3.29)). The weighting function, γ used in front of the necessity measure is always one when the necessity measure has a value other than zero (see Equations (3.33), (3.34), and (3.35)). When the necessity measure is zero, γ is also zero. Whereas, the function used to weigh the possibility measure, β , in the membership range [0, $\mu_{\tilde{R}}(C_{comp})$], has values in the range (0,1) since there is evidence which supports both ***P*** and ***not P*** . As given below, combination of a weighted possibility measure and a weighted necessity measure identifies the risk tolerance measure. We used the ratio of evidence which supports ***P*** to the total available evidence to weigh the possibility measure, Equation (3.30). A more detailed explanation of the weighting functions, β and γ used in this approach is given below.

Incorporating the above concepts into the enumeration of the risk tolerance measure, $T(\boldsymbol{P})$ and normalizing this measure to a range [0,1] yields:

$$T(\boldsymbol{P}) = \begin{cases} 0 & C_{comp} \le L_{\tilde{R}}(0) \\ \frac{1}{2}\left(\beta \times Poss\left(\boldsymbol{P}\right) + \gamma \times Nec\left(\boldsymbol{P}\right)\right) & L_{\tilde{R}}(0) < C_{comp} < U_{\tilde{R}}(0) \\ 1 & C_{comp} \ge U_{\tilde{R}}(0) \end{cases} \tag{3.29}$$

where in the range $L_{\tilde{R}}(0) < C_{comp} < U_{\tilde{R}}(0)$, β and γ are defined as follows:

$$\beta = \frac{A_{poss\_l}}{A_{poss\_T}} \tag{3.30}$$

weighted areas for β are defined as:

$$A_{poss\_l} = \int_{0}^{\mu_{\tilde{R}}(C_{comp})} \alpha[C_{comp} - L_{\tilde{R}}(\alpha)]d\alpha \tag{3.31}$$

$$A_{poss\_T} = \int_{0}^{\mu_{\tilde{R}}(C_{comp})} \alpha[U_{\tilde{R}}(\alpha) - L_{\tilde{R}}(\alpha)]d\alpha \tag{3.32}$$

weighted areas for γ are defined as:

$$\gamma = \frac{A_{nec\_l}}{A_{nec\_T}} \tag{3.33}$$

$$A_{nec\_l} = \begin{cases} 0 & if\ L_{\tilde{R}}(\mu_{\tilde{R}}(C_{comp})) = C_{comp} \\ \int_{\mu_{\tilde{R}}(C_{comp})}^{1} \alpha[U_{\tilde{R}}(\alpha) - L_{\tilde{R}}(\alpha)]d\alpha & if\ U_{\tilde{R}}(\mu_{\tilde{R}}(C_{comp})) = C_{comp} \end{cases} \tag{3.34}$$

$$A_{nec\_T} = \int_{\mu_{\tilde{R}}(C_{comp})}^{1} \alpha[U_{\tilde{R}}(\alpha) - L_{\tilde{R}}(\alpha)]d\alpha \tag{3.35}$$

$L_{\tilde{R}}(\alpha)$ and $U_{\tilde{R}}(\alpha)$ for both β and γ are defined as:

$$\begin{aligned}
&L_{\tilde{R}}(\alpha) = \begin{cases} \inf\{x \mid x \in R_\alpha\} & if\ \alpha \in (0,1] \\ \inf\left\{x \mid x \in Supp\left(\tilde{R}\right)\right\} & if\ \alpha = 0 \end{cases} \\
&U_{\tilde{R}}(\alpha) = \begin{cases} \sup\{x \mid x \in R_\alpha\} & if\ \alpha \in (0,1] \\ \sup\left\{x \mid x \in Supp\left(\tilde{R}\right)\right\} & if\ \alpha = 0 \end{cases} \\
&\mu_{\tilde{R}}(x) : X \to [0,1] \\
&\alpha - cut\ of\ \tilde{R},\ R_\alpha = \{x \in X \mid \mu_{\tilde{R}}(x) \geq \alpha\}
\end{aligned} \tag{3.36}$$

where $Supp\left(\tilde{R}\right) = \left\{x \in X \mid \mu_{\tilde{R}}(x) > 0\right\}$ is the support of the fuzzy risk $\tilde{R}$. In Figure 3.13, for $\alpha = 0.1$, the $\alpha - cut$ of the fuzzy risk $\tilde{R}$ is the crisp set [a, b]. Thus, $L_{\tilde{R}}(0.1) = a$ and $U_{\tilde{R}}(0.1) = b$.

As can be seen from Equations (3.30), (3.31), and (3.32), β is a ratio of two weighted areas. The weighted area under the membership function to the left of the compliance criteria below the compliance criteria α-cut (i.e., $\mu_{\tilde{R}}(C_{comp})$) is A_{poss_l} and the total weighted area under the membership function below the compliance criteria α-cut (i.e., $\mu_{\tilde{R}}(C_{comp})$) is A_{poss_T}. We interpret the ratio of these weighted areas as an indicator of the strength of the evidence supporting $\widetilde{R} \leq \widetilde{C}$ relative to the total evidence available within the membership range [0, $\mu_{\widetilde{R}}(C_{comp})$]. In defining the parameter β we modified the ambiguity concept proposed in Delgado et al. (1998b). Since we selected the weighting factor for the areas as α , Equations (3.31) and (3.32) define the first moment of the areas under the membership function with respect to risk axis, which assign less importance to the α *levels* of a fuzzy number when α is near zero, and more significance to those α *levels* when α is near one.

In a similar fashion, we defined γ as the ratio of two weighted areas in the range [$\mu_{\widetilde{R}}(C_{comp})$,1] as given in Equation (3.33). A_{nec_l} is zero when the C_{comp} is located on the left leg of the membership function and A_{nec_l} equals A_{nec_T} when C_{comp} is located on the right leg of the membership function (Equation (3.34)). Thus, when C_{comp} is located on the left of the membership function (i.e., $Nec(\boldsymbol{P}) = 0$) γ is zero, and when C_{comp} is located on the right of the membership function, γ is one. Thus, when $Nec(\boldsymbol{P})$ is positive (i.e., greater than zero) it is always weighted with a factor of one.

In the risk tolerance measure proposed in this study, the possibility measure is weighted with a factor of β and the necessity measure is weighted with a factor of γ . However, γ is always zero when $Nec(\boldsymbol{P}) = 0$ and γ is always one when $Nec(\boldsymbol{P}) \neq 0$. Thus, $Nec(\boldsymbol{P})$ is always weighted with a factor of one. As can be seen from Equation (3.29), when the fuzzy risk, $\widetilde{R}$ lies completely on the left, or on the right of the compliance guideline, C_{comp}, $T(\boldsymbol{P}) = 1$ (fully acceptable health risk) or $T(\boldsymbol{P}) = 0$ (unacceptable health risk), respectively. When C_{comp} lies within the support of $\widetilde{R}$, then the risk tolerance measure ranges between $0 < T(\boldsymbol{P}) < 1$. Therefore, this approach leaves the regulatory agencies to define an acceptable $T(\boldsymbol{P})$ value between zero and one such that the resulting fuzzy risk may be evaluated as acceptable or not acceptable with respect to the compliance guideline.

3.5.3 Effect of the Shape of Membership Function on the Possibility, the Necessity, and the Risk Tolerance Measures

As pointed out before, the number of fuzzy input parameters, their shapes and supports, and the type of analysis conducted (e.g., sole possibilistic or hybrid approaches) effect the shape of the resulting fuzzy risk. In this section, our goal is to compare the outcome and the use of the possibility, the necessity, and the risk tolerance measures in decision-making for various membership functions of fuzzy risks. First, for simplicity, we will restrict the analysis to three different hypothetical membership functions (i.e., Case 1, Case 2, and Case 3) as the resulting fuzzy risks, Figure 3.14.

The chosen fuzzy risks have same supports and the same risk value corresponding to a membership value of one. Thus, the resulting risks in all three cases indicate that: (i) Risk values lower than 2.94×10^{-5} or greater than 2.44×10^{-4} are not possible; and, (ii) the most likely risk is 1.37×10^{-4}.

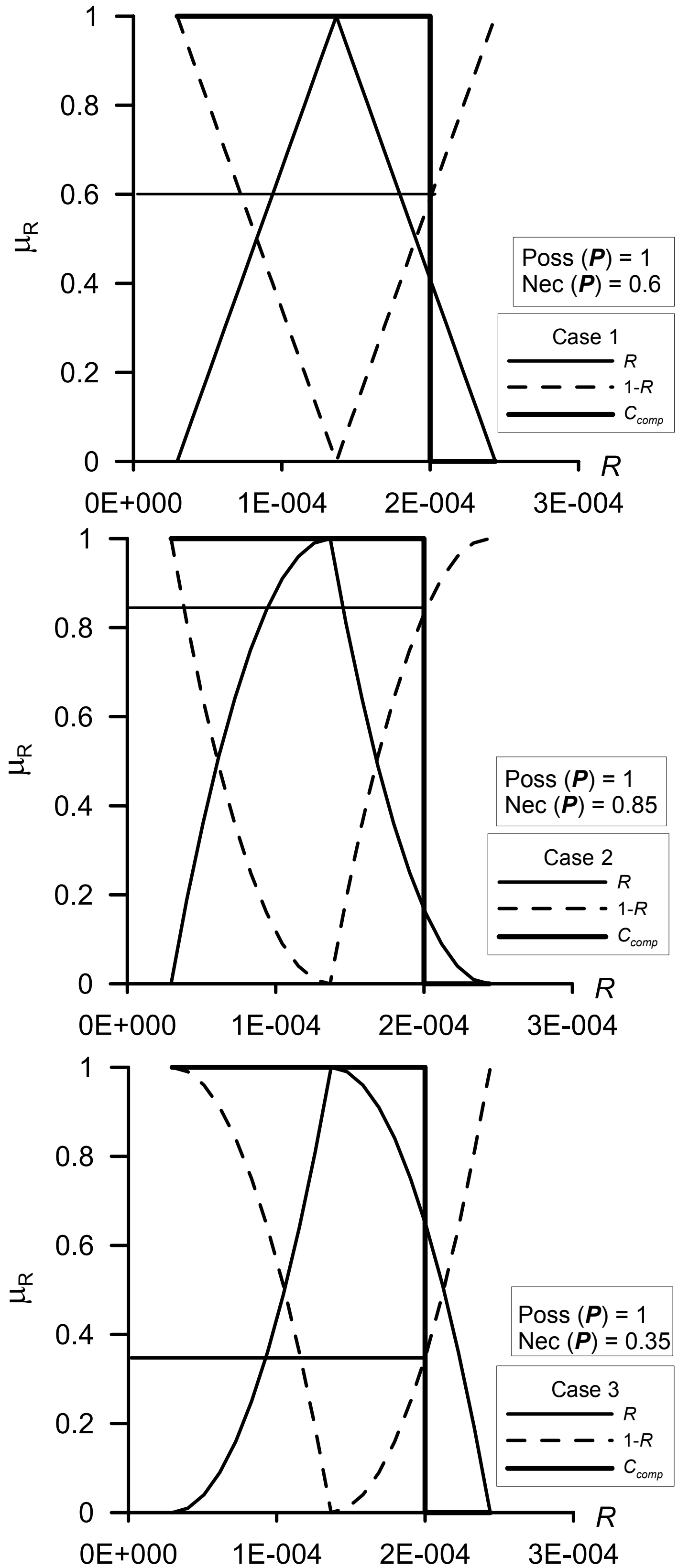


Figure 3.14 Hypothetical fuzzy risk membership functions

We further assume that the crisp compliance guideline C_{comp} is $2x10^{-4}$. If all the parameters in the risk equation were modeled as crisp numbers then the calculated risk, R would be a crisp number, and in order to decide if R is acceptable, it would be sufficient to check if R is smaller than or equal to C_{comp}. However, modeling certain parameters of the risk equation as fuzzy numbers will result in a fuzzy risk, $\widetilde{R}$ (see the results provided in Sections 3.3.4 and 3.4.4).

In order to compare the resulting fuzzy risk with a crisp compliance guideline, C_{comp}, one alternative is to use one of the two measures of the possibility theory (i.e., the possibility measure or the necessity measure). For example, the guideline may require the estimated risk to be less than or equal to C_{comp} for a possibility measure of 1.0, or for a necessity measure of 0.7. Let's consider the following alternative guidelines and check if fuzzy risks for each case satisfy these guidelines:

Guideline 1: Risk should be less than or equal to $2x10^{-4}$ for a possibility measure of 1.0.

Guideline 2: Risk should be less than or equal to $2x10^{-4}$ for a necessity measure of 0.4.

Guideline 3: Risk should be less than or equal to $2x10^{-4}$ for a necessity measure of 0.5.

Guideline 4: Risk should be less than or equal to $2x10^{-4}$ for a necessity measure of 0.7.

Guideline 5: Risk should be less than or equal to $2x10^{-4}$ for a necessity measure of 1.0.

Acceptability of each of the fuzzy risks given in Figure 3.14 is evaluated with respect to these five guidelines and the results are summarized in Table 3.2. As can be seen from Table 3.2, although the supports and most likely risk values are the same for all three fuzzy risks, the crisp compliance guideline of $2x10^{-4}$ is satisfied for necessity measures up to 0.7 for Case 2 while it is not even satisfied for a necessity measure of 0.4 for Case 3. For Case 3, Guidelines 2, 3, and 4 will be satisfied if the compliance criteria were $2.33x10^{-4}$. If the compliance guideline was $2.44x10^{-4}$ then it would be satisfied by all of the cases for a necessity measure of 1.0. As the compliance guideline decreases from $2.44x10^{-4}$ to $2.0x10^{-4}$ the necessity measure decreases from 1.0 to 0.35 for Case 3 while it decreases from 1.0 to 0.85 for Case 2. The regulatory agency must be aware of these facts and according to the context of the health risk assessment study utilization of the most appropriate measure for making decisions should be enforced. By context, we mean the features of the system, the expectations and needs of both the exposed population and the decision-maker.

Table 3.2 Compliance of fuzzy risk for each case with respect to various guidelines

	Guideline 1 $Poss(\boldsymbol{P})=1$	Guideline 2 $Nec(\boldsymbol{P})=0.4$	Guideline 3 $Nec(\boldsymbol{P})=0.5$	Guideline 4 $Nec(\boldsymbol{P})=0.7$	Guideline 5 $Nec(\boldsymbol{P})=1$
Case 1 $Poss(\boldsymbol{P})=1$ $Nec(\boldsymbol{P})=0.6$	Acceptable	Acceptable	Acceptable	Not Acceptable	Not Acceptable
Case 2 $Poss(\boldsymbol{P})=1$ $Nec(\boldsymbol{P})=0.85$	Acceptable	Acceptable	Acceptable	Acceptable	Not Acceptable
Case 3 $Poss(\boldsymbol{P})=1$ $Nec(\boldsymbol{P})=0.35$	Acceptable	Not Acceptable	Not Acceptable	Not Acceptable	Not Acceptable

It is worth remembering that, the necessity measure will always provide a more conservative estimate and may be preferred with respect to the possibility measure in decision-making for human health risk assessment studies. However, in our opinion, using only the necessity measure in decision-making will yield in loss of some important information conveyed by the possibility measure. A combination of these two measures may provide more comprehensive information about the acceptability of the fuzzy risk. The risk tolerance measure we propose in this study serves this purpose.

The risk tolerance measure is a combination of the possibility and the necessity measures. As can be seen from Equations (3.30), (3.31), and (3.32) β is always in the range (0, 1). Since C_{comp} is on the right leg of the membership function (Figure 3.14) for all three cases, $Poss(\boldsymbol{P}) = 1$, $0 < Nec(\boldsymbol{P}) < 1$, and the risk tolerance measure is in the range (0, 1). The $T(\boldsymbol{P})$ values for each case discussed above are calculated and given in Table 3.3.

Table 3.3 The risk tolerance measures for Cases 1, 2, and 3

	β	γ	$T(\boldsymbol{P})$
Case 1	0.92	1.0	0.76
Case 2	0.92	1.0	0.89
Case 3	0.96	1.0	0.66

Lets assume that the compliance standard requires a risk tolerance measure of at least 0.7 for the guideline "risk should be less than or equal to $2\text{x}10^{-4}$." According to this guideline, fuzzy risk for Case 3 does not satisfy the requirement, thus it is not acceptable. Fuzzy risks for Cases 1 and 2 satisfy the requirement and they are both acceptable. As can be seen from Figure 14, the possibility of the proposition $\boldsymbol{P}$ is one for both Case 1 and Case 2. β values for Case 1 Case 2 are the same too. Thus, contribution of the possibility measure to $T(\boldsymbol{P})$ is the same for Case 1 and Case 2. But since the necessity of

Case 2 is much higher than that of Case 1, the overall $T(\boldsymbol{P})$ value for Case 2 is higher than that of Case 1.

Now lets compare the acceptability of the three fuzzy risks for the guideline "risk should be less than or equal to $2x10^{-4}$" for a possibility, a necessity, and a risk tolerance measures of 0.7. As can be seen from Table 3.2, all three risks satisfy the guideline for a possibility of 1.0. This indicates that, they all satisfy the guideline for every possibility measure in [0,1] range. This is the most optimistic case. However, only Case 2 satisfies the guideline for a necessity measure of 0.7 (see Table 3.2) and this is the most pessimistic case. If a risk tolerance measure of 0.7 is required, then as can be observed from Table 3.3, both Case 1 and Case 2 satisfy the guideline. It can be concluded that integrating the information conveyed by the possibility and necessity measures into a single measure, the risk tolerance measure, resulted in the acceptability of the fuzzy risk for Case 1 to change from "not acceptable" (i.e., when only necessity measure is considered) to "acceptable." The risk tolerance measure integrates available information imbedded in the possibility and the necessity measures into a single criterion and results in a more comprehensive decision criterion.

In order to further investigate the impact of various membership functions on the risk tolerance measure, we work on four more examples by which we will examine in more detail how the risk tolerance measure differs from the possibility and the necessity measures. First, we examine the case when the compliance guideline, C_{comp} is on the right leg of the fuzzy risk. Consider the fuzzy risks, $\widetilde{R_1}$ and $\widetilde{R_2}$ in Figure 3.15. Notice that the left legs of the membership functions of both risks are the same while the right legs are different. For both fuzzy risks $Poss\left(\widetilde{R}_1 \le \widetilde{C}\right) = Poss\left(\widetilde{R}_2 \le \widetilde{C}\right) = 1$ and since $\mu_{\widetilde{R}_1}(C_{comp}) = \mu_{\widetilde{R}_2}(C_{comp}) = \mu_{\widetilde{R}}(C_{comp})$, $Nec\left(\widetilde{R}_1 \le \widetilde{C}\right) = Nec\left(\widetilde{R}_2 \le \widetilde{C}\right) = 1 - \mu_{\widetilde{R}}(C_{comp})$. Although the amount of evidence which supports $\widetilde{R}_1 > C_{comp}$ is smaller than the amount of evidence which supports $\widetilde{R}_2 > C_{comp}$ (i.e., $U_{\widetilde{R}_1}(\alpha)$ is smaller than $U_{\widetilde{R}_2}(\alpha)$ for $0 < \alpha < \mu_{\widetilde{R}}(C_{comp})$) both the possibility measures and the necessity measures for $\widetilde{R}_1 \le \widetilde{C}$ and $\widetilde{R}_2 \le \widetilde{C}$ are the same. Thus, if one of these measures are used for determining the acceptability of the $\widetilde{R}_1$ and $\widetilde{R}_2$ with respect to the compliance guideline, C_{comp}, as proposed in the literature, both of them will result in the same decision. However, in our opinion, degree of compliance of $\widetilde{R}_1$ is greater than that of $\widetilde{R}_2$. The risk tolerance measure reflects this idea. When the possibility and necessity measures of each fuzzy risk is weighted with corresponding β and γ values, respectively, $T\left(\widetilde{R}_1 \le \widetilde{C}\right) > T\left(\widetilde{R}_2 \le \widetilde{C}\right)$.

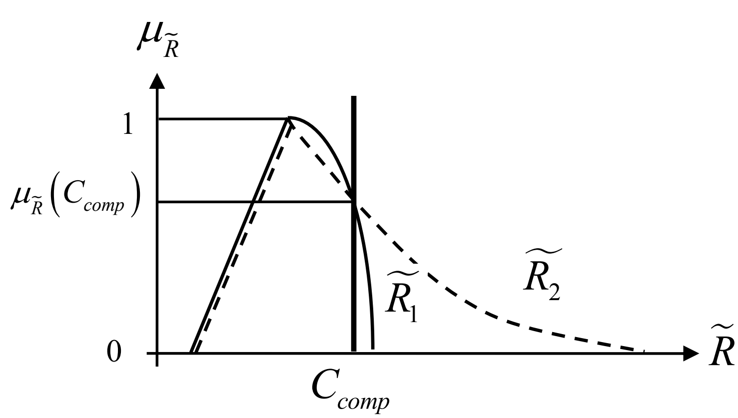


Figure 3.15 Fuzzy risks $\widetilde{R}_1$ and $\widetilde{R}_2$ with C_{comp}

Now let's consider fuzzy risks $\widetilde{R_2}$ and $\widetilde{R_3}$ in Figure 3.16. This time, right legs of the fuzzy risks are the same but left legs are different. As can be seen from Figure 3.16, Again $Poss\left(\widetilde{R_2} \le \widetilde{C}\right) = Poss\left(\widetilde{R_3} \le \widetilde{C}\right) = 1$ and $Nec\left(\widetilde{R_2} \le \widetilde{C}\right) = Nec\left(\widetilde{R_3} \le \widetilde{C}\right) =$ $1 - \mu_{\widetilde{R}}(C_{comp})$. As can be observed from Figure 3.16 β increases as the weighted area below $\mu_{\widetilde{R}}(C_{comp})$ between the left leg of the membership function and C_{comp} increases. This is logically correct since such an increase indicates an increase in the amount of evidence which supports $\widetilde{R} \le \widetilde{C}$. Thus, $T\left(\widetilde{R_3} \le \widetilde{C}\right) > T\left(\widetilde{R_2} \le \widetilde{C}\right)$.

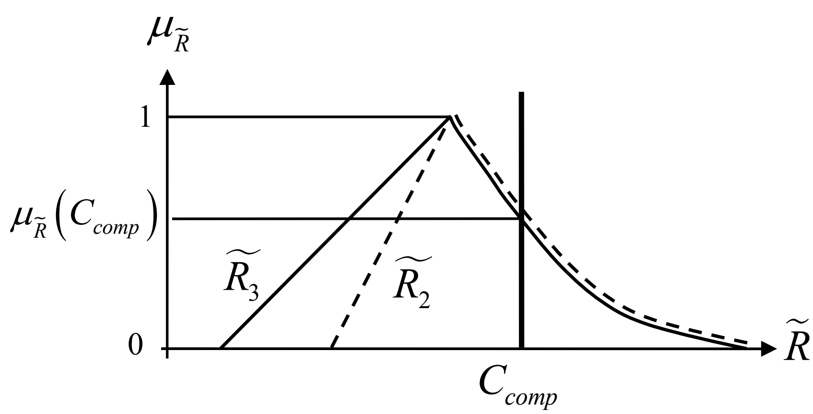


Figure 3.16 Fuzzy risks $\widetilde{R_2}$ and $\widetilde{R_3}$ with C_{comp}

When the compliance guideline is located on the left leg of the membership function of the fuzzy risk, the possibility measure takes values between zero and one and the necessity measure of the proposition is always zero. Thus, the risk tolerance measure can not have a value greater than 0.5. Now let's examine how the risk tolerance measure differs from the possibility and the necessity measures when the compliance guideline is on the left leg of the fuzzy risk.

Consider again $\widetilde{R}_1$ and $\widetilde{R}_2$, however a smaller value for the compliance guideline, C'_{comp} (see Figure 3.17). The proposition becomes $\boldsymbol{P}'$: "the fuzzy risk $\widetilde{R}$ is smaller than or equal to the compliance guideline C'_{comp}." Note that, $\mu_{\widetilde{C'}}(x) =$ membership function of compliance criteria for any value x (note: $\mu_{\widetilde{C'}}(x)=1 \quad if\ x \le C'_{comp}$ and $\mu_{\widetilde{C'}}(x)=0 \quad if\ x > C'_{comp}$). For this case $Poss\left(\widetilde{R}_1 \le \widetilde{C'}\right) = Poss\left(\widetilde{R}_2 \le \widetilde{C'}\right) = \mu_{\widetilde{R}}\left(C'_{comp}\right)$, $Nec\left(\widetilde{R}_1 \le \widetilde{C'}\right) = Nec\left(\widetilde{R}_2 \le \widetilde{C'}\right) = 0$. The amount of evidence which support $\widetilde{R}_1 < C'_{comp}$ and $\widetilde{R}_2 < C'_{comp}$ are the same. However, the amount of evidence which supports $\widetilde{R}_1 > C'_{comp}$ is smaller than the amount of evidence which supports $\widetilde{R}_2 > C'_{comp}$ thus, β for $\widetilde{R}_1$ is greater than that of $\widetilde{R}_2$. Consequently, $T\left(\widetilde{R}_1 \le \widetilde{C'}\right) > T\left(\widetilde{R}_2 \le \widetilde{C'}\right)$.

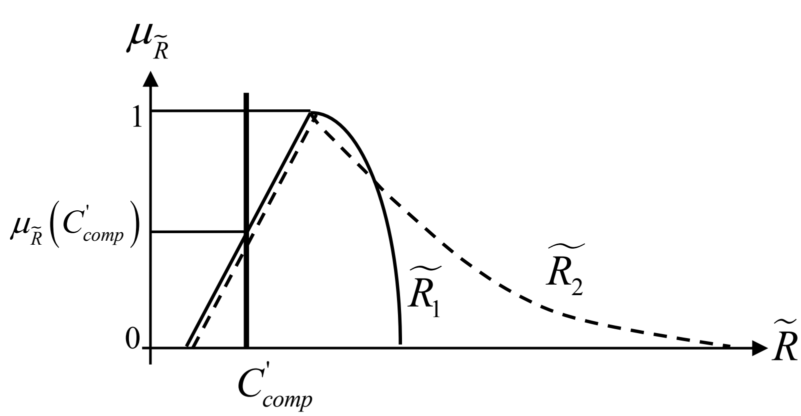


Figure 3.17 Fuzzy risks $\widetilde{R}_1$ and $\widetilde{R}_2$ with C'_{comp}

Now let's examine the case for fuzzy risks $\widetilde{R_2}$ and $\widetilde{R_3}$ when the compliance guideline is C'_{comp} as given in Figure 3.18. The possibility measures associated with the compliance of $\widetilde{R_2}$ and $\widetilde{R_3}$ with C'_{comp} are different: $Poss\left(\widetilde{R_2} \le \widetilde{C'}\right) = \mu_{\widetilde{R_2}}\left(C'_{comp}\right)$, $Poss\left(\widetilde{R_3} \le \widetilde{C'}\right) = \mu_{\widetilde{R_3}}\left(C'_{comp}\right)$. Both necessity measures are still zero: $Nec\left(\widetilde{R_1} \le \widetilde{C'}\right) = Nec\left(\widetilde{R_2} \le \widetilde{C'}\right) = 0$.

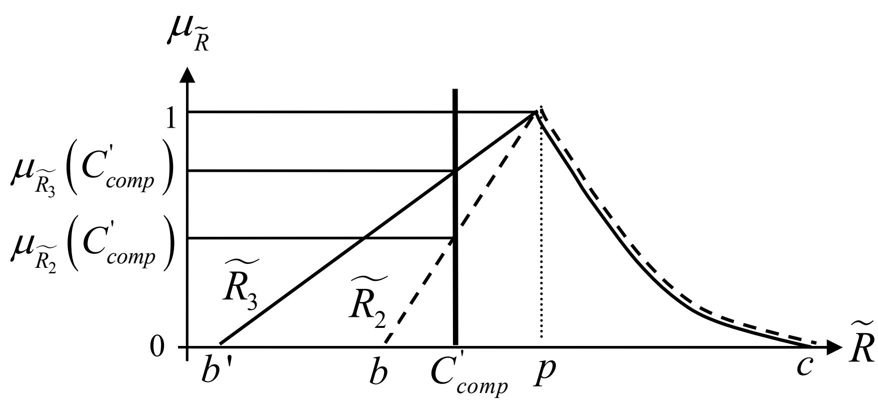


Figure 3.18 Fuzzy risks $\widetilde{R_2}$ and $\widetilde{R_3}$ with C'_{comp}

It can be shown that when $L_{\widetilde{R}}(\alpha)$ and $U_{\widetilde{R}}(\alpha)$ (see Equation (3.36)) are represented by general functions such as $L_{\widetilde{R}}(\alpha) = (p-b)\alpha^m + b$ and $U_{\widetilde{R}}(\alpha) = c-(c-p)\alpha^n$ respectively, β in Equation (3.30) increases as b' decreases from b to zero for all m and n (see Appendix C). Thus, in Figure 3.18, β for $\widetilde{R_2}$ is smaller than that of $\widetilde{R_3}$. Since $Poss\left(\widetilde{R_2} \le \widetilde{C'}\right)$ is also smaller than $Poss\left(\widetilde{R_3} \le \widetilde{C'}\right)$, it can be concluded that the risk tolerance measure for $\widetilde{R_2}$ is smaller than the risk tolerance measure of $\widetilde{R_3}$ for all these cases $\left(\text{i.e.,} T\left(\widetilde{R_3} \le \widetilde{C'}\right) > T\left(\widetilde{R_2} \le \widetilde{C'}\right)\right)$. This outcome is reasonable since there is more evidence for $\widetilde{R_3}$ than $\widetilde{R_2}$ that supports fuzzy risk being smaller than or equal to the compliance criterion.

3.5.4 DEFUZZIFICATION

An alternative method for comparing a fuzzy risk with a crisp compliance guideline, C_{comp}, is to convert the fuzzy risk into a crisp number and compare the resulting crisp risk with the compliance guideline. The process of converting a fuzzy number into a crisp number is called defuzzification. A number of defuzzification methods leading to different results have been proposed in the literature: maximum method, center of gravity method, center of maxima method, mean of maxima method, etc (Klir and Yuan 1995; Wang 1997).

Two of the defuzzification methods used in the literature are discussed below (Klir and Yuan 1995; Wang 1997). Suppose $\widetilde{R}$ is the risk expressed as a fuzzy set and $x^* \in X$ is the defuzzified value of this risk. Conceptually, the task of defuzzification is to specify a point in X that best represents the fuzzy set $\widetilde{R}$.

(i) Center of gravity method

$$x^* = \frac{\int_X x\mu_{\widetilde{R}}(x)dx}{\int_X \mu_{\widetilde{R}}(x)dx} \tag{3.37}$$

One weakness of the center of gravity method is that it can not distinguish between the fuzzy sets which may have same centroid but differ in their degree of fuzziness. For two symmetrical fuzzy risks which have different supports, the defuzzified values might be the same and lead to the same decision. However, the fuzzy risk with larger support may have higher membership values for higher risk values. The center of gravity method does not account for this effect.

(ii) Maximum method

Define the set:

$$hgt\left(\widetilde{R}\right) = \left\{x \in X \mid \mu_{\widetilde{R}}(x) = \sup_{x \in X} \mu_{\widetilde{R}}(x)\right\} \tag{3.38}$$

where height, $hgt\left(\widetilde{R}\right)$ is the set of all points in X at which $\mu_{\widetilde{R}}(x)$ achieves its maximum value. The maximum method defines x^* as an arbitrary element in $hgt\left(\widetilde{R}\right)$, that is,

$$x^{*} = \text{any point in } hgt\left(\widetilde{R}\right) \tag{3.39}$$

The maximum method assigns the risk with the highest membership value as the defuzzified value. The shape of the membership function is totally neglected. Maximum method may result in optimistic or pessimistic defuzzified risk values depending on the shape of the fuzzy risk.

Let's consider the three different fuzzy risks given in Figure 3.14 and compare the defuzzified results. The maximum method produces the same crisp risk value, 1.37×10^{-4} for all three fuzzy risks. Thus, if the compliance guideline is again set as 2.0×10^{-4}, and defuzzified risk according to maximum method is used as the representative risk, it can be concluded that all of the fuzzy risks comply with this criteria. When center of gravity method is used, the defuzzified risk values for Cases 1, 2, and 3 are 1.37×10^{-4}, 1.19×10^{-4} and, 1.62×10^{-4}, respectively. All these defuzzified risk values are smaller than 2.0×10^{-4} so they also satisfy the crisp guideline of 2.0×10^{-4}. Although, the compliance guideline of 2.0×10^{-4} resulted in the same conclusion for these three risk values for both of the defuzzification methods, it might not always be the case. For example, if the compliance guideline was 1.5×10^{-4} then standard will not be satisfied for Case 3 while it will be satisfied for Cases 1 and 2.

3.5.5 CONCLUSION FOR RISK TOLERANCE MEASURE FOR DECISION-MAKING IN FUZZY HEALTH RISK ASSESSMENT

Treatment of uncertainty in health risk assessment studies is crucial. In order to make a decision on the compliance of a fuzzy risk with respect to a compliance guideline, the fuzzy risk which may result from the possibilistic or hybrid approach need to be compared with the compliance guideline. Comparison of a fuzzy risk with respect to a crisp compliance guideline is not a straightforward process. The simplest approach is to convert the fuzzy risk into a crisp number (i.e., defuzzification) and then to compare it with the crisp guideline. However, this causes some of the information integrated in the membership function of the fuzzy risk to be lost. The defuzzification process results in a crisp risk estimate and the comparison of this risk estimate with the compliance guideline provides a yes/no type of conclusion: the resulting risk exceeds the guideline or it does not. Such a comparison does not provide any information about the degree of compliance of the fuzzy risk with respect to the guideline. In order to achieve this, the possibility theory may be used.

Although, fuzzy set theory and possibility theory has been applied to health risk assessment studies by various researchers, the decision-making phase has not been investigated in detail. One approach proposed by Guyonnet et al. (1999) and Guyonnet et al (2003) uses the possibility and the necessity measures to check the acceptability of the fuzzy risk with respect to the compliance guideline.

We selected three hypothetical membership functions and the effect of the shapes of these functions on the possibility, and the necessity measures, and also on decision-making process is discussed first. Although the most likely risk and the support of the membership function are the same for each of these fuzzy risks, the possibility and the necessity measures of a proposition vary significantly. Using three different cases chosen in this study, it is shown that when the compliance criterion is located on the right leg of the fuzzy risk, the necessity measure for each case varies significantly. The possibility measures for all three fuzzy risks (see Figure 3.14) are one.

For the situations where the compliance guideline is bigger than the risk corresponding to a membership function of one, the possibility measure is always one regardless of the shape of the membership function. However, if the compliance guideline is smaller than the most likely risk (i.e., risk value that has a membership value of one) the possibility measure is affected by the shape of the membership function too (see Figure 3.18). In the case where possibility measure is less than one, all the evidence that supports the proposition also supports the complement of the proposition too, and this results in a necessity measure of zero. A necessity measure of zero indicates that the evidence that supports the impossibility of the complement of the proposition is zero. In general, possibility measure is based on the available non-negative evidence (i.e., any evidence that supports the proposition) thus it provides an optimistic measure. Depending on if the compliance guideline is located on the left or the right leg of the fuzzy risk, some or all of the evidence that supports the proposition may well support the complement as well. Thus, utilizing only the possibility measure for health risk assessment studies may not be reasonable. The necessity measure provides a more conservative assessment, so it might be preferred to the possibility measure in health risk assessment studies. However, the possibility measure also conveys some valuable information about the fuzzy risk. Hence, a fuzzy measure which utilizes both the possibility and the necessity measures might be more informative. One such measure, the risk tolerance measure is proposed in this study.

The risk tolerance measure combines the possibility and the necessity measures into a single measure. It generates results which are more optimistic than those generated with only the necessity measure and more pessimistic than those generated with only the possibility measure since it weights the possibility measure with a factor smaller than one and also combines the possibility and the necessity measures. Using only the necessity measure is the most conservative approach, however it ignores the possibility measure totally. The possibility measure conveys some valuable information by considering any evidence that supports the proposition even though some may support the complement at the same time (Kikuchi and Pursula 1998). Our goal in proposing the risk tolerance measure, was to include such evidence into the decision-making process. The risk tolerance measure weighs the possibility measure according to the ratio of the evidence which supports the proposition, $\boldsymbol{P}$ to the total evidence. Establishment of a standard procedure for evaluating compliance of a fuzzy risk with respect to a compliance guideline requires careful examination of the results of real case possibilistic and hybrid health risk assessment studies.

3.6 CONCLUSIONS

In health risk assessment studies, it is very important to include all available information into the mathematical models. Traditionally, the available information is interpreted in a probabilistic sense and probability theory has been used to integrate this information into mathematical models. However, the form of the information must guide the analyst in deciding which mathematical tool to use. The probability theory, the possibility theory, a combination of these two, or another method may be more appropriate for the specific study.

We proposed two hybrid approaches which allow both probabilistic and fuzzy information about the parameters of the model to be integrated into health risk estimates. These approaches will be useful when model parameters may best be characterized by a combination of random variables, fuzzy variables, or random variables with fuzzy parameters.

When possibilistic or hybrid models are used in human health risk assessment studies the resulting risk is characterized by a fuzzy number. In order to evaluate acceptability of the resulting fuzzy risk, it has to be compared with respect to a crisp compliance guideline. Since possibilistic and hybrid methods are relatively new compared to probabilistic methods, decision-making in human health risk assessment studies which result in fuzzy risks is a recently developing research area as well. We proposed a new measure, the risk tolerance measure, to evaluate the compliance of a fuzzy health risk with a crisp guideline. In our opinion, the risk tolerance measure which is a combination of the possibility measure and the necessity measure is more informative compared to the existing alternatives. Usefulness of the risk tolerance measure needs to be strengthened by utilizing it in real-world problems.

The rest of the thesis deals with uncertainty modeling in groundwater resources management problem. Various approaches which utilizes fuzzy set theory concepts are proposed and applied to the groundwater resources management problem in the Savannah region.

4 GROUNDWATER RESOURCES MANAGEMENT IN THE SAVANNAH REGION

In Chapter 3, we investigated solutions to various problems in health risk assessment area and proposed a new method which may be useful in measuring acceptability of a human health risk under uncertainty using fuzzy analysis. In the second part of this thesis the uncertainty issues associated with a groundwater resources management problem are investigated. Thus, in this chapter we present a site specific application and propose a coupled simulation-optimization model that may be used in evaluating optimum additional groundwater potential in the Savannah region. A framework for fuzzy multi-objective decision-making which is coupled with the deterministic optimal solution is also included in this chapter.

4.1 INTRODUCTION

The Upper Floridan Aquifer (UFA) of southeast Georgia is susceptible to saltwater intrusion. This aquifer is a primary source of drinking and industrial process water throughout 24 counties of coastal Georgia. Pumping from this aquifer at various locations has already lowered groundwater levels, resulting in saltwater intrusion of the aquifer from underlying strata containing highly saline water at Brunswick, GA, and encroachment of seawater into the aquifer at the northern end of Hilton Head Island, S.C. (Clarke and Krause 2000; Clarke and Krause 2001; Garza and Krause 1996). This saltwater contamination has constrained further development of the UFA in the coastal area and created competing demands for the limited supply of water (Leeth et al. 2003). Nevertheless, the coastal area of Georgia continues to grow and the best water resources management strategy needs to be identified in order to sustain continuous source of potable water supply in the region.

Coastal Georgia is divided into three subareas (i.e., northern, central, and southern) which are separated primarily because of their geologic characterization (EPD 1997). The Georgia Environmental Protection Division (EPD), in cooperation with the United States Geological Survey (USGS) conducted various groundwater management studies in the region to better define the mechanisms of groundwater flow, to understand intrusion of saltwater in the UFA, and to assess long-term groundwater supply and quality. As components of these studies, various groundwater flow models have been developed. The Savannah Area Model is one of these models which was developed in 1996 (Garza and Krause 1996) to gain greater resolution for the Savannah – Hilton Head Island area. The 24-county coastal Georgia area and the Savannah Area Model boundary are given in Figure 4.1. Here we are only concerned with groundwater resources management problem in the Savannah Area Model domain (hereafter referred to as the Savannah region).

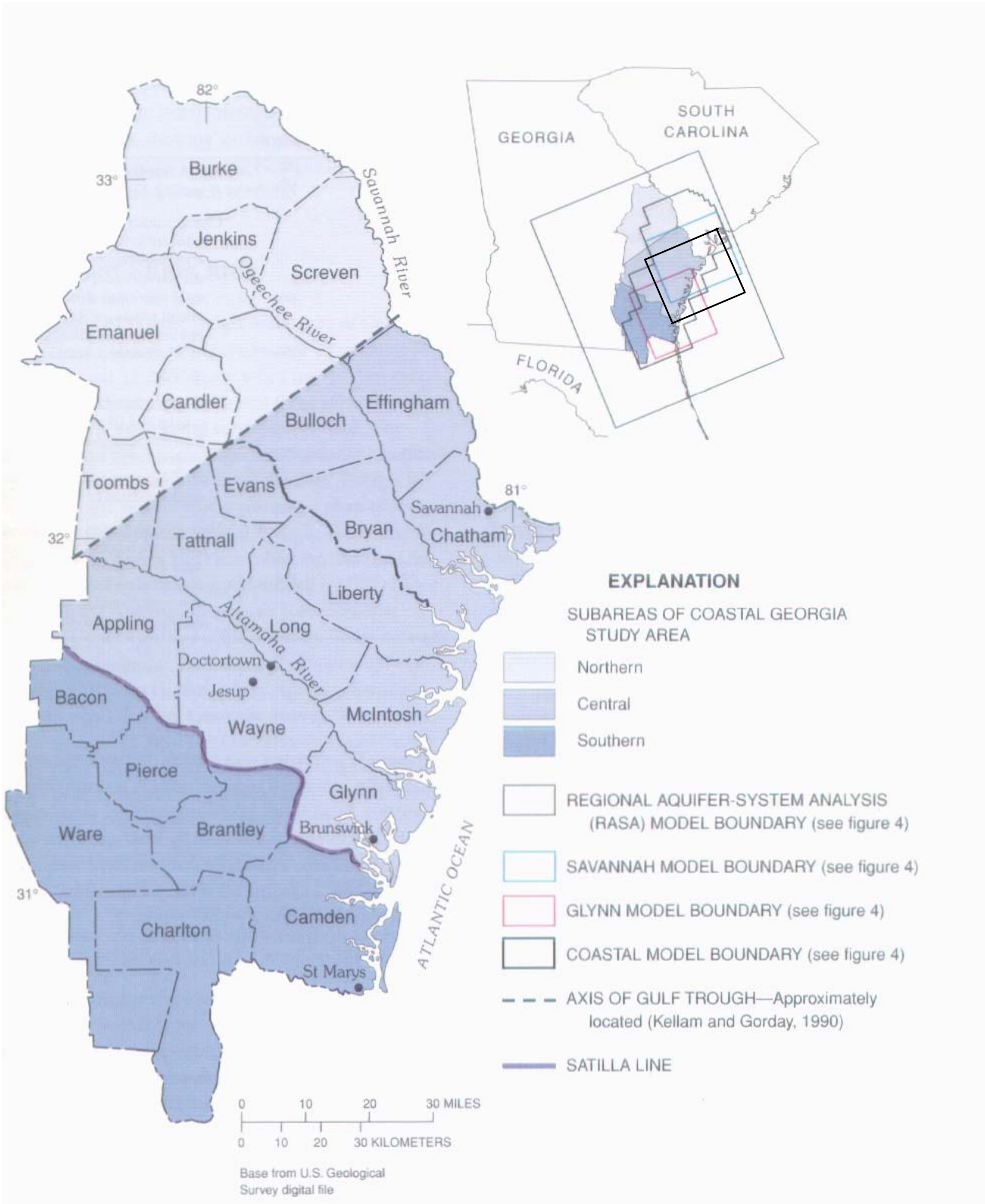


Figure 4.1 The 24-county coastal Georgia, and the Savannah Area Model boundary (Clarke and Krause 2000)

In the Savannah region, there is one confirmed location of saltwater intrusion: the northern end of Hilton Head Island (Clarke and Krause 2000; Clarke and Krause 2001; EPD 1997; Garza and Krause 1996). Further south, near the eastern end of Bull Island in South Carolina, geological conditions are favorable for saltwater to enter the aquifer as well. According to USGS and EPD studies some wells in this area show higher than expected salinity levels. Furthermore, based on groundwater modeling, the USGS reports that saltwater may also be entering the aquifer offshore from Tybee Island (EPD 1997). Thus, these two locations, Bull and Tybee Islands, can also be identified as critical locations at which saltwater intrusion may occur. According to EPD, since saltwater encroachment at the northern end of Hilton Head Island (hereafter referred to as indicator site) is confirmed, development of groundwater in the UFA in the region is constrained by the criteria that further development in the region should not yield additional drawdown at the indicator site (Clarke and Krause 2000; EPD 1997; Garza and Krause 1996).

In this chapter, our first goal is to develop a coupled simulation-optimization model which can be used in evaluating additional groundwater withdrawal potential of the UFA, the LFA, or both in the Savannah region without further aggravating the saltwater intrusion at the northern end of Hilton Head Island. Our second goal is to use the results of the coupled simulation-optimization model in a fuzzy multi-objective decision-making framework to determine the best groundwater resources management strategy for the region. In order to not deviate significantly from earlier studies we have selected the primary constraint in our study similar to the previously conducted studies (Clarke and

Krause 2000; Garza and Krause 1996) and current management strategies of the region. That is “not aggravating the saltwater contamination problem at the northern end of Hilton Head Island” is the main constraint in our simulation-optimization model. We have specifically chosen to keep this criterion in order to evaluate what can and cannot be done in the region while satisfying this condition. This choice of ours does not imply that we agree with the premise behind this strategy as a saltwater intrusion control measure for the region. To us, it is the current practice for the region which was put into force by EPD after considerable work. As users we would like to identify what the management options are if we work within the bounds of this strategy. In our analysis we assume that the existing conditions (i.e., locations and pumping rates of existing wells) will remain the same and we calculate the additional optimum pumping rates in the selected hypothetical well locations.

To understand this criterion and saltwater intrusion problem in the Savannah region better, in Section 4.2, we first review the current groundwater management strategies developed by EPD and the recent USGS studies which were conducted to assist the development of these management strategies. In this review, we point out the critical issues identified by USGS and EPD. In light of these studies, the goals of the current study and our motivation are also given in Section 4.2. A short summary of the hydrogeology of the region and the coupled simulation-optimization modeling approach we propose are provided in Sections 4.3 and 4.4, respectively. The results of the coupled simulation-optimization model are provided in Section 4.5. The fuzzy multi-objective decision-making framework which is proposed to evaluate the best management strategy is given in Section 4.6, and Section 4.7 concludes this chapter.

4.2 Current Groundwater Management Practice in the Savannah Region and Goals of This Study

First, we review two recent studies conducted by U.S. Geological Survey in the Savannah region and Georgia EPD's Interim Strategies related with managing groundwater resources in the region. Then, we state our goals in this study and our motivation in choosing them in this section.

4.2.1 Review of Recent U.S. Geological Survey Studies

As mentioned previously, EPD collaborated with USGS to evaluate the potential for obtaining additional groundwater withdrawal in the Savannah region. USGS conducted various studies (Clarke and Krause 2000; Clarke et al. 2004; Garza and Krause 1996), and the States of Georgia and South Carolina and other stakeholders used the results of these studies to formulate regulatory actions and management plans for the Floridan Aquifer system. Two recent studies conducted by USGS for the Savannah region are "Water-Supply Potential of Major Streams and the Upper Floridan Aquifer in the Vicinity of Savannah, Georgia" (Garza and Krause 1996) and "Design, revision, and application of ground-water flow models for simulation of selected water-management scenarios in the coastal area of Georgia and adjacent parts of South Carolina and Florida"

(Clarke and Krause 2000). The analysis used in these studies is summarized in the following paragraphs because Georgia EPD uses the results of these studies to evaluate groundwater withdrawal permit requests and to formulate interim water management strategies for coastal Georgia (EPD 1997). More recent groundwater modeling studies currently under investigation by USGS in the region, such as multi-phase modeling efforts, are not included in this review since the outcome of these studies were not in the public domain yet.

The first study referenced above (Garza and Krause 1996) is conducted by USGS and the Chatham County-Savannah Metropolitan Planning Commission (MPC). In that study, to evaluate groundwater resources in the Savannah region, a groundwater flow model, the Savannah Area Model, is developed. The water supply potential of the UFA is constrained by groundwater piezometric head declines at indicator sites (two in Brunswick area and one in Hilton Head Island). One of these indicator sites, the northern end of Hilton Head Island where seawater encroachment is observed lies inside the Savannah Area Model boundary. The indicator sites in Brunswick, GA are outside the Savannah Area Model boundary. The groundwater model developed by USGS (Garza and Krause 1996) is used in the study, as well as others, to estimate the maximum additional pumping rate that can be applied to the UFA without lowering the piezometric head at the indicator sites. Since simulation of "zero change" is not practical to measure because of computational accuracy, a simulated value of groundwater level decline of less than or equal to 0.05 ft at indicator sites is adopted to represent a "no change" or "zero change" condition. In the USGS study, 24 randomly distributed and areally dispersed pumping well locations are selected. Then at each one of these potential pumping well locations the pumping rate is increased until the drawdown at one of the indicator sites exceeds 0.05 ft. The final pumping rate that satisfies this criterion is recorded as groundwater potential at that location. As a result of this analysis, a map showing iso-contours of groundwater withdrawal potential is constructed. It should be emphasized here that in this approach for the development of additional groundwater withdrawal potential iso-contours, the analysis considers only one of the 24 potential wells pumping at a time. Groundwater extraction from multiple wells in the region and their hydraulic interactions with each other are not considered, and this is an important drawback of their approach.

In the second study (Clarke and Krause 2000) the revisions, modifications, and updates to three of the most recently developed coastal models – the Regional Aquifer-System Analysis (RASA) Model (Krause and Randolph 1989), the Glynn County Model (Randolph and Krause 1990), and the Savannah Area Model (Garza and Krause 1996) are described. Here, we only summarize the Savannah Area Model section of the study.

Together with the revision and modifications, the report (Clarke and Krause 2000) documents the results of 32 computer simulations of hypothetical pumping scenarios. The scenarios are developed by EPD and the Chatham County-Savannah Metropolitan Planning Commission to evaluate the water management alternatives in coastal Georgia. The potential for additional development of groundwater from the UFA is again constrained by drawdown at the northern end of Hilton Head Island for the Savannah region. Groundwater pumping changes for various scenarios are developed based on changes in permitted or actual withdrawals from the UFA. Fifteen of these scenarios are developed by EPD for the Savannah – Hilton Head Island area to quantify the reduction in groundwater withdrawal that is necessary to allow sustainable use of the UFA at Savannah. Sustainable, in this case, refers to the following condition: "Saltwater would not be flowing toward Savannah from the point of encroachment at Port Royal Sound." One of the conclusions of the study is that the potential for additional development of groundwater from the UFA is constrained by groundwater level declines at known locations of saltwater contamination (i.e., Hilton Head Island for the Savannah Area Model). It is also concluded that the farther away the pumping is located from Hilton Head Island, the smaller will be the effect on the groundwater levels in the UFA at Hilton Head Island and thus on saltwater intrusion. This is the expected outcome which makes hydrologic and engineering sense.

4.2.2 Georgia Environmental Protection Division's Interim Strategies and Critical Issues in the Savannah Region

In February, 1996, in order to protect the UFA from further saltwater intrusion within the Southeast Georgia region, EPD developed an "Interim Strategy for Managing Salt Water Intrusion in the UFA of Southeast Georgia." This strategy is primarily based on the USGS studies. Between March and April 1996, EPD held nine public meetings and received over 400 responses to this proposed strategy. Among other concerns, a primary demand from the users was on "clarification of the cost-benefit impact of the proposed

Interim Strategy." In the review process, there was the perception that the proposed interim strategy could create adverse economic impacts on users. Another important issue raised in the comments was that the proposed Interim Strategy was unfair to some categories of users, particularly those that are close to Hilton Head Island. After receiving these comments, EPD decided to conduct further research and analysis. On December 20, 1996 a Revised Interim Strategy was released by EPD. Three public meeting were held in January 1997, and around 90 comments were received. Based on these comments, EPD conducted other studies in corporation with USGS and the School of Policy Studies of Georgia State University to improve the proposed Interim Strategy. At the conclusion of these studies, EPD released the final version of the Interim Strategy on April 23, 1997.

In the "Interim Strategy for Managing Salt Water Intrusion in the Upper Floridan Aquifer of Southeast Georgia" (EPD 1997), EPD has identified an area for enforced protection near the City of Savannah. This critical region has been termed the northern-capped area and included all of Chatham County, Effingham County south of Highway 119, and Bryan County south of Fort Stewart (EPD 2004). Thus, since 1997, EPD has not issued permits for groundwater withdrawal from the UFA in the northern-capped area unless such water is reallocated from a permit reduction elsewhere within the northern-capped area (EPD 2004). Outside the northern-capped area, EPD did not cap withdrawals to any specific level, but decided to allow reasonable additional pumping from the UFA until such time as it can be shown that such withdrawals will not result in unacceptable adverse influence on the two critical locations (i.e., Hilton Head Island and Brunswick).

In the "Supplement to the Interim Strategy for Managing Salt Water Intrusion in the Upper Floridan Aquifer of Southeast Georgia" (EPD 2001), the LFA is listed as one of the alternative water sources in coastal Georgia. Thus, to manage additional groundwater withdrawals from the LFA, EPD issued the "Interim Strategy for Permitting Lower Floridan Withdrawals In Coastal Georgia" (EPD 2003). In this Interim Strategy, EPD proposed that any new permit to the LFA may be approved only if the applicant meets a standard of causing no net negative impact to the UFA (i.e., no net decrease in the amount of available water in the UFA measured in terms of additional local piezometric head declines). EPD required all permit applicants to demonstrate this no net negative impact criteria to the UFA in their permit application.

4.2.3 Goals of the Current Study and Motivation

One of the key issues identified in the USGS study (Clarke and Krause 2000) is that the negative impact that maybe observed on groundwater levels in the UFA at Hilton Head Island due to pumping is a function of the pumping well location. This conclusion is important in determining spatial distribution of groundwater availability throughout the Savannah region. This information may provide guidance for future industrial groundwater users in their site selection processes. In order to demonstrate the spatial distribution of additional groundwater availability, the model domain is divided into subareas and each one of these subareas is represented by a potential well. Using the coupled simulation-optimization model the optimum additional groundwater withdrawal rates from each one of these potential wells are determined assuming that they operate simultaneously. The coupled simulation-optimization model and the results of this approach are given in Sections 4.4.2.1 and 4.5.1, respectively. It is important to note here that this approach considers multiple extraction wells which are potentially operating at the same time.

Our aim in the first part of our analysis is to provide the spatial distribution of the additional groundwater availability in the Savannah region. The results obtained from this analysis will shed light on managing long-term planning goals for future public and commercial groundwater users. In the first analysis, we used 70 simultaneous potential pumping wells that are distributed in the region. The purpose is to evaluate the outcome of the additional groundwater extraction from evenly distributed simultaneous extraction points in the region. This choice may be viewed as having considered demand locations beyond the current needs of the region. We are aware of that; however, coastal area of Georgia has been experiencing a fast growth in population and commerce (Martin et al. 2005; Provost et al. 2005; U.S. Geological Survey 2000) and we are of the opinion that the water demand in the region will continue to increase and long-term solutions, or at least information on long-term demands should be the key in future planning and management studies. Evaluation of the additional groundwater availability at a limited number of specific demand locations is considered as the scope of the second part of our analysis.

Another key concern indicated by the USGS (Garza and Krause 1996) is that in their analysis they assumed that all additional groundwater withdrawal will occur from only one cell. The simulation results of cell-by-cell extraction are aggregated to yield the final outcome, i.e., the development potential map. Thus, simultaneous pumping from different locations is not considered. This approach does not provide the total amount of the additional groundwater potential that maybe available to multiple users in the Savannah region. We realize that calculating the total additional groundwater withdrawal potential in the Savannah region is important in terms of providing guidance in evaluating multiple groundwater withdrawal permit applications. In order to evaluate simultaneous additional groundwater withdrawal potential with a number of wells less than the ones used in our first stage analysis, we have considered a hypothetical groundwater withdrawal application case for a limited number of future users. Each one of these demand points is represented by a potential well location. Then a coupled simulation-optimization procedure is formulated to calculate allowable additional groundwater withdrawal rates from each one of these potential well locations assuming that all of them will be extracting groundwater at the same time. The analysis and the results of this second stage are given in Sections 4.4.2.2 and 4.5.2, respectively.

Finally, EPD, in the proposed Interim Strategies, clearly indicates that groundwater withdrawals from the UFA in the vicinity of Savannah (i.e., the northern-capped area) have a high risk of increasing saltwater intrusion at Hilton Head Island. Thus, additional groundwater withdrawal from this region is not allowed. Based on this observation, one of our goals in this study is to identify the critical region from which additional groundwater extraction will cause aggravation of the saltwater encroachment problem at Hilton Head Island. We will refer to this zone as the “zero pumping zone” in the rest of this study. According to our definition, within the "zero pumping zone" no additional groundwater extraction will be allowed since it will aggravate the saltwater intrusion problem at the indicator site. In the analysis provided here, we are interested in understanding what factors contribute to the extent of the "zero pumping zone" or how the size of this zone can be managed effectively. More importantly we want to evaluate the effect of the extent of the "zero pumping zone" on the distribution of groundwater extraction outside this zone or vice-versa. Moreover, there may be regions in the Savannah area where demand exceeds available additional groundwater withdrawal potential, which is determined as the outcome of optimal pumping analysis provided in this study. Thus, alternative sources of water supply have to be considered for the Savannah region. EPD identifies the LFA as an alternative source of water supply in the region. However, EPD also enforces various restrictions for the utilization of the LFA as described in Section 4.2.2. Since, the LFA is suggested as an alternative water supply for the region, it is also our goal to demonstrate the attractiveness or overall potential of the LFA in supplying additional groundwater to the region as an alternative source to the UFA.

In this study, based on these concepts, the additional groundwater withdrawal from the Floridan aquifer system is evaluated using two consecutive approaches: (i) coupled simulation-optimization model to identify optimum groundwater withdrawal from the UFA, the LFA and, UFA+LFA; and, (ii) fuzzy multi-objective decision-making analysis which imbeds the optimal solutions obtained in the first stage into the additional management objectives that are important in the Savannah region. For this purpose, a heuristic approach (i.e. a fuzzy multi-objective decision-making process in which the objectives are identified in terms of linguistic terms) is designed to evaluate overall performances of the alternative management strategies (i.e., groundwater extractions from the UFA, the LFA, and UFA+LFA) with respect to the various heuristic objectives that we identified considering the current needs of the region. This decision-making framework is explained in Section 4.6.

4.3 HYDROGEOLOGY IN THE SAVANNAH REGION

The Savannah area is underlain by several thousand feet of consolidated sedimentary rocks and unconsolidated sediments that range in age from Late Cretaceous to Holocene (Miller 1986). The rocks and sediments dip seaward and generally thicken in that direction. The principal hydrogeologic units in this area, in descending order are the surficial aquifer, the upper confining unit, and the Floridan aquifer system (Garza and Krause 1996). Aquifers and confining units in Savannah, GA, and in Hilton Head Island, S.C. are shown in Figure 4.2. Hydrogelological conditions in the aquifers and confining units are summarized below.

The surficial aquifer consists of interbedded sand, clay, and limestone of Miocene and younger age (Garza and Krause 1996). The aquifer is generally under water table conditions; however locally it is semiconfined to confined (Clarke and Krause 2000). The surficial aquifer is used primarily for domestic lawn irrigation, and is the principal source of drinking water in some rural areas (Garza and Krause 1996).

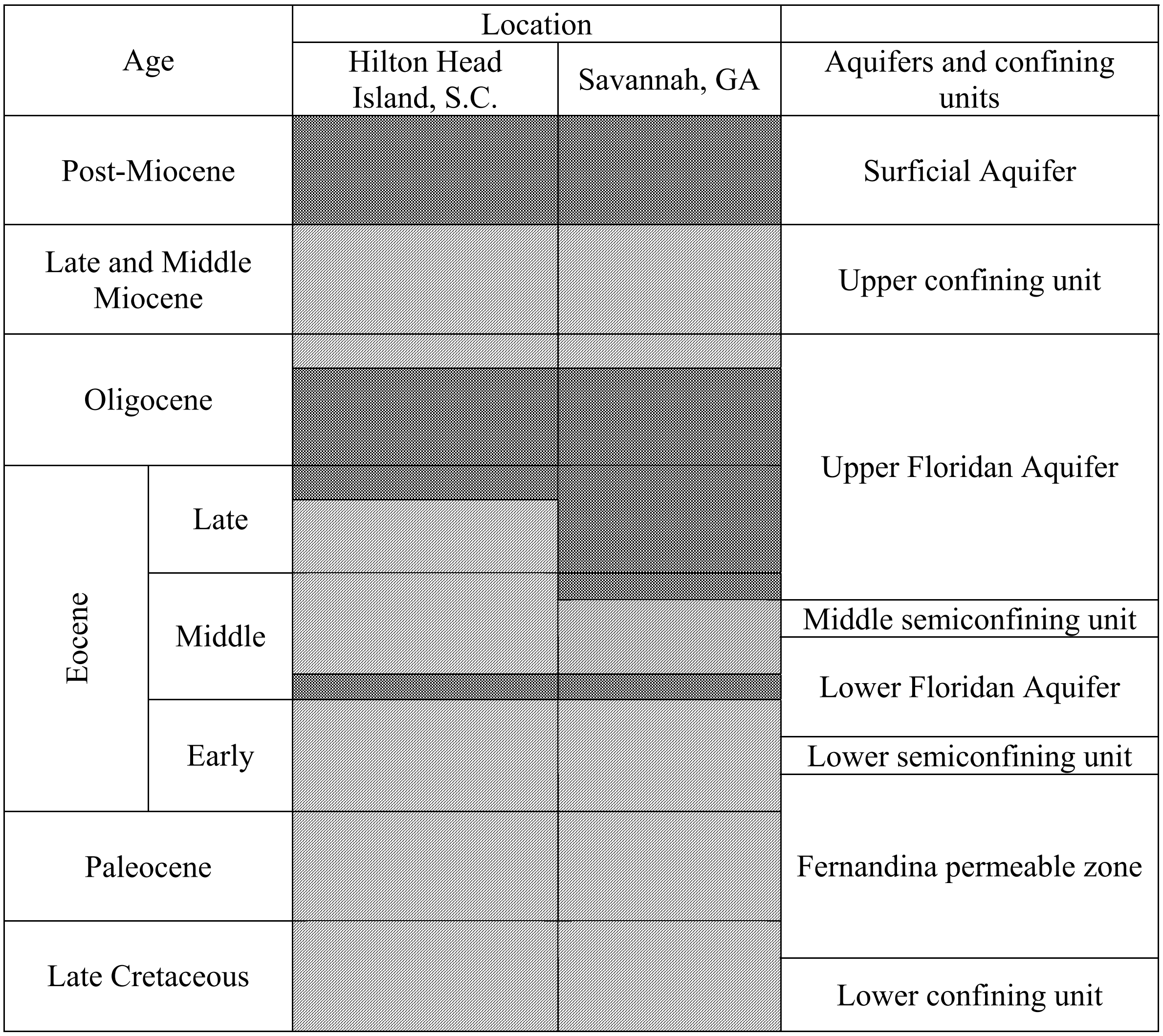


Figure 4.2 Aquifers and confining units in Savannah, Ga., and Hilton Head Island, S.C. (modified from Krause and Randolph (1989))

The upper confining unit lies under the surficial aquifer. The upper confining unit consists of clay and other clastic sediments of low to moderate permeability (Randolph and Krause 1990). The thickness of the upper confining unit ranges from 50 ft in northern Screven County, GA and in coastal South Carolina to about 400 ft (Garza and Krause 1996). Although the unit includes water bearing zones (Krause and Randolph 1989), because these zones are present only locally, the unit is considered as a confining unit in the Savannah Area Model (Garza and Krause 1996).

The Floridan Aquifer System consists of two water-bearing units, the Upper Floridan Aquifer (UFA) and the Lower Floridan Aquifer (LFA) (Garza and Krause 1996; Krause and Randolph 1989; Miller 1986; Payne et al. 2005). The Floridan Aquifer System consists primarily of carbonate rocks of Oligocene and Eocene age (Garza and Krause 1996). As can be seen from Figure 4.2, the UFA and the LFA are separated by a semiconfining unit, the middle semiconfining unit.

The UFA consists mainly of carbonate rocks of Oligocene and late Eocene age that crop out northwest of the Savannah Area Model domain (Garza and Krause 1996). Depth to the top of the UFA in the study area ranges from less than 100 ft to about 450 ft, and increases toward the south (Miller 1986). The aquifer thickness in the model area ranges from less than 1 ft in the northern part to about 600 ft in the southern part (Miller 1986). The UFA is highly productive. Ranges of transmissivities in the vicinity of Savannah, in Hilton Head Island, and generally in the model domain is summarized in Table 4.1. The UFA is the major source of freshwater in the Savannah region.

Table 4.1 Range of transmissivities

Location	Transmissivity (ft^2/day)
The vicinity of Savannah	25,000-50,000[a]
Hilton Head Island	around 50,000[b]
Northern part of the model domain	5,000-10,000[c]
Southern part of the model domain	> 100,000[c]
Model Domain	860-205,000[d]

[a] Bush and Johnston (1988) and Krause and Randolph (1989)
[b] Smith (1988)
[c] Krause and Randolph (1989)
[d] Clarke and Krause (2000)

The UFA is underlain by a semiconfining unit which separates it from the LFA. The middle semiconfining unit consists of low-permeability limestone and dolomite of middle to late Eocene age (Garza and Krause 1996). Thickness of this unit within the model domain ranges from less than 100 ft to more than 600 ft (Miller 1986).

The LFA lies under the middle semiconfining unit. The LFA is composed mainly of dolomitic limestone of early and middle Eocene age (Garza and Krause 1996; Payne et al. 2005). Based on simulations by Krause and Randolph (1989) transmissivity of the LFA range from about 2,000 to 80,000 ft^2/day. Depth to the top of the LFA in the model area ranges from about 600 to 1,000 ft (Miller 1986). The LFA is not widely used for water supply in coastal Georgia because it is deeply buried and contains saltwater in places; however the LFA is a source of freshwater in the Savannah area (Garza and Krause 1996). In the northeastern part of the study area, where the UFA is thin or absent, the LFA is a major source of water supply (Garza and Krause 1996). More detailed information about the hydrogeology in the Savannah region can be found in Garza and Krause (1996) and Kentel et al. (2005).

4.4 Coupled Simulation-Optimization Model For Savannah Region

The coupled simulation-optimization model is used to identify optimum additional groundwater withdrawal potential in the region. First, the simulation model is explained. Then, the coupled simulation-optimization model and the methodology we used to couple these two models are provided.

4.4.1 Simulation Model

The Upper Floridan and the Lower Floridan aquifers compose the Floridan aquifer system in the Savannah region. This aquifer system is the primary source of freshwater in the coastal area of the State of Georgia. Development of the aquifer system as a freshwater supply source began in about 1880, and the freshwater extraction increased as the region continued to grow. In their modeling study, Garza and Krause (1996) reported a total extraction rate of 120.2 MGal/day of which 109.2 MGal/day is from the UFA. More recent pumping rates in the region are identified as 102.1 MGal/day from the UFA and 11.1 MGal/day from the LFA. In the present study, we used the latter extraction rates which yield a total extraction of 113.2 MGal/day. These extraction rates describe the current conditions in the Savannah region.

As reviewed earlier, the USGS has developed a groundwater simulation model for the region, identified here as the Savannah Area Model (Garza and Krause 1996) which utilizes the MODFLOW simulation code (McDonald and Harbaugh 1988). We use this model and its database without any modification in a desktop computer platform utilizing Processing MODFLOW (PMWIN) computational environment (Lee et al. 2000). A brief explanation of the MODFLOW computer code, three-dimensional groundwater flow equations and finite difference formulation used in MODFLOW to simulate groundwater flow are presented in Appendix D.

The USGS Savannah Area Model treats the UFA and the LFA as two semiconfined layers while the unconfined unit above the UFA is treated as a fixed head boundary layer. In the USGS studies, steady state simulations are conducted to represent long-term response of the aquifer system. The Savannah Area Model is extensively calibrated with the field data to determine the aquifer parameters to be used for the aquifer system (Clarke and Krause 2000; Garza and Krause 1996) and it is a well accepted management tool for the region. The Floridan aquifer system is represented as a two-layer, semiconfined aquifer system, and due to the nature of the resulting partial differential equations, the model response can be interpreted as linear. By linearity we mean, the responses (i.e., piezometric head) at the observation points are linearly proportional to extraction rate at the potential wells that may be distributed in the aquifer system.

The study area used in this thesis covers an area of 6,680 square miles (mi^2) and it is the same as the one used in Garza and Krause (1996) (see Figure 4.3). In the USGS model, a 76x88 finite difference grid with square elements is used to idealize the region. Each square element has an area of 1 mi^2. We use the same grid in our model.

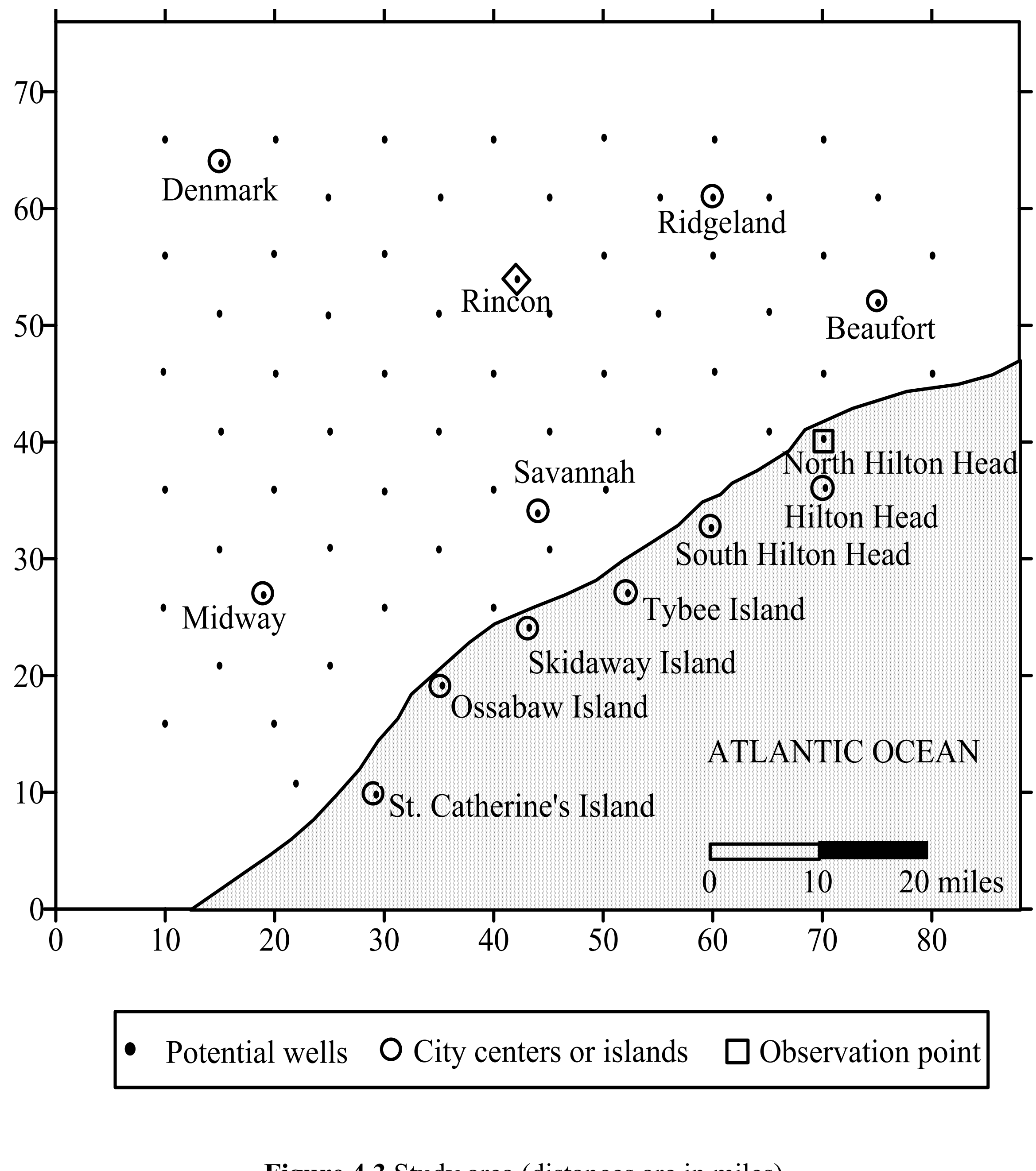


Figure 4.3 Study area (distances are in miles)

Drainage features, hydrogeologic setting, water use, and water quality data for this model can be found in Garza and Krause (1996).

Similar to many other investigative USGS studies, (Hayes 1979; Krause and Randolph 1989; Randolph and Krause 1984; Randolph and Krause 1990; Randolph et al. 1991; Smith 1988) the groundwater flow in the Floridan aquifer system is assumed to be under steady-state conditions representing long-term conditions in the aquifer. Other aspects of the numerical model used in this study, i.e. the database and the numerical model used to solve the system (finite difference formulation in MODFLOW), are identical to those of the USGS studies that are referenced above.

4.4.2 Coupled Simulation-Optimization Model

In this study we are concerned with the additional groundwater withdrawal potential in the Savannah region. As discussed earlier, a total groundwater extraction rate of 113.2 MGal/day already exists in the Savannah region, and the coastal area of Georgia continues to grow causing an increase in this freshwater demand. Since the UFA is currently the primary source of freshwater in the region, potential of the additional groundwater extraction from the UFA needs to be identified. To achieve this goal a coupled simulation-optimization model is proposed.

The coupled simulation-optimization model is used for two different goals: (i) determining the spatial distribution of additional groundwater withdrawal potential within the model domain; and, (ii) evaluating multiple groundwater withdrawal permit applications. The coupled simulation-optimization model that is used in this study is described in the following sections.

4.4.2.1 Coupled simulation-optimization model for determining the spatial distribution of the additional groundwater supply potential in the region

In order to determine the spatial distribution of the additional groundwater withdrawal potential throughout the study area, we divide the model domain into subareas and represent each subarea by a potential well. A total of 70 potential pumping wells are used to represent the subareas in the Savannah region. As can be seen from Figure 4.3, the potential wells are distributed evenly throughout the model domain including several additional wells representing the main cities and islands. We think this arrangement of potential wells may provide a good representation of the subareas considered in the region.

In both of the USGS studies summarized earlier and all of the Interim Strategies developed by EPD, the potential for the additional groundwater development in the Savannah region is constrained by the drawdown at the northern end of Hilton Head Island. Saltwater encroachment has been identified at this indicator site, and the drawdown at this location is used as an indication of a saltwater intrusion problem in the Savannah region. In our optimization model formulation we use the same constraint in order to not diverge from the criteria selected by EPD. A drawdown less than or equal to

0.05 ft in the UFA at the indicator site when compared with existing conditions is considered to be acceptable in the sense of not aggravating the saltwater contamination problem (Georgia Department of Natural Resources 2002).

The response matrix approach is used to embed results of the simulation model into the optimization model while handling the drawdown constraint at the northern end of Hilton Head Island. The response matrix approach is based on the principle of superposition. It is applicable when the system response is linear or approximately linear and the boundary conditions are homogeneous (Das and Datta 2001).

The main objective of the optimization model is maximizing the additional groundwater withdrawal potential in the Savannah region while providing equal opportunity to each potential well location without increasing risk of saltwater intrusion at the northern end of Hilton Head Island. Based on this criterion, the proposed optimization model can be described as:

$$\begin{aligned} &\text{maximize} && \sum_{i=1}^{N}\left[Q_i - w_i \times \left(Q_{avg} - Q_i\right)^2\right] \\ &\text{subject to} && s_{HH}\left(Q_i\right) \le s_{\max} \\ &&& 0 \le Q_i \le Q_{\max} \quad i = 1,2,3,\ldots,N \end{aligned} \tag{4.1}$$

where Q_i is the pumping rate in MGal/day at the potential well i, N is the total number of potential wells, Q_{avg} is the ideal average pumping rate in MGal/day, w_i is the weighting factor, s_{HH} is the total drawdown at the northern end of Hilton Head Island in ft and it is a linear function of pumping at the potential wells, $s_{\max}$ is the maximum drawdown that is allowed at the northern end of Hilton Head Island and, $Q_{\max}$ is the maximum pumping that can be assigned to a pumping well. In our example, N is 70, $s_{\max}$ is 0.05 ft, and $Q_{\max}$ is 10 MGal/day.

As identified in Section 4.2.2, one of the concerns raised in response to the "Interim Strategy for Managing Salt Water Intrusion in the UFA of Southeast Georgia" proposed by EPD in February, 1996 was that the proposed Interim Strategy was unfair to some categories of users, particularly those that are close to Hilton Head Island. In order to avoid such perceptions and implement equal right of groundwater withdrawal throughout the model domain, an ideal average pumping rate, Q_{avg}, is defined. Q_{avg} is approximated using the following formula:

$$Q_{avg} = \frac{\sum_{i=1}^{N} Q_i}{N} \tag{4.2}$$

The objective function used in Equation (4.1) is composed of two terms. The first term maximizes total pumping. The second term in the objective function, $\left(Q_{avg} - Q_i\right)^2$, is a penalty term. The function of this term is to penalize any pumping rate different from Q_{avg}, so that the additional optimum pumping rates from the potential wells are forced to be as close as possible to each other in magnitude. However, as expected, even though a penalty term is used, equal pumping rates throughout the region can not be obtained since the drawdown at Hilton Head Island constraint favors pumping from potential wells which are further inland from Hilton Head Island. This is observed in our modeling results and also reported in USGS studies earlier. In order to control the magnitude of penalty imposed, a weighting factor, w_i, is used in front of the penalty term.

In this study we investigate the impact of using a constant (i.e., $w_i = 1;\ i = 1, 2, ..., N$) or various functions as the weighting factor. In the second case, the term w_i is selected as a function which dependents on the distance between the potential well and the northern end of Hilton Head Island (i.e., indicator site). The function we propose for w_i is as follows:

$$w_i = \left(B - \frac{d_i}{d_{\max}} \right) \tag{4.3}$$

where B is a constant greater than one, d_i is the distance between the potential well i, and the northern end of Hilton Head Island, and $d_{\max}$ is the maximum of all d_i's.

The weighting factor given in Equation (4.3) increases as the distance between the potential well and the indicator site decreases for a given B. Thus, pumping rates different than Q_{avg} at wells close to the indicator site are penalized more. Due to the drawdown constraint at Hilton Head Island, the additional optimum pumping rates from potential wells close to Hilton Head are expected to be lower than Q_{avg}. Thus, according to our model, pumping less than Q_{avg} at wells close to the indicator site will be highly penalized. In the optimization model constructed above, this term emphasizes the importance of equal groundwater extraction rights of users located in the region close to Hilton Head Island.

As the outcome of the coupled simulation-optimization model (when 70 potential well locations are used) roughly circular zones of increasing magnitudes of pumping rates around the indicator site (i.e., the northern end of Hilton Head Island) are obtained. The inner circular zone which is constrained by the zero pumping contour and the shoreline represents the "zero pumping zone." The weighting term given in Equation (4.3) can be used to adjust the size of the "zero pumping zone" around the indicator site. B is a constant greater than 1, and it allows the user to assign various degrees of importance to the weighting factor. For example, if B is chosen as 2 then the weighting factor ranges between 1 and 2. If B is chosen as 6 then the weighting factor ranges between 5 and 6. As the value of the penalty term increases additional optimum pumping rates get close to each other in magnitude (i.e., as w_i increases the additional optimum pumping rates gets closer to Q_{avg}) and the size of the "zero pumping zone" will decrease at the same time. The optimization model will maintain a more uniform pumping rate distribution in the region by shifting some of the additional available groundwater from outer zones towards areas close to Hilton Head Island. As an end effect, the "zero pumping zone" will get smaller while extraction wells pump higher near the indicator site compared to the cases in which the penalty term is smaller. The impact of using different values for weighting factors on the additional optimum pumping rates is provided in Sections 4.5.1 and 4.5.2.

The first constraint of the coupled simulation-optimization model given in Equation (4.1) forces the drawdown at the indicator site to be less than $s_{\max}$. Drawdown at the indicator site due to pumping at potential wells, s_{HH}, is calculated using the response matrix approach. The second constraint restricts the pumping rates to be positive and less than the maximum pumping, $Q_{\max}$.

Summation of the additional optimum pumping rates from the potential wells, $\sum_{i=1}^{N} Q_i$, yield the total additional groundwater withdrawal potential in the Savannah region. Thus the total groundwater withdrawal in the Savannah region can be calculated by adding the current groundwater extraction rate, 113.2 MGal/day, to this sum.

4.4.2.2 COUPLED SIMULATION-OPTIMIZATION MODEL FOR EVALUATING MULTIPLE GROUNDWATER WITHDRAWAL PERMIT APPLICATIONS

The purpose of the model described above is to identify the additional groundwater supply potential of the region. As described above, in this first analysis, 70 pumping locations that are distributed within the region are used. While choosing 70 pumping locations our intention is not to represent the actual demand locations of the region, it is to estimate the overall groundwater supply potential of the region in the long run. Assuming that: (i) exponential growth in the region will eventually require utilization of all the available groundwater in the region; (ii) availability of groundwater to all of its users should be independent of their location; and, (iii) implementation of groundwater management strategies considering only the current needs of the region will constrain sustainable development in the region, we discretize the model domain into representative subareas and calculate simultaneous pumping rates from each one of these subareas. The spatial distribution of the resulting available groundwater supply potential is presented as equal pumping rate contours in Section 4.5.1. We are aware of the fact that the future development, consequently the water demand, in the region will vary spatially. However, in terms of long-term planning, all the subareas within the model domain have potential for future development, thus should have equal opportunities for access to groundwater. Our analysis will provide preliminary guidance in evaluating the local groundwater availability and help investors plan their future developments in the region. The second goal of this analysis is to provide a methodology as guidance in granting specific groundwater permits for a few users in the region. Since we do not have access to the real groundwater permit applications, we demonstrate this analysis for a hypothetical case as described below.

Hypothetical Case: There are six groundwater withdrawal permit applications in the Savannah region for the coming planning period. These permit applications are at Rincon, Bloomingdale, Marlow, Ridgeland, Denmark, and Hinesville (hereafter referred as demand locations). Bloomingdale, Denmark, and Hinesville apply for 2 MGal/day while Marlow and Ridgeland apply for 1 MGal/day additional groundwater withdrawal. The City of Rincon applies for additional groundwater withdrawal to extend its municipal water supply system. However, the municipal water supply of Rincon is given as a function of its population among other variables. In the literature, various probabilistic population forecasting methods have been developed (Alho 1997; Alho and Spencer

1985; Lutz et al. 2001; Sanderson et al. 2003). These probabilistic population forecasting methods may be used to determine probabilistic water demand of a city. Let's assume that the City of Rincon has conducted extended probabilistic population forecast studies and evaluated its additional groundwater demand as a stochastic process. Based on these studies let's assume that discrete demands and their associated probabilities are available for Rincon: 1.5 MGal/day, 1.3 MGal/day, and 1.1 MGal/day with probabilities 0.7, 0.2, and 0.1, respectively. This set of six permit applications constitutes our hypothetical case.

The coupled simulation-optimization model given in Equation (4.1) can be used to evaluate the additional groundwater withdrawal potential for these six demand locations (i.e., $N = 6$ in Equation (4.1)). For this hypothetical case, groundwater withdrawals at these six demand locations are calculated for three different situations: (i) groundwater is extracted from the UFA; (ii) the LFA; and, (iii) UFA+LFA. Again the analysis is conducted using various w_i values for each one of these cases. The results of the numerical simulations are given in Section 4.5.2. In the final decision-making process for the hypothetical case other objectives are also included into the analysis and a heuristic multi-objective decision-making framework to select the best groundwater management strategy among available alternatives (i.e., groundwater is extracted from the UFA, the LFA, UFA+LFA using various weighting factors for the penalty term) is used.

4.5 RESULTS OF COUPLED SIMULATION-OPTIMIZATION MODEL

In this section, we provide the results of the numerical simulations conducted using the models described above. The following section provides the results for the spatial distribution of the additional groundwater supply potential of the region considering various constraints. Then the results of groundwater supply potential for multiple permit applicants, i.e. the hypothetical case study given in Section 4.4.2.2, are provided.

4.5.1 THE SPATIAL DISTRIBUTION OF THE ADDITIONAL GROUNDWATER EXTRACTION POTENTIAL IN THE SAVANNAH REGION

The coupled simulation-optimization model given in Equation (4.1) is used to determine the additional optimum pumping rates (MGal/day) from the UFA at each one of the 70 potential wells. First, a constant value of one is used for all weighting factors, w_i. The additional optimum pumping rates obtained are represented as equal pumping rate contours in Figure 4.4.

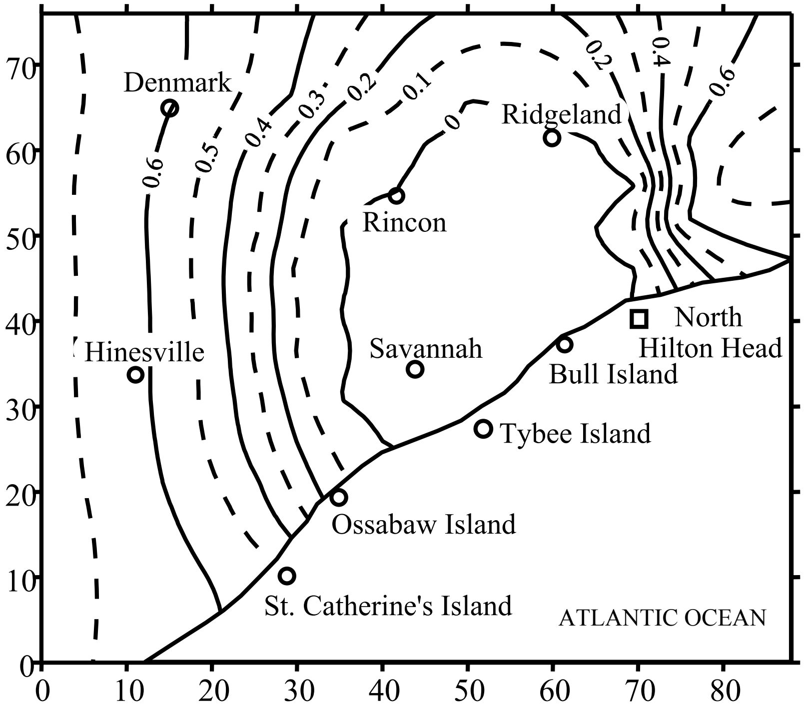


Figure 4.4 Equal pumping rate contours (MGal/day) for the UFA using $w_i = 1, \forall i$ (distances are in miles)

As can be seen from Figure 4.4, as the outcome of the optimization model some of the potential wells are assigned zero pumping, thus a zone from which no pumping is allowed (i.e., "zero pumping zone") is formed around Hilton Head Island. For this case the maximum pumping rate is about 0.7 MGal/day represented by the contour to the west of the City of Hinesville and southeast of the model domain.

To investigate the impact of various weighting functions, three additional simulations are conducted using $w_i = \left(2 - d_i / d_{\max}\right)$, $w_i = \left(4 - d_i / d_{\max}\right)$, *and* $w_i = \left(6 - d_i / d_{\max}\right)$ (hereafter abbreviated as $B = 2$, $B = 4$, and $B = 6$, respectively). The additional optimum pumping rates are calculated using each of these w_i's in the objective function given in Equation 4.1. The equal pumping rate contours for these three cases are given in Figure 4.5.

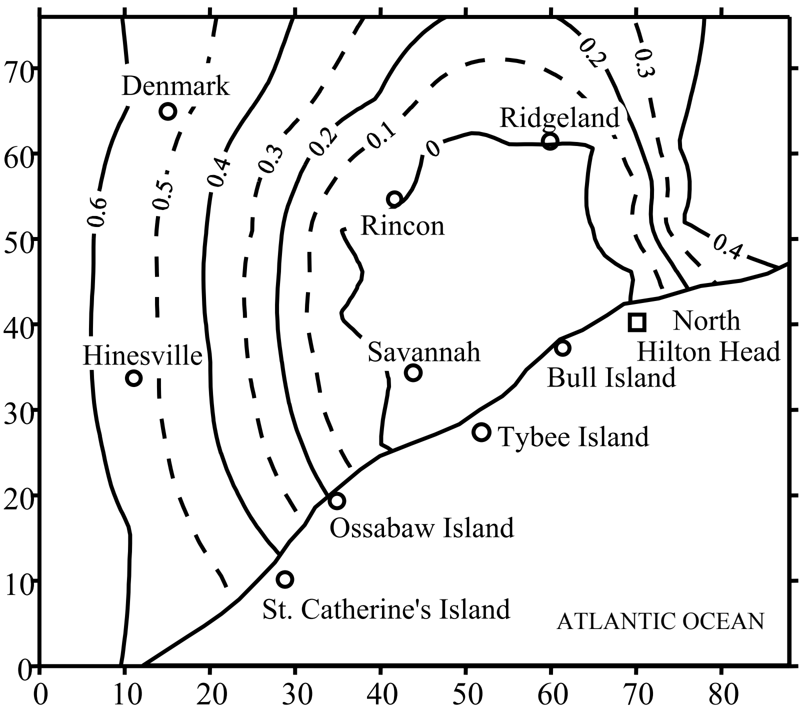
As can be seen from Figure 4.5, utilizing a function which dependents on the distance between the potential well and the northern end of Hilton Head Island as w_i results in the following when B increases:

i. a decrease of the areal extent of the "zero pumping zone,"

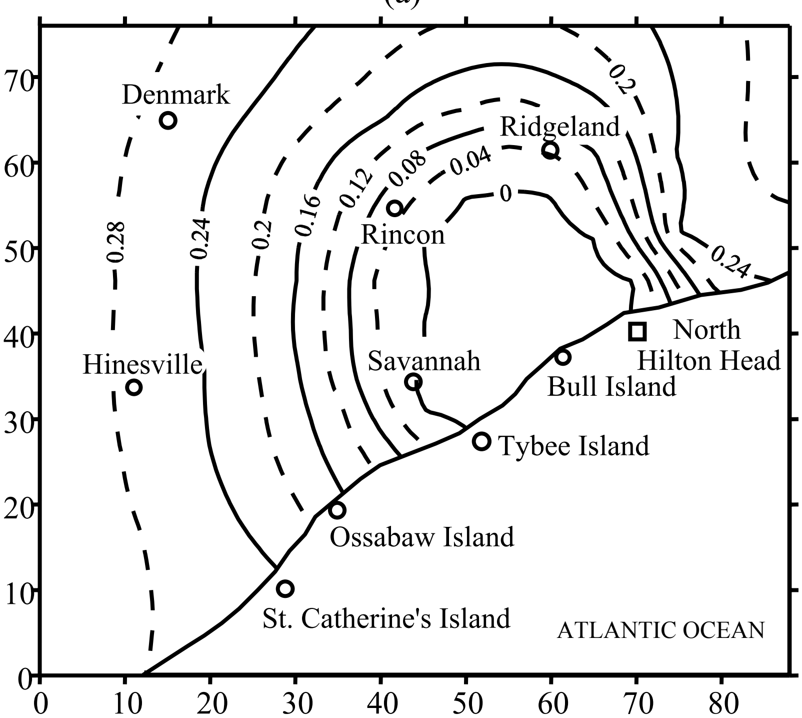
ii. a more uniform pumping rate distribution throughout the model domain,

iii. a decrease of the total amount of additional water that can be withdrawn from the UFA.

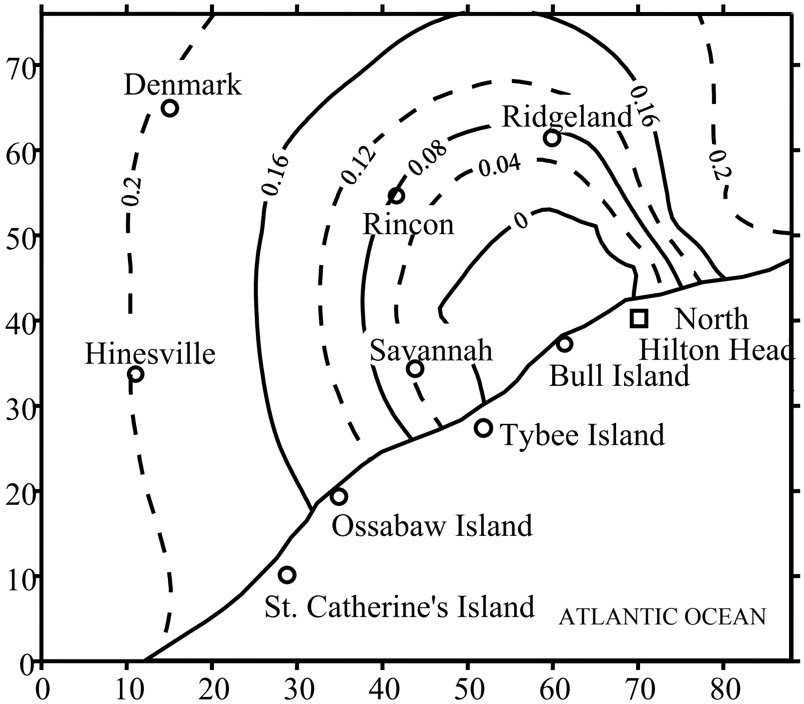
As w_i increases (i.e., as B increases), the pumping rates different than Q_{avr} are penalized more. Thus, the i get closer to each other in magnitude. However, the increase in pumping rates from the potential wells that are close to the northern end of Hilton Head Island causes the drawdown at the northern end of Hilton Head Island to increase significantly. This increase has to be balanced by decreasing the pumping rates from the wells that are located farther away from Hilton Head Island. Thus, there is a trade off between decreasing the size of the "zero pumping zone" and the total additional amount of groundwater withdrawal in the region. As B increases both the "zero pumping zone" and the total additional amount of groundwater withdrawal decrease in the region.


Denmark
Ridgeland
Rincon
Hinesville
Savannah
North
Hilton Head
Bull Island
Tybee Island
Ossabaw Island
St. Catherine's Island
ATLANTIC OCEAN
0.6
0.5
0.4
0.3
0.2
0.1
0
0.2
0.3
0.4
0 10 20 30 40 50 60 70 80
0 10 20 30 40 50 60 70

(a)


Denmark
Ridgeland
Rincon
Hinesville
Savannah
North
Hilton Head
Bull Island
Tybee Island
Ossabaw Island
St. Catherine's Island
ATLANTIC OCEAN
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
0.2
0.24
0 10 20 30 40 50 60 70 80
0 10 20 30 40 50 60 70

(b)

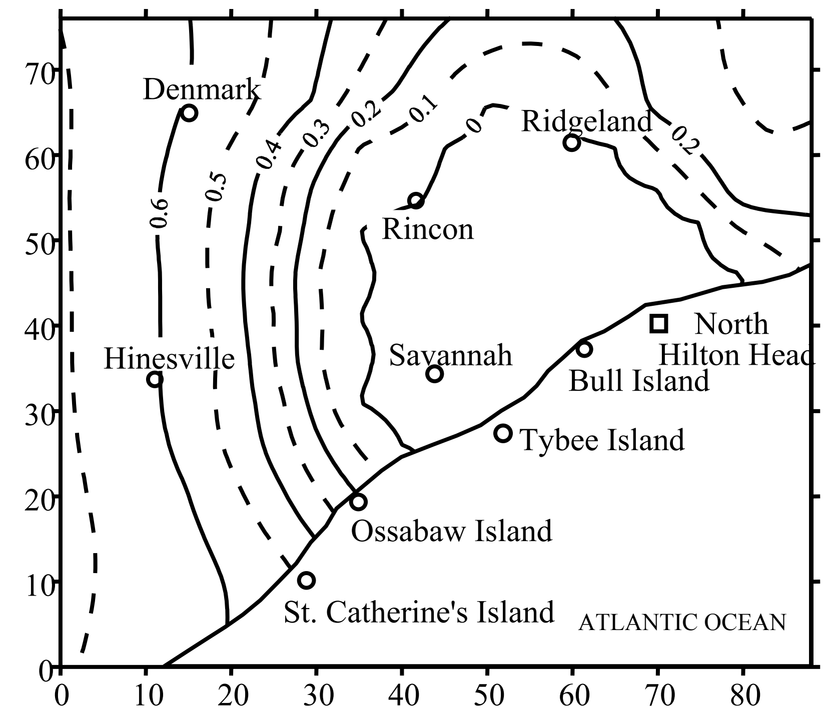


(c)

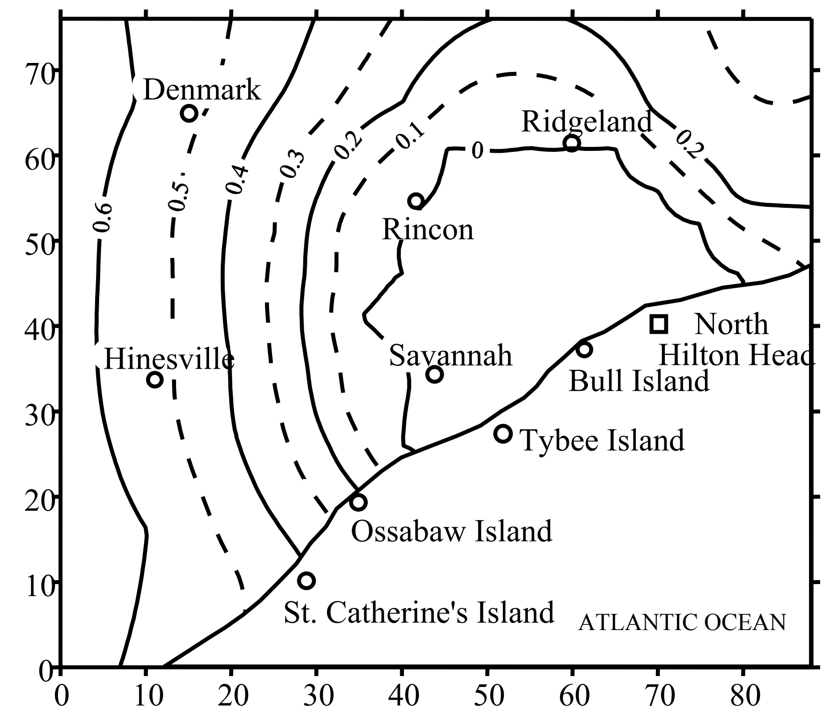
Figure 4.5 Equal pumping rate contours (MGal/day) for the UFA using
(a) $B = 2$; (b) $B = 4$; (c) $B = 6$ (distances are in miles)

In order to explore the spatial distribution of additional groundwater availability from the LFA and UFA+LFA similar analysis are conducted for these two cases as well. The results for the LFA are very similar to those obtained by the UFA. Equal pumping rate contours (MGal/day) obtained using various w_i 's for the LFA are given in Figure 4.6.

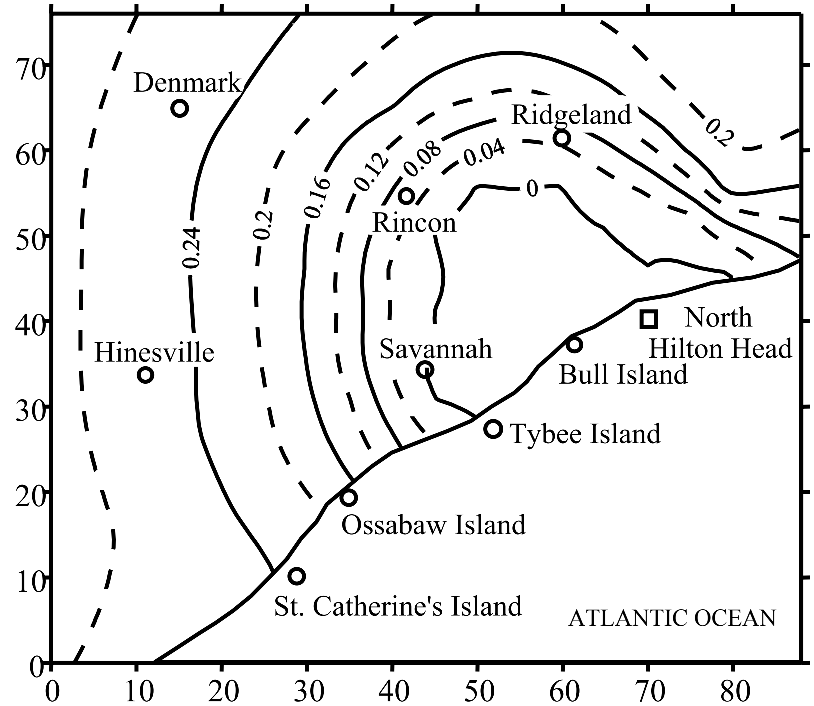
When groundwater withdrawal is allowed from both the UFA and the LFA simultaneously, the total amount of additional groundwater withdrawal increases compared to the cases that utilize UFA or the LFA alone. Equal pumping rate contours from the UFA and the LFA are plotted separately for the UFA+LFA case in Figure 4.7.


Denmark
Ridgeland
Rincon
Hinesville
Savannah
North
Hilton Head
Bull Island
Tybee Island
Ossabaw Island
St. Catherine's Island
ATLANTIC OCEAN
0.6
0.5
0.4
0.3
0.2
0.1
0
0.2

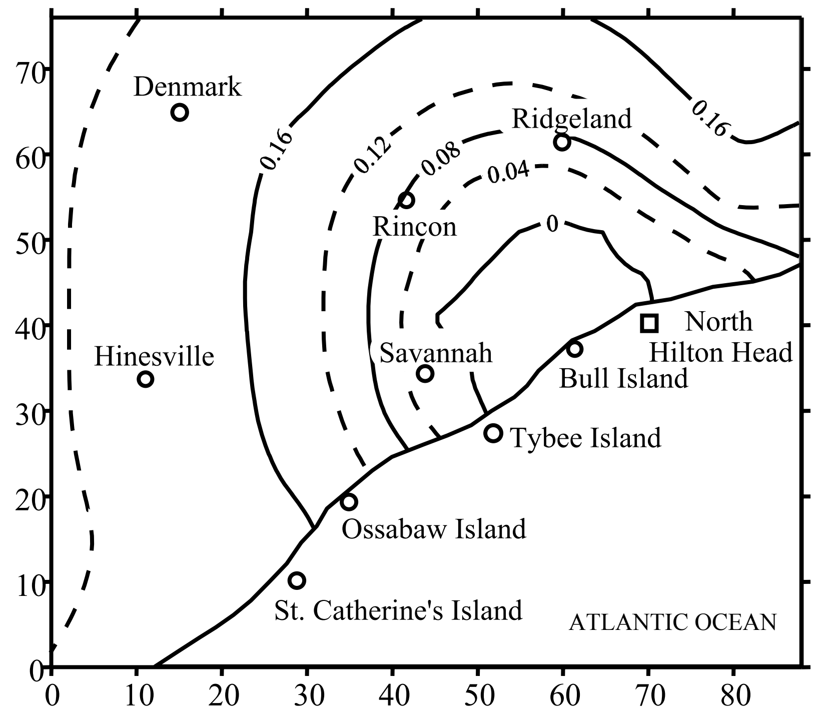
(a)


Denmark
Ridgeland
Rincon
Hinesville
Savannah
North
Hilton Head
Bull Island
Tybee Island
Ossabaw Island
St. Catherine's Island
ATLANTIC OCEAN
0.6
0.5
0.4
0.3
0.2
0.1
0
0.2

(b)

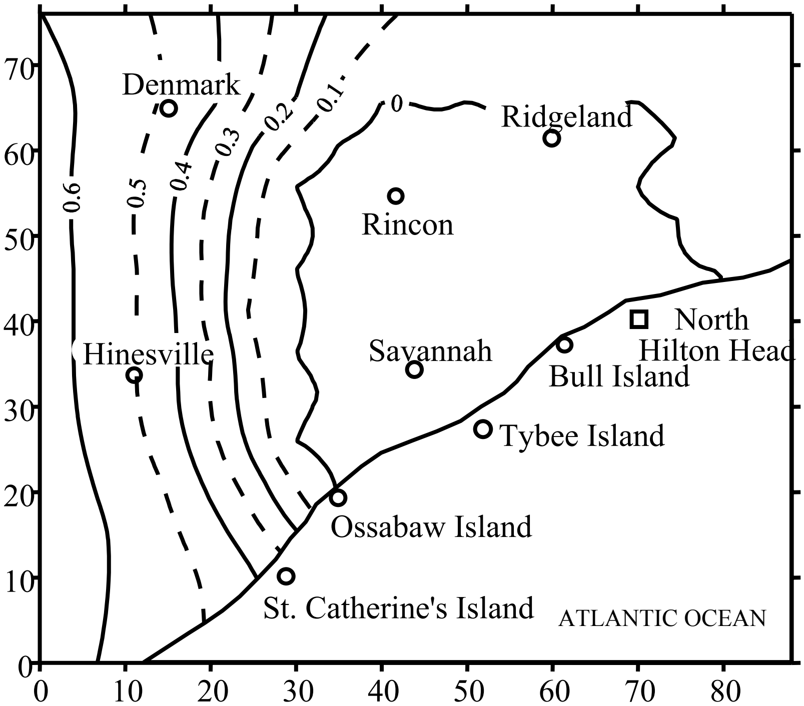


(c)

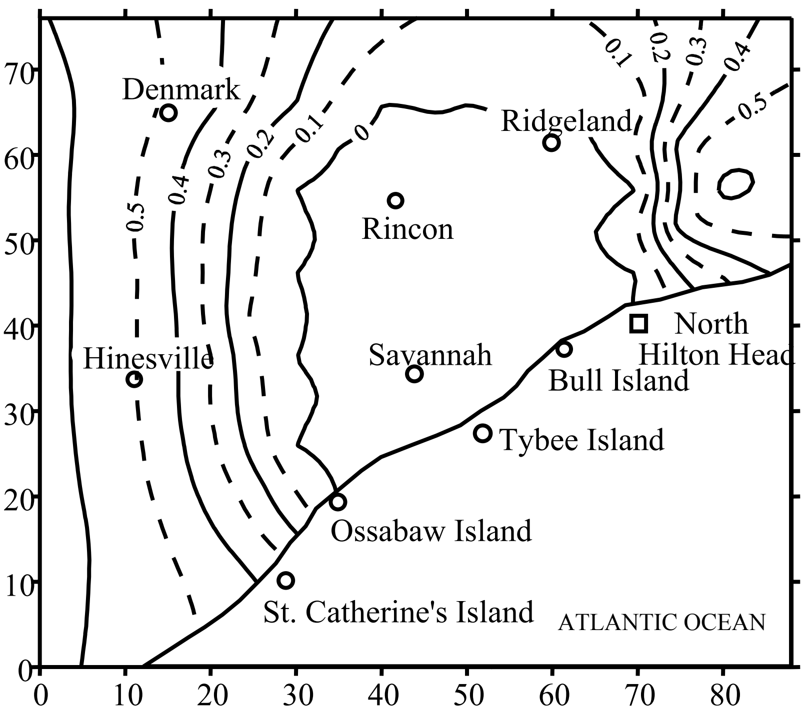


(d)

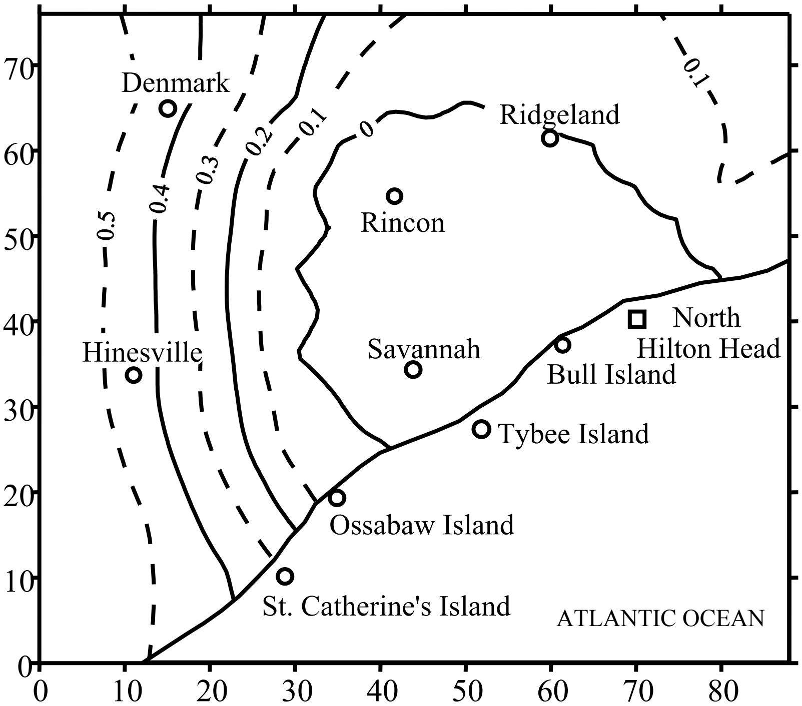
Figure 4.6 Equal pumping rate contours (MGal/day) for the LFA using $(a)\ w_i = 1, \forall i$; $(b)\ B = 2$; $(c)\ B = 4$; $(d)\ B = 6$ (distances are in miles)



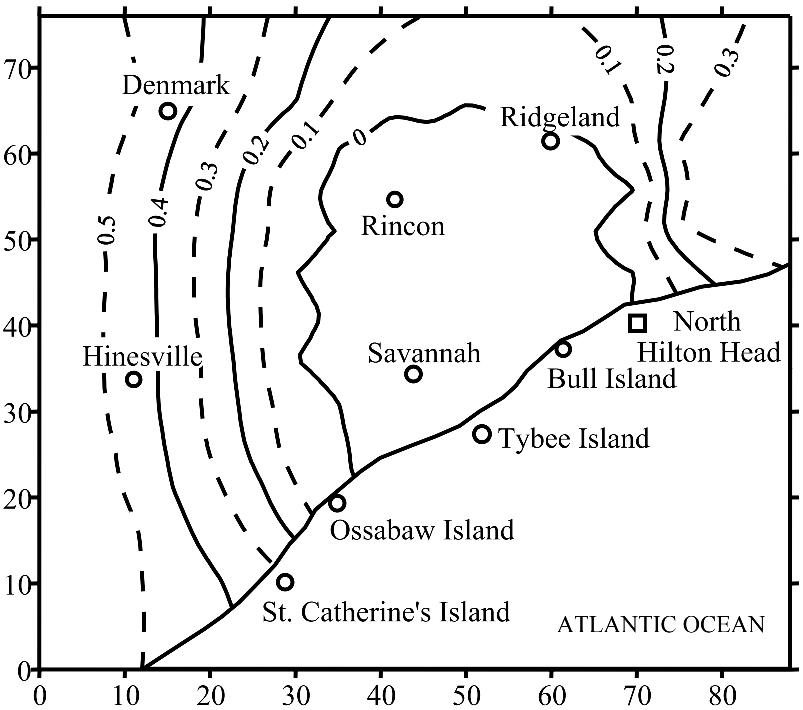
(a1) For LFA



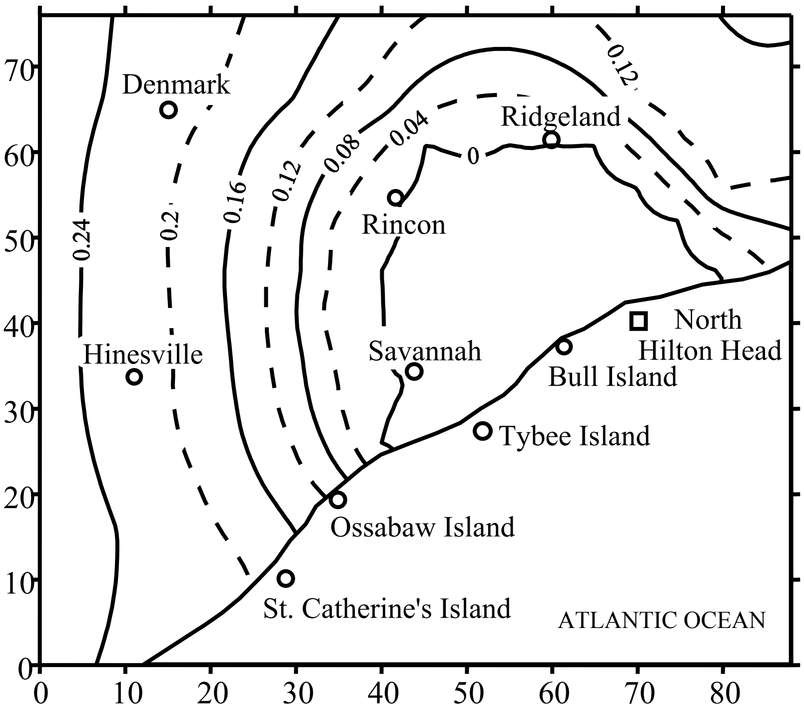
(a2) For UFA



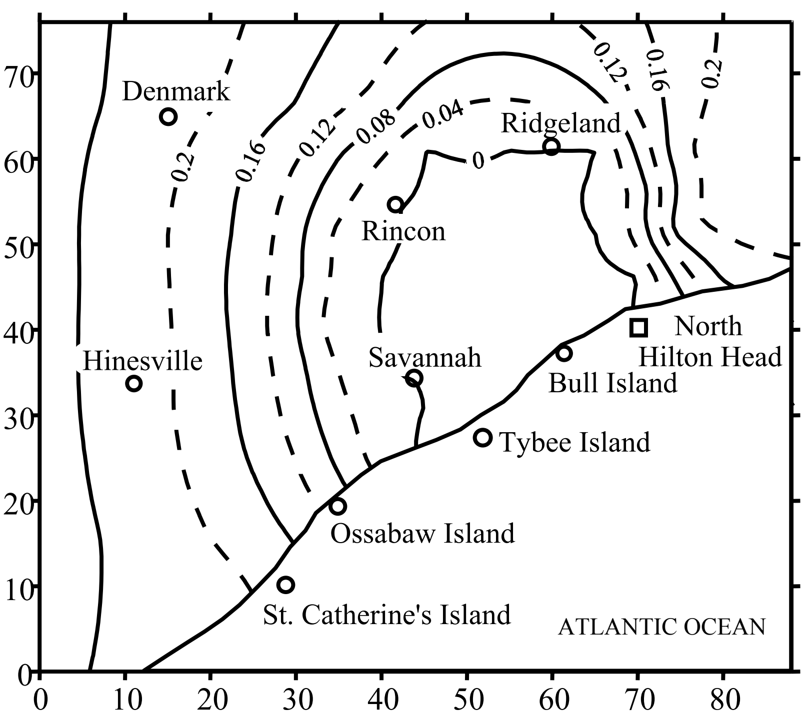
(b1) For LFA



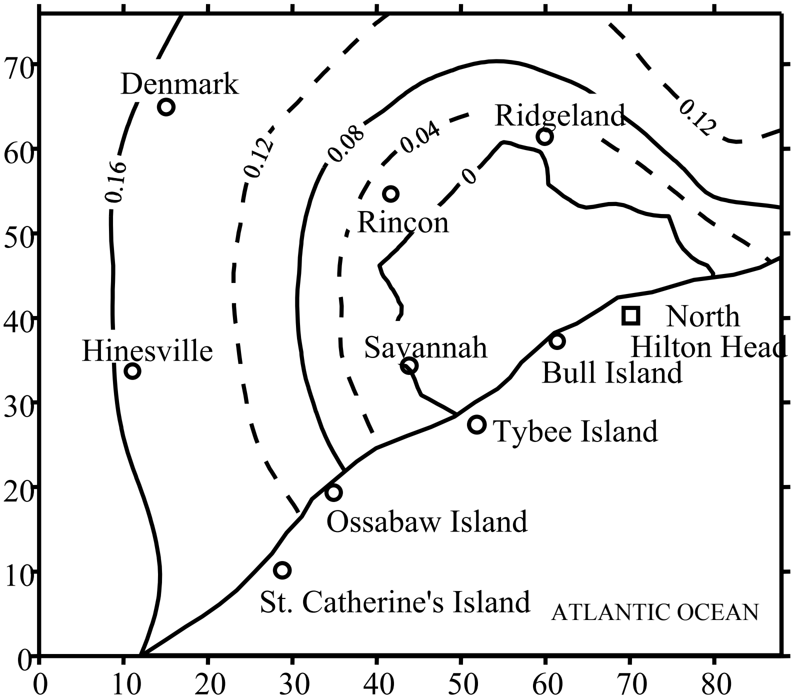
(b2) For UFA



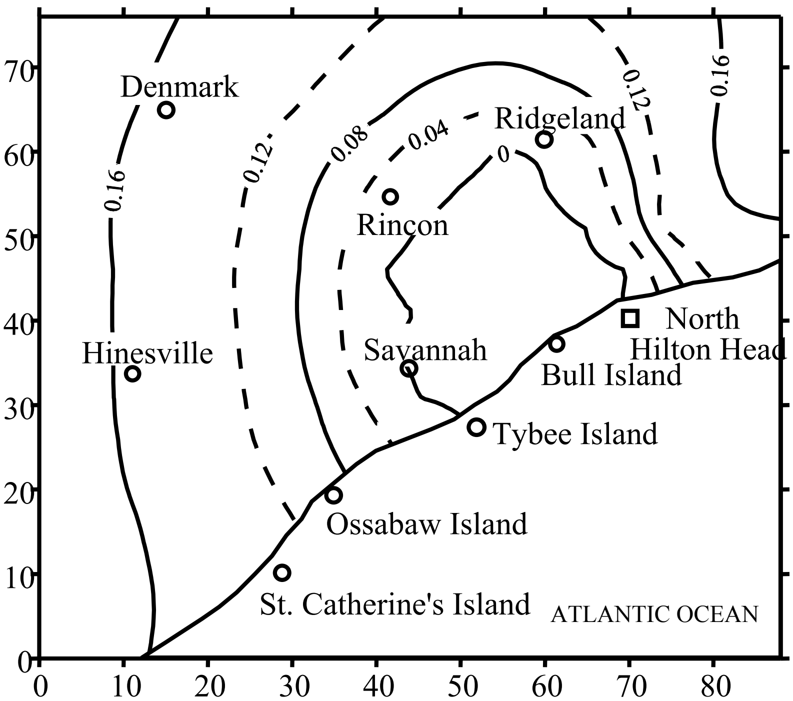
(c1) For LFA



(c2) For UFA



(d1) For LFA



(d2) For UFA

Figure 4.7 Equal pumping rate contours (MGal/day) for UFA+LFA using
$(a1)\, w_i = 1, \forall i$ for LFA; $(b1)\ B = 2$ for LFA; $(c1)\ B = 4$ for LFA; $(d1)\ B = 6$ for LFA;
$(a2)\, w_i = 1, \forall i$ for UFA; $(b2)\ B = 2$ for UFA; $(c2)\ B = 4$ for UFA; $(d2)\ B = 6$ for UFA
(distances are in miles)

Optimum pumping rates for each of the 70 well locations when groundwater is withdrawn from the UFA, the LFA, and from UFA+LFA using various w_i's are provided in Appendix F. For the UFA+LFA case, at each well location two pumping rates, one from the LFA and one from the UFA, are provided. Equal pumping rate contours for the LFA, the UFA, and UFA+LFA using various w_i's are provided in Figures 4.4 to 4.7. We will refer to the area which lies in between two solid line contours in these figures as a "contour band." The pumping rate associated with the dashed contour (i.e., the average of the solid contours) within each contour band will be used in referring to that specific contour band. For example, the contour band identified by 0.0 MGal/day and 0.2 MGal/day contours will be referred as 0.1 MGal/day contour band (see Figure 4.4).

The number of potential wells that may fall within each contour band given in Figures 4.4 through 4.7 are different for each case (i.e., LFA, UFA, UFA+LFA using various w_i values). The total amount of additional groundwater that can be withdrawn from each contour band can be approximated by multiplying the number of potential wells located in a contour band by the pumping rate corresponding to the dashed contour in between the solid contours. Based on this approach, using Figures 4.4 through 4.7, the number of wells that fall within each contour band are counted (note that in order not to complicate the figures the well locations are not provided on each figure, however, the well locations are given in Figure 4.3) and the total amount of groundwater that will be available from each of these bands are calculated. The results are given in Table 4.2.

Table 4.2 Number of wells in between various contour intervals and associated amount of additional available groundwater withdrawal

Aquifer	w_i	Number of wells in between contours of equal pumping rates in MGal/d				Total available groundwater (MGal/d) in between contours				Total available groundwater from aquifer (MGal/d)	
		0.0-0.2	0.2-0.4	0.4-0.6	0.6-0.8	0.0-0.2	0.2-0.4	0.4-0.6	0.6-0.7	Sum of contours	Model results
LFA	1	5	10	14	6	0.5	3	7	4.2	14.7	14.7
UFA	1	7	7	16	8	0.7	2.1	8	5.6	16.4	16.5
UFA+LFA, LFA	1	5	7	14	0	0.5	2.1	7	0	9.6	9.4
UFA+LFA, UFA	1	4	9	17	0	0.4	2.7	8.5	0	11.6	11.1
		0.0-0.2	0.2-0.4	0.4-0.6	0.6-0.8	0.0-0.2	0.2-0.4	0.4-0.6	0.6-0.7		
LFA	$2-d_i/d_{\max}$	7	14	17	0	0.7	4.2	8.5	0	13.4	13.5
UFA	$2-d_i/d_{\max}$	9	13	19	0	0.9	3.9	9.5	0	14.3	14.7
UFA+LFA, LFA	$2-d_i/d_{\max}$	10	14	7	0	1	4.2	3.5	0	8.7	9.0
UFA+LFA, UFA	$2-d_i/d_{\max}$	9	17	8	0	0.9	5.1	4	0	10.0	10.0
		0.0-0.08	0.08-0.16	0.16-0.24	0.24-0.28	0.0-0.08	0.08-0.16	0.16-0.24	0.24-0.28		
LFA	$4-d_i/d_{\max}$	9	9	19	14	0.36	1.08	3.8	3.64	8.9	8.9
UFA	$4-d_i/d_{\max}$	5	10	19	17	0.2	1.2	3.8	4.42	9.6	9.8
UFA+LFA, LFA	$4-d_i/d_{\max}$	5	14	21	0	0.2	1.68	4.2	0	6.1	6.1
UFA+LFA, UFA	$4-d_i/d_{\max}$	7	12	25	0	0.28	1.44	5	0	6.7	6.7
		0.0-0.08	0.08-0.16	0.16-0.24		0.0-0.08	0.08-0.16	0.16-0.20			
LFA	$6-d_i/d_{\max}$	15	21	22		0.6	2.52	4.4		7.5	7.3
UFA	$6-d_i/d_{\max}$	12	18	25		0.48	2.16	5.0		7.6	8.1
UFA+LFA, LFA	$6-d_i/d_{\max}$	12	33	2		0.48	3.96	0.4		4.8	5.0
UFA+LFA, UFA	$6-d_i/d_{\max}$	14	35	2		0.56	4.2	0.4		5.2	5.4

For example, the number of wells located inside 0.1 MGal/day contour band for the case in which groundwater is withdrawn from the UFA using $w_i = 1, \forall i$ (Figure 4.4) is seven. Thus, the total amount of additional groundwater that can be withdrawn from the UFA for $w_i = 1, \forall i$ from 0.1 MGal/day contour band is 7x0.1=0.7 MGal/day (see Table 4.2). However, it should be noted here that this total amount (i.e., 0.7 MGal/day) represents the additional available groundwater from the whole area identified by 0.0 MGal/day and 0.2 MGal/day contours. Thus, this amount can not be withdrawn from a single location within 0.1 MGal/day contour band. Withdrawing the total amount of additional available groundwater (i.e., 0.7 MGal/day) from a single location may violate the drawdown constraint at Hilton Head Island. Thus, the results presented in Table 4.2 may only be interpreted in terms of the spatial distribution of the additional available groundwater.

For each case (i.e., groundwater being withdrawn from the LFA, the UFA, and UFA+LFA for $w_i = 1, \forall i \; or \; B = 2, 4, or\, 6$) the total amount of groundwater that can be withdrawn from the model domain is approximated by adding the estimated amount of the additional groundwater withdrawals from the contour bands used for that case. For example, when the groundwater is withdrawn from the UFA using $w_i = 1, \forall i$, the total amount of additional groundwater that can be withdrawn from the model domain can be calculated as 7x0.1+7x0.3+16x0.5+8x0.7=16.4 MGal/day (Table 4.2). This is an approximation. Summation of the additional optimum pumping rates obtained by solving the coupled-simulation optimization model yields the exact amount of additional available groundwater in the model domain (see Appendix F). Optimum pumping rates obtained from the coupled simulation-optimization model is summed and represented in the last column of Table 4.2. As can be seen from the comparison of the last two columns of Table 4.2, the total amount of additional available groundwater calculated by summing the additional available groundwater estimated by using the contour bands are very similar to those obtained directly from the coupled simulation-optimization model.

The results presented in this section may be useful in identifying spatial distribution of additional groundwater availability throughout the Savannah region and may be used in site selection processes of the prospective industries which are planning to apply for groundwater withdrawal permits.

4.5.2 Case Study: Multiple Groundwater Withdrawal Permit Applications

As a hypothetical case, the coupled simulation-optimization model is solved by using only the six demand locations. It is assumed that the groundwater withdrawals are from the UFA. Initially all w_i's are selected as one and the additional optimum pumping rates are calculated. The additional optimum pumping rates that can be withdrawn from the UFA at six demand locations using $w_i = 1, \forall i$ are given in Table 4.3. The results are also represented in a bubble graph as shown in Figure 4.8. The total amount of groundwater withdrawal from the UFA when $w_i = 1, \forall i$ is approximately 5.9 MGal/day. As can be seen from Table 4.3 combined withdrawal available at Denmark and Hinesville accounts for approximately half of this total amount (i.e., 2.8 MGal/day).

Table 4.3 The additional optimum pumping rates that can be extracted from the UFA at six demand locations using $w_i = 1, \forall i$ and distances between the hypothetical well and Hilton Head Island

Location	Optimum pumping rate (MGal/day)	Distance (miles)
Rincon	0.622	31.30
Bloomingdale	0.882	36.35
Marlow	1.013	40.31
Ridgeland	0.584	23.26
Denmark	1.393	60.01
Hinesville	1.411	59.41
Total	**5.905**	**-**

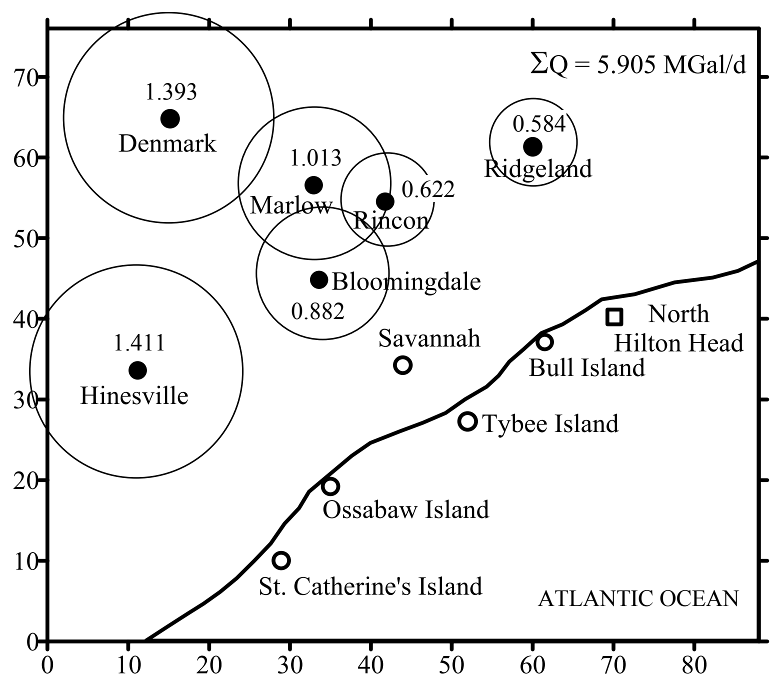


Figure 4.8 The additional optimum pumping rates from the UFA for six demand locations using $w_i = 1, \forall i$ (distances are in miles)

The minimum amount of groundwater extraction is assigned to Ridgeland while the maximum amount is assigned to Hinesville. As can be seen from Figure 4.8, the optimum amount of additional groundwater that is available at a demand location is a function of the distance between the northern end of Hilton Head Island (i.e., indicator site where drawdown is restricted to be smaller than or equal to 0.05 ft) and the demand location. The size of the bubbles in Figure 4.8 is proportional to the amount of the additional available groundwater pumping rate at that location. As can be seen from Figure 4.8, the amount of the additional available groundwater from six demand locations varies considerably. The additional optimum pumping rates tend to increase as the distance between the demand location and the indicator site increases.

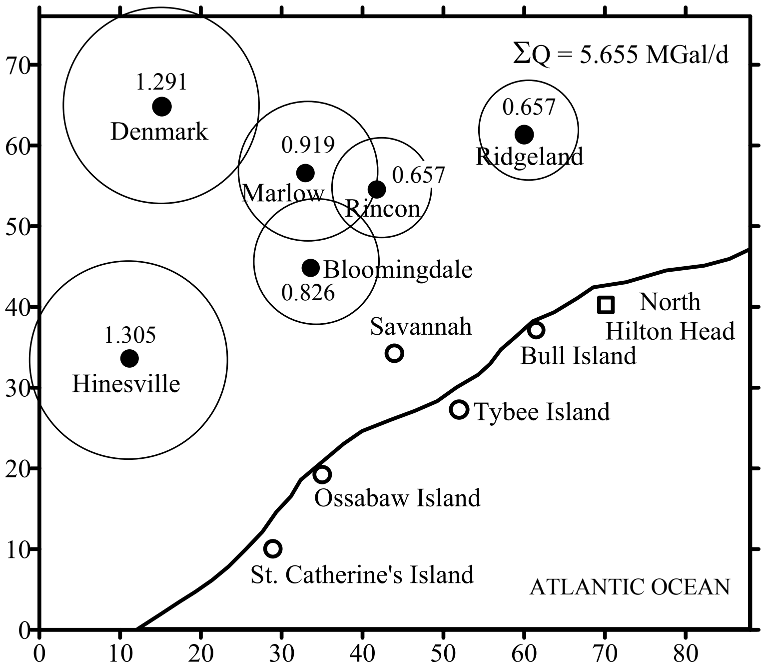
As discussed earlier, EPD developed an “Interim Strategy for Managing Salt Water Intrusion in the UFA of Southeast Georgia” in Feb 1996. One of the major comments raised in the public meetings conducted following this Interim Strategy was that the proposed Interim Strategy was unfair to some users, particularly those that are close to the indicator site (i.e., Hilton Head Island).

Our analysis determines the optimum additional pumping rates available at various demand locations and how these additional optimum pumping rates decrease as a function of distance between the demand location and the indicator site. Accordingly, the results obtained by using $w_i = 1, \forall i$ (see Table 4.3 and Figure 4.8) may be interpreted as unfair. As explained earlier, the weighting factor, w_i, in front of the penalty term in Equation (4.1) can be adjusted so that the coupled simulation-optimization model yields a more uniform distribution of pumping rates. In order to investigate the effect of utilizing different weighting factors on the additional optimum pumping rates that can be extracted from the UFA, the same weighting functions identified as cases $(a)\, B=2, (b)\, B=4, (c)\, B=6$ are used. The optimum additional pumping rates obtained using these different weighting factors are given in Figure 4.9.

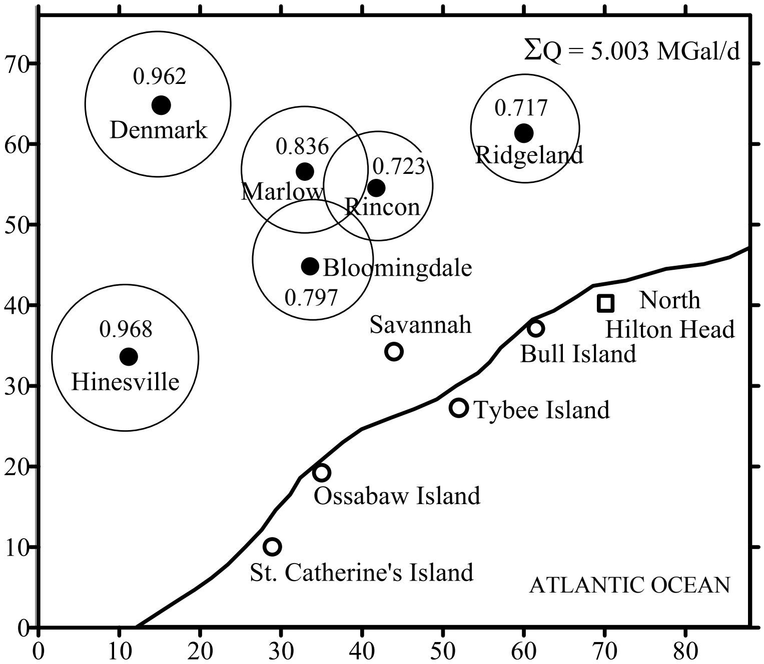
Again as can be observed from Figure 4.9, as w_i increases the magnitude of the additional optimum pumping rates get closer to each other, however the total pumping rate decreases. The optimal results for the UFA obtained by using different weighting factors are also given in Table 4.4.

Table 4.4 The additional optimum pumping rates that can be extracted from the UFA at six demand locations using various weighting factors

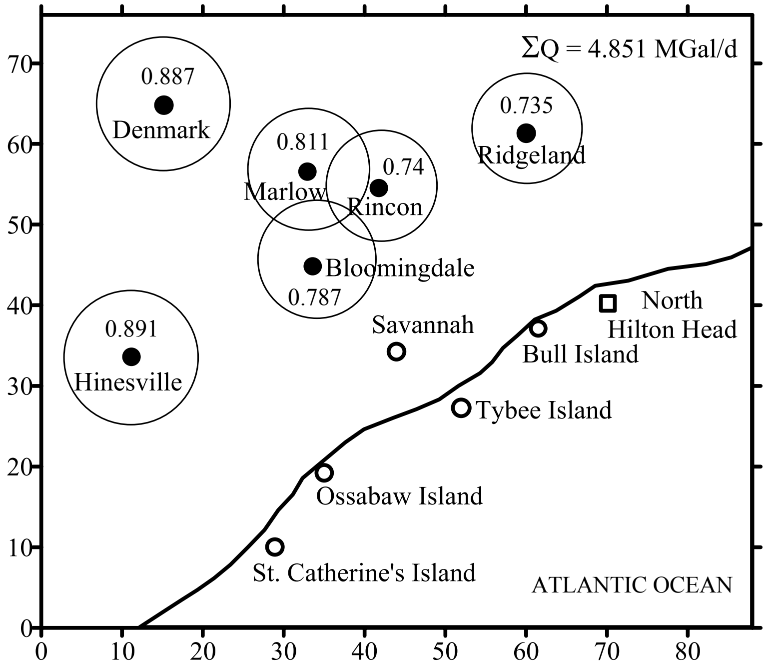
Location	Optimum pumping rate (MGal/day)			
	$w_i = 1$ $i = 1, 2, ..., 6$	$w_i = \left(2 - \frac{d_i}{d_{\max}}\right)$	$w_i = \left(4 - \frac{d_i}{d_{\max}}\right)$	$w_i = \left(6 - \frac{d_i}{d_{\max}}\right)$
Rincon	0.622	0.657	0.723	0.740
Bloomingdale	0.882	0.826	0.797	0.787
Marlow	1.013	0.919	0.836	0.811
Ridgeland	0.584	0.657	0.717	0.735
Denmark	1.393	1.291	0.962	0.887
Hinesville	1.411	1.305	0.968	0.891
Total	**5.905**	**5.655**	**5.003**	**4.851**


ΣQ = 5.655 MGal/d
1.291
Denmark
0.919
Marlow
0.657
Rincon
0.657
Ridgeland
Bloomingdale
0.826
1.305
Hinesville
Savannah
North
Hilton Head
Bull Island
Tybee Island
Ossabaw Island
St. Catherine's Island
ATLANTIC OCEAN

(a)


ΣQ = 5.003 MGal/d
0.962
Denmark
0.836
Marlow
0.723
Rincon
0.717
Ridgeland
Bloomingdale
0.797
0.968
Hinesville
Savannah
North
Hilton Head
Bull Island
Tybee Island
Ossabaw Island
St. Catherine's Island
ATLANTIC OCEAN

(b)



(c)

Figure 4.9 The additional optimum pumping rates from the UFA for six demand locations using $(a)\,B = 2$; $(b)\,B = 4$; $(c)\,B = 6$ (distances are in miles)

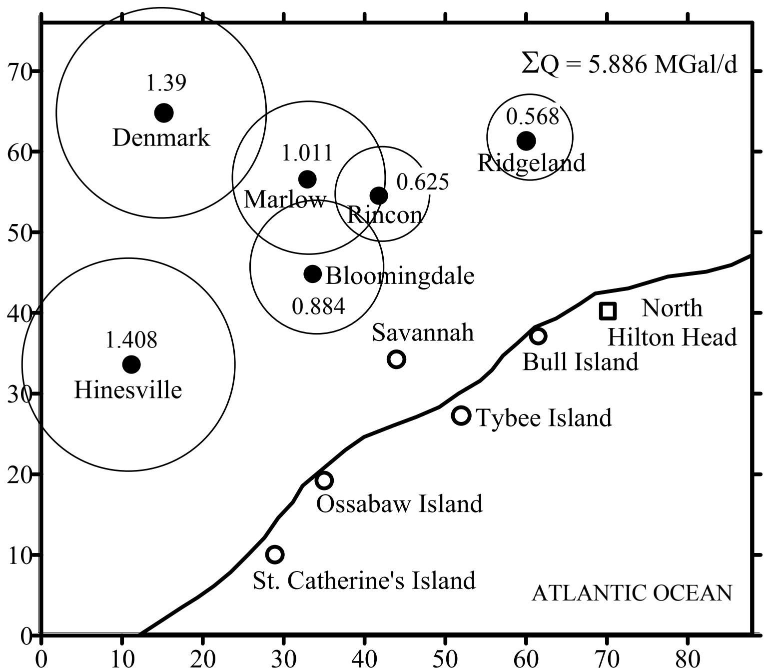
As stated earlier, EPD has restricted groundwater withdrawal permits for the UFA. In the "Supplement to the Interim Strategy for Managing Salt Water Intrusion in the Upper Floridan Aquifer of Southeast Georgia" (EPD 2001), the LFA is listed as one of the alternative water sources in the coastal area. Thus, we conduct the same analysis described above for the LFA as well. This case considers that all the withdrawals are from the LFA. The additional optimum pumping rates (MGal/day) at six demand locations using various weighting factors are provided in Table 4.5.

Table 4.5 The additional optimum pumping rates that can be extracted from the LFA at six demand locations using various weighting factors

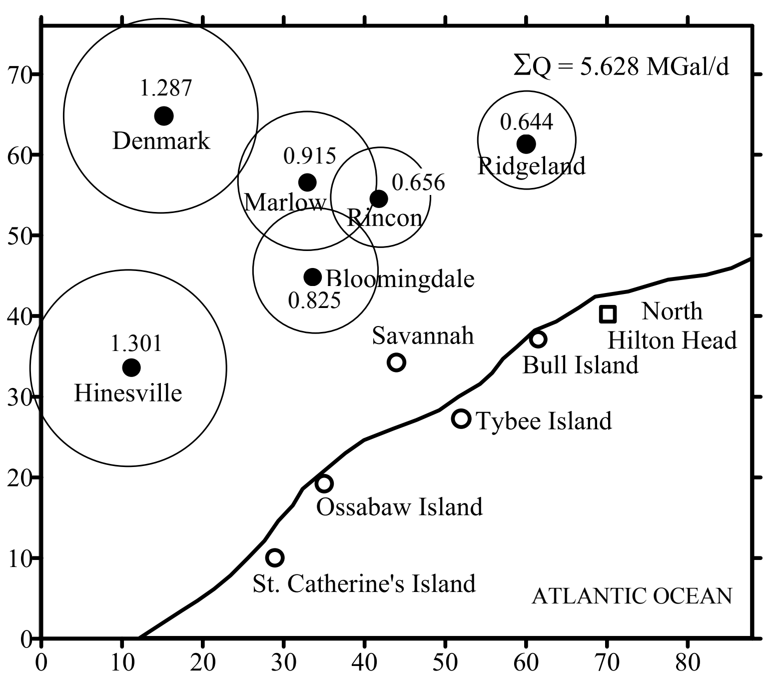
Location	Optimum pumping rate (MGal/day)			
	$w_i = 1$ $i = 1, 2, ..., 6$	$w_i = \left(2 - \frac{d_i}{d_{\max}}\right)$	$w_i = \left(4 - \frac{d_i}{d_{\max}}\right)$	$w_i = \left(6 - \frac{d_i}{d_{\max}}\right)$
Rincon	0.625	0.656	0.720	0.736
Bloomingdale	0.884	0.825	0.794	0.783
Marlow	1.011	0.915	0.831	0.806
Ridgeland	0.568	0.644	0.708	0.727
Denmark	1.390	1.287	0.958	0.882
Hinesville	1.401	1.301	0.963	0.886
Total	**5.886**	**5.628**	**4.974**	**4.820**

Comparison of Tables 4.4 and 4.5 indicates that the optimum additional pumping rates from the UFA and the LFA are very similar to each other. These results show that piezometric head decline response at the northern end of Hilton Head Island due to groundwater withdrawal from the UFA and the LFA at the selected demand locations are similar, thus the additional optimum pumping rates are similar. This indicates the leaky nature of the confining unit as it is modeled in the Savannah Area Model. Optimum pumping rates obtained for the LFA at six demand locations using various w_i's are given as bubble plots in Figure 4.10.

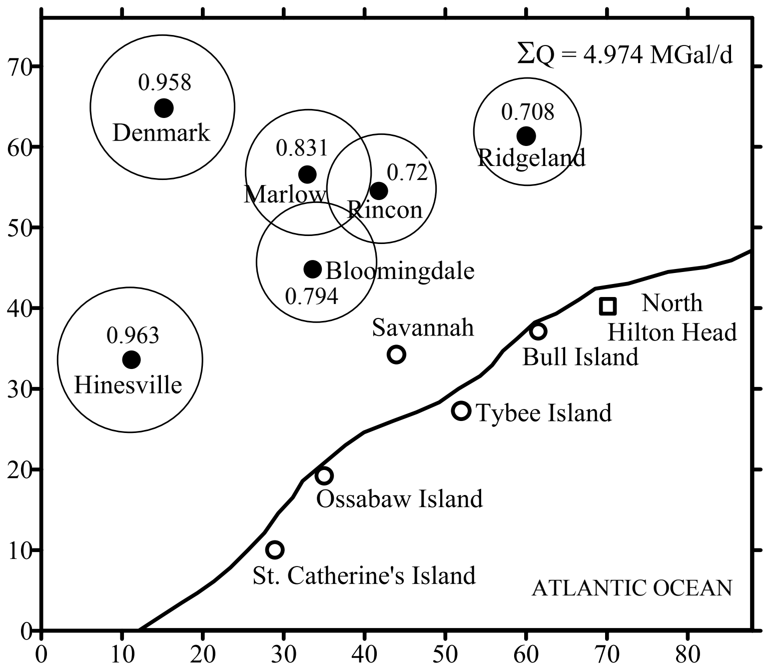
We also evaluated the impact of simultaneous pumping from the LFA and the UFA (i.e., UFA+LFA). In order to implement this case, two different variables, one to represent groundwater pumping rate from the LFA and the other to represent pumping from the UFA are used at each one of the six demand locations. Thus, a total of twelve additional optimum pumping rates are calculated. These results are given in Table 4.6.


ΣQ = 5.886 MGal/d
1.39
Denmark
1.011
Marlow
0.625
Rincon
0.568
Ridgeland
Bloomingdale
0.884
1.408
Hinesville
Savannah
North
Hilton Head
Bull Island
Tybee Island
Ossabaw Island
St. Catherine's Island
ATLANTIC OCEAN
0 10 20 30 40 50 60 70 80
0 10 20 30 40 50 60 70

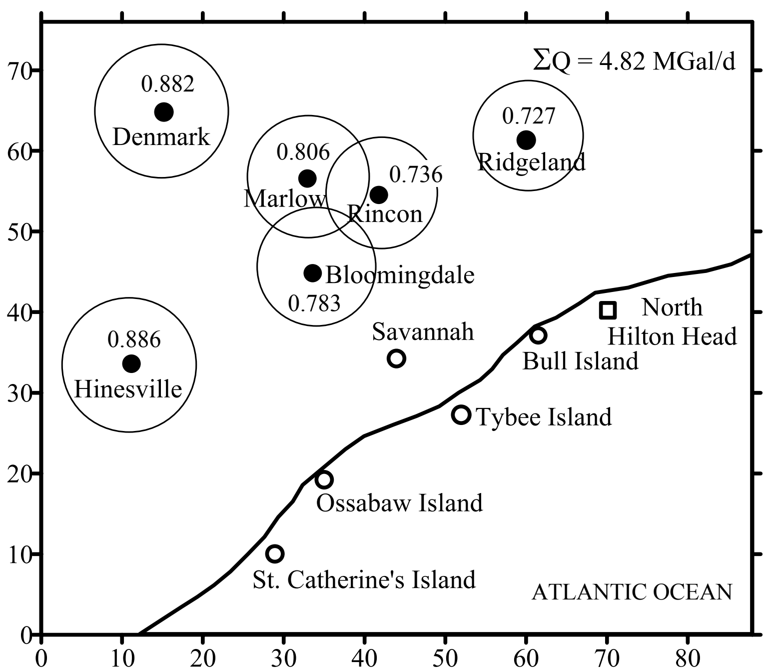
(a)


ΣQ = 5.628 MGal/d
1.287
Denmark
0.915
Marlow
0.656
Rincon
0.644
Ridgeland
Bloomingdale
0.825
1.301
Hinesville
Savannah
North
Hilton Head
Bull Island
Tybee Island
Ossabaw Island
St. Catherine's Island
ATLANTIC OCEAN
0 10 20 30 40 50 60 70 80
0 10 20 30 40 50 60 70

(b)



(c)



(d)

Figure 4.10 The additional optimum pumping rates from the LFA for six demand locations using $(a)\ w_i = 1, \forall i$; $(b)\ B = 2$; $(c)\ B = 4$; $(d)\ B = 6$ (distances are in miles)

Table 4.6 The additional optimum pumping rates that can be extracted from UFA+LFA at six demand locations using various weighting factors

Location	Optimum pumping rate (MGal/day)							
	$w_i = 1$, $i = 1, 2, \ldots, 6$		$w_i = \left(2 - \dfrac{d_i}{d_{\max}}\right)$		$w_i = \left(4 - \dfrac{d_i}{d_{\max}}\right)$		$w_i = \left(6 - \dfrac{d_i}{d_{\max}}\right)$	
	LFA	UFA	LFA	UFA	LFA	UFA	LFA	UFA
Rincon	0.241	0.241	0.274	0.274	0.339	0.339	0.355	0.356
Bloomingdale	0.501	0.501	0.444	0.443	0.413	0.413	0.403	0.403
Marlow	0.629	0.631	0.534	0.536	0.451	0.452	0.426	0.427
Ridgeland	0.184	0.203	0.262	0.274	0.328	0.333	0.347	0.350
Denmark	1.010	1.009	0.907	0.907	0.578	0.578	0.503	0.503
Hinesville	1.027	1.027	0.921	0.921	0.583	0.583	0.506	0.506
Sum from LFA and UFA	3.592	3.612	3.342	3.355	2.692	2.698	2.540	2.545
Total	**7.204**		**6.697**		**5.390**		**5.085**	

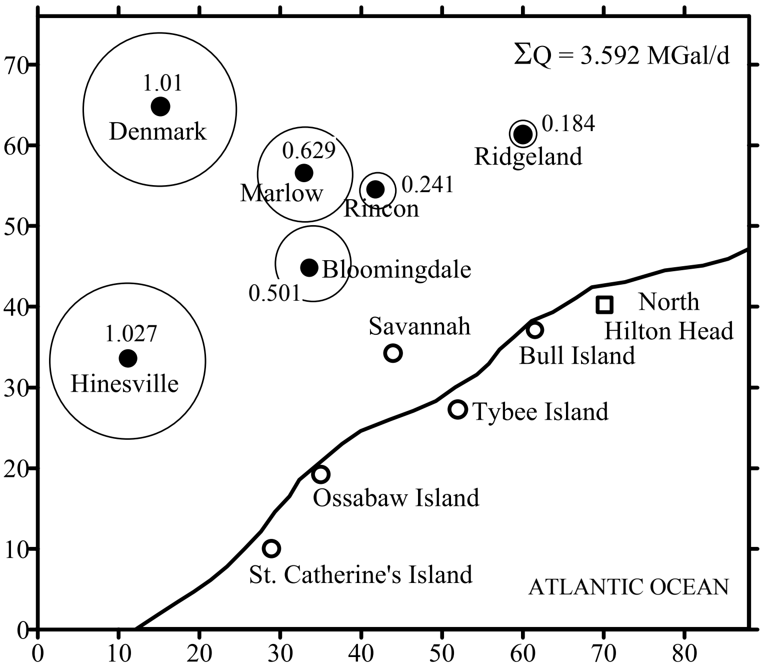
The total amount of groundwater that can be withdrawn from the aquifer is determined by summing the withdrawal rates from the LFA and the UFA. The last row of Table 4.6 shows the total amount of additional groundwater that can be obtained from six demand locations for each w_i. As can be seen from Table 4.6, when both LFA and UFA (i.e., UFA+LFA) are utilized, the total amount of groundwater withdrawal is higher compared to those of the LFA and the UFA cases.

Individual additional groundwater withdrawal potentials at each of the six permit locations are calculated for UFA, LFA, and UFA+LFA separately. As can be seen from Tables 4.4, 4.5, and 4.6 various w_i's resulted in different optimum additional pumping rates, consequently different total additional pumping rates. Optimum pumping rates obtained for UFA+LFA case at six demand locations using various w_i's are given in

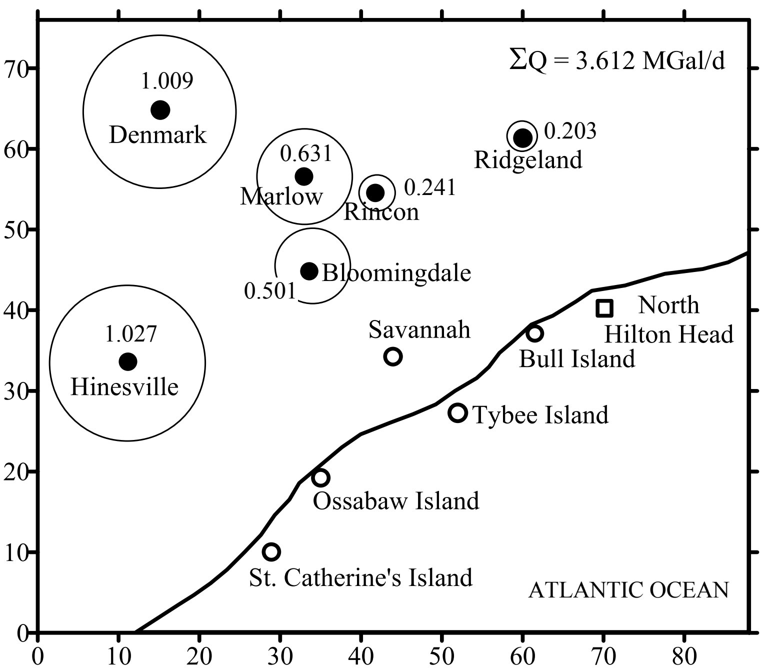
Figure 4.11. Groundwater withdrawal rates from the LFA and from the UFA are plotted separately in Figure 4.11.

Optimum additional groundwater withdrawal rates obtained when the groundwater is extracted from the UFA, the LFA, and UFA+LFA for various w_i's show that, when B (see Equation (4.3)) increases (i.e., more uniform groundwater extraction rates obtained) the total amount of groundwater that can be extracted from the aquifer decreases. Thus, there is a trade-off between enforcing "fair" groundwater management strategies and maximizing total groundwater withdrawal.

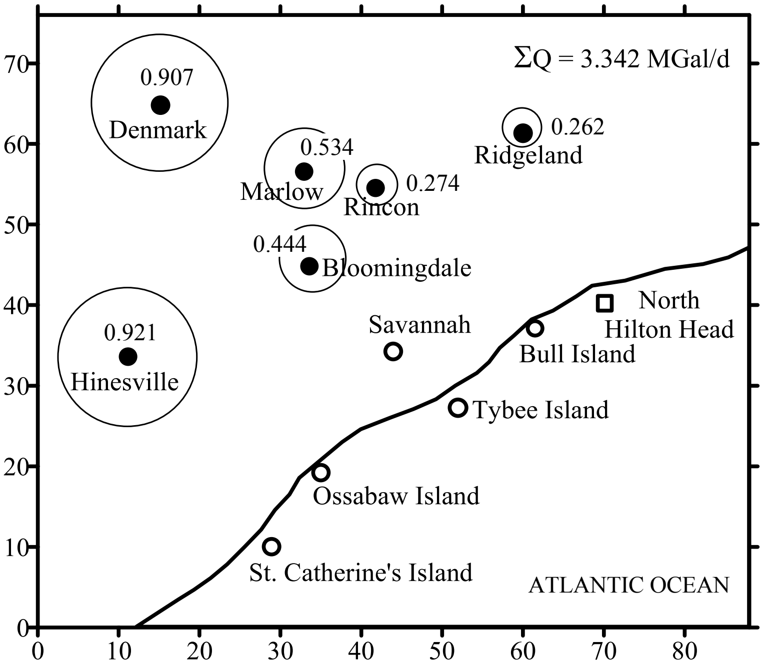
The coupled simulation-optimization model considers only a hydrological constraint (i.e., drawdown at the northern end of Hilton Head Island should not exceed 0.05 ft). However, in selecting best groundwater management strategy in a coastal aquifer may require consideration of additional environmentally, politically, and economically motivated objectives. In order to decide which strategy (i.e., withdrawing groundwater from the UFA, the LFA, or UFA+LFA for various weighting factors) is better, a decision-making analysis is performed considering possible additional objectives of the specific problem. If decision-maker's goal is to maintain highest total pumping rate then obviously UFA+LFA with $w_i = 1,\ i = 1, 2, ..., 6$ is the best groundwater management strategy. However, water resources management problems, especially those around coastal areas, are usually more complex than this and most of the time, additional conflicting objectives may need to be considered. This subject is discussed in the following section.



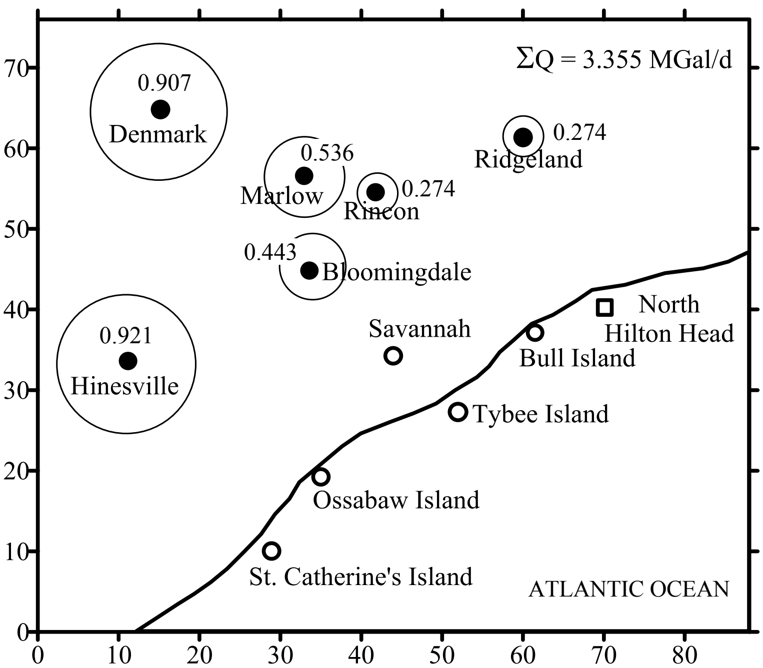
(a1) For LFA



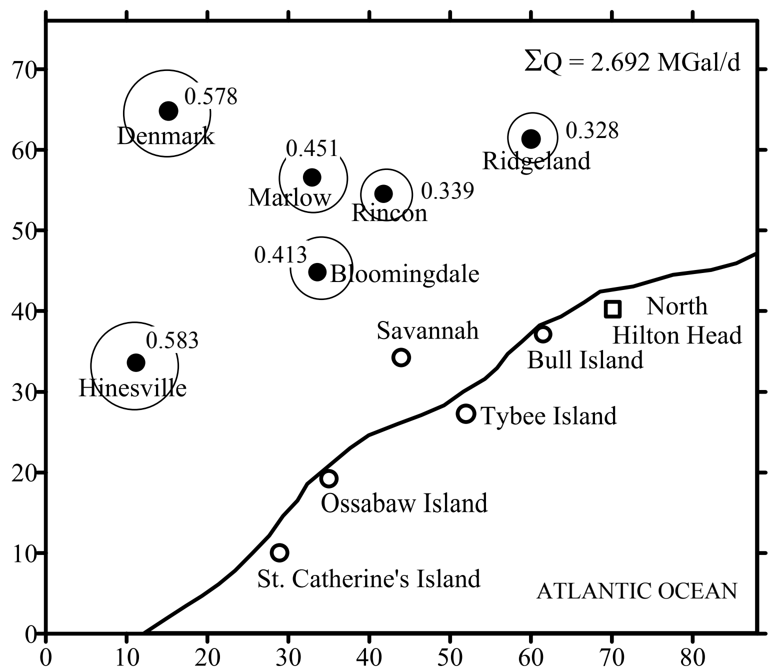
(a2) For UFA



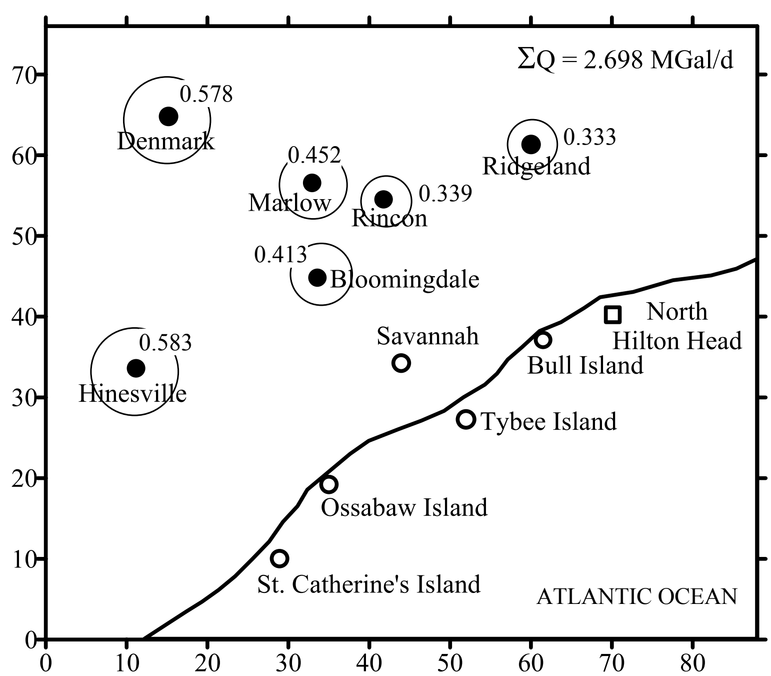
(b1) For LFA



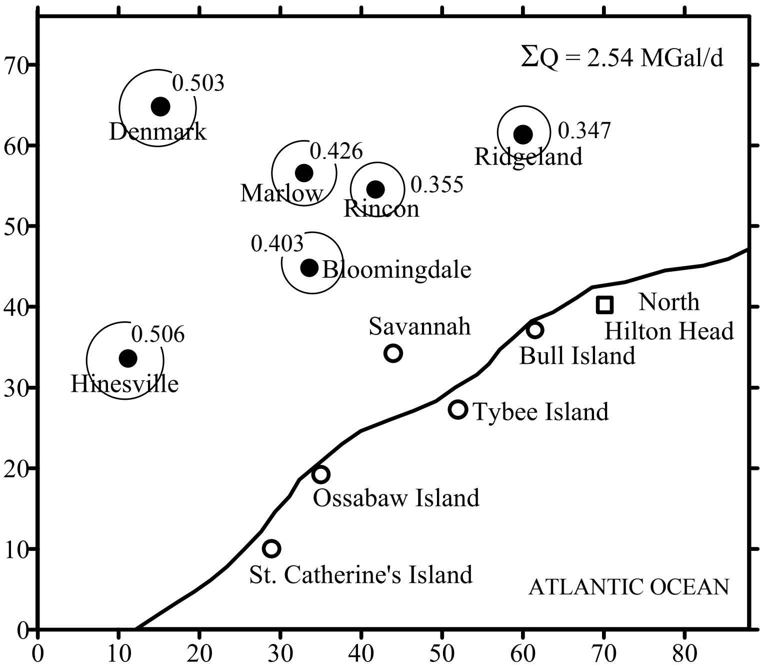
(b2) For UFA



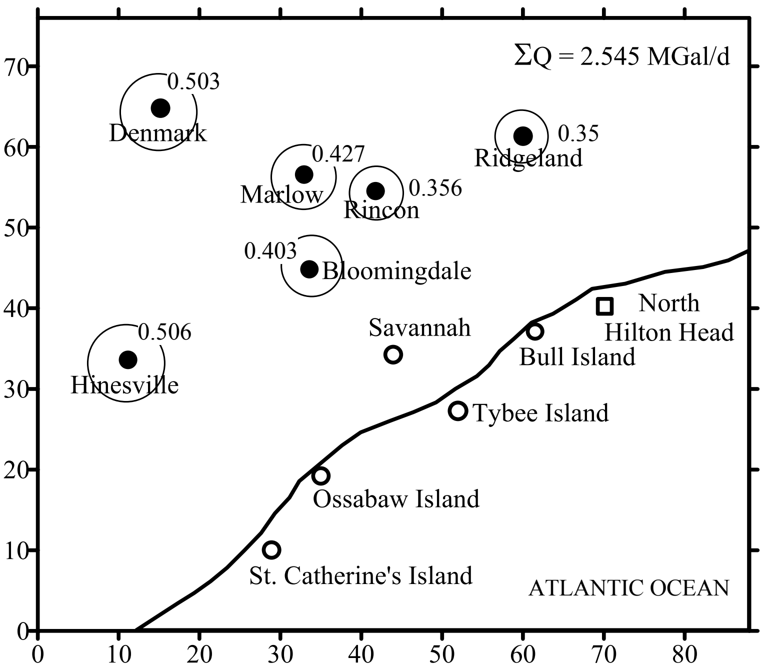
(c1) For LFA



(c2) For UFA



(d1) For LFA



(d2) For UFA

Figure 4.11 The additional optimum pumping rates from UFA+LFA for 6 demand locations using $\begin{matrix} (a1)\, w_i = 1, \forall i; \text{LFA} & (b1)\, B = 2; \text{LFA} & (c1)\, B = 4; \text{LFA} & (d1)\, B = 6; \text{LFA} \\ (a2)\, w_i = 1, \forall i; \text{UFA} & (b2)\, B = 2; \text{UFA} & (c2)\, B = 4; \text{UFA} & (d2)\, B = 6; \text{UFA} \end{matrix}$

(distances are in miles)

4.6 MULTI-OBJECTIVE DECISION-MAKING FRAMEWORK TO EVALUATE ALTERNATIVE GROUNDWATER MANAGEMENT STRATEGIES

For a typical groundwater resources management problem, depending on the additional water demand in the region, environmental restrictions, and available resources such as time and money other, objectives can be identified. Generally, these objectives are vague, thus it is easier to represent them as heuristic objectives using fuzzy sets. Each management strategy will satisfy each one of the fuzzy objectives to a certain degree. These individual satisfaction degrees have to be aggregated into a single overall performance value. Fuzzy aggregation process is explained in Appendix G. A decision-making framework which uses fuzzy set concepts is proposed in the following sections to determine the best groundwater management strategy among the optimal alternatives identified above.

A fuzzy multi-objective decision-making procedure is applied to the hypothetical case (see Section 4.4.2.2) to determine the best groundwater management strategy to supply the additional demands at six demand locations (i.e., Rincon, Bloomingdale, Marlow, Ridgeland, Denmark and Hinesville). As a result of the proposed decision-making procedure the best management strategy is determined with respect to a set of fuzzy objectives.

Different management strategies may be enforced by the authorities to guide the future groundwater utilization pattern of the area. Since different management strategies may lead to different utilization patterns (i.e., demand locations and additional optimum pumping rates at these demand locations are used as indicators of the utilization pattern in this study) they will satisfy the management objectives to different degrees. It is the manager's responsibility to determine the best management strategy which will satisfy the multiple objectives to the best degree.

4.6.1 Alternative Groundwater Management Strategies and the Procedure to Select the Best Management Strategy

We have already identified three management strategies for this study: additional groundwater is supplied from (i) the UFA; (ii) the LFA; and, (iii) UFA+LFA. In order to evaluate the impact of the penalty function in the decision-making process, four substrategies are also considered under each one of these three management strategies. The substrategies consider various w_i's: (i) $w_i = 1;\ i = 1, 2, \ldots, 6$; (ii) $w_i = \left(2 - d_i / d_{\max}\right)$; (iii) $w_i = \left(4 - d_i / d_{\max}\right)$; and, (iv) $w_i = \left(6 - d_i / d_{\max}\right)$. Thus, a total of 12 management strategies are evaluated optimally as discussed in the previous sections. These management strategies are summarized in Table 4.7. The fuzzy multi-objective decision-making framework proposed in this section is used to select the best groundwater resources management strategy among these 12 alternatives. Various combinations of fuzzy objectives are considered and the best management strategy for each case is determined.

Table 4.7 Management strategies

	Aquifer groundwater is extracted from	Penalty term, w_i
MS_1a	UFA	$w_i = 1;\ i = 1, 2, ..., 6$
MS_1b	UFA	$w_i = (2 - d_i / d_{\max})$
MS_1c	UFA	$w_i = (4 - d_i / d_{\max})$
MS_1d	UFA	$w_i = (6 - d_i / d_{\max})$
MS_2a	LFA	$w_i = 1;\ i = 1, 2, ..., 6$
MS_2b	LFA	$w_i = (2 - d_i / d_{\max})$
MS_2c	LFA	$w_i = (4 - d_i / d_{\max})$
MS_2d	LFA	$w_i = (6 - d_i / d_{\max})$
MS_3a	UFA+LFA	$w_i = 1;\ i = 1, 2, ..., 6$
MS_3b	UFA+LFA	$w_i = (2 - d_i / d_{\max})$
MS_3c	UFA+LFA	$w_i = (4 - d_i / d_{\max})$
MS_3d	UFA+LFA	$w_i = (6 - d_i / d_{\max})$

The process of selecting the best management strategy can be summarized as follows:

i. Identify fuzzy objectives, i.e., $\widetilde{F}_k;\ k = 1, 2, 3, ..., m$, where m is the total number of fuzzy objectives.

ii. Determine the membership value of each management strategy for each fuzzy objective (i.e., individual satisfaction degree of each management strategy for each fuzzy objective) i.e., $\mu_{s,k}$, $s = 1, 2, 3, ..., r$ *and* $k = 1, 2, 3, ..., m$ where r is the total number of strategies.

iii. Calculate an overall representative degree of performance for each management strategy with respect to all fuzzy objectives, i.e., D_s, $s = 1, 2, 3, ..., r$.

iv. Choose the best management strategy, i.e., the management strategy with the highest D_s.

4.6.2 FUZZY OBJECTIVES AND INDIVIDUAL SATISFACTION DEGREES OF THE MANAGEMENT STRATEGIES FOR EACH FUZZY OBJECTIVE

Management objectives need be determined by the managers of the resource. For the hypothetical study, the following four fuzzy objectives are selected as the critical goals that need to be considered for the Savannah area permit applications:

i. Maintain high satisfaction of the sum of individual demands, $\widetilde{HSD} = \widetilde{F}_1$;
ii. Maintain low drawdowns at critical locations other than Hilton Head Island indicator site (i.e., Tybee Island and Bull Island), $\widetilde{LD} = \widetilde{F}_2$ (note: critical drawdown condition at the indicator site is already satisfied for all cases based on our optimal solution methodology);
iii. Maintain fair groundwater withdrawals (i.e., uniform additional groundwater withdrawals from each demand location), $\widetilde{FG} = \widetilde{F}_3$; and,
iv. Maintain low cost, $\widetilde{LC} = \widetilde{F}_4$

Although the analysis is carried out for a hypothetical example, the fuzzy objectives identified above are representative of groundwater management objectives in the Savannah region. These choices represent our interpretation of the environmental, political, and economical conditions for the region, and by no means, is an interpretation of possible objectives of EPD. Although the proposed analysis is demonstrated by using these four fuzzy objectives, the proposed approach is general and can be applied to other fuzzy objectives that can be identified for the region with relative ease. Four fuzzy objectives we identified above are explained in more detail in the following paragraphs.

i. Maintain high satisfaction of the sum of individual demands, $\widetilde{HSD} = \widetilde{F}_1$

Each groundwater permit applicant demands a certain amount of additional groundwater that will be pumped from the Floridan aquifer system, Dem_i in MGal/day. The subscript i refer to a permit applicant. In this study, six potential applicants are considered as identified earlier. As a result of the coupled simulation-optimization model, the additional available groundwater withdrawals, Q_i in MGal/day, are calculated at each demand location i. As an indicator of the overall satisfaction of the management strategy, the sum of the percent individual satisfactions at each demand location is defined as follows:

$$Satisfaction = \sum_{i=1}^{N} \frac{Q_i}{Dem_i} \times 100 \tag{4.4}$$

where $N = 6$. If the optimum additional amount of groundwater allocated by the optimization model is higher than the demand (i.e., $Q_i \geq Dem_i$) the percent satisfaction of that demand location is taken as 100%. Thus, satisfaction value calculated by using Equation (4.4) ranges between 0 and $N\text{x}100$ (i.e., 600 for the hypothetical case of six demand location selected in this application). Thus, the range 0-600 is used as the domain of the fuzzy objective of high satisfaction of the sum of individual demands. The membership function of $\widetilde{HSD} = \widetilde{F}_1$ is given in Figure 4.12 (a).

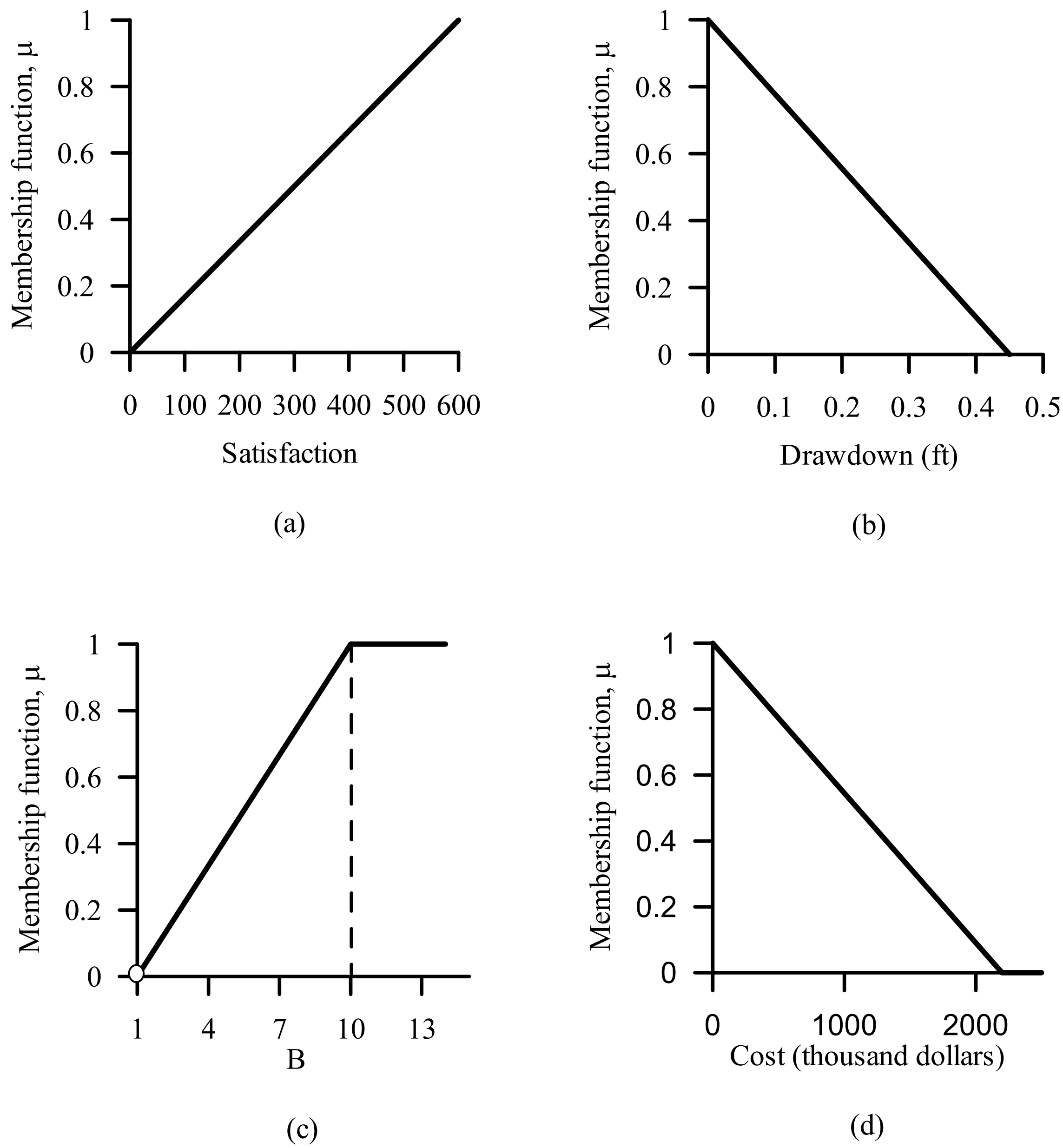


Figure 4.12 Membership function of (a) high satisfaction, $\widetilde{HSD}$, (b) low piezometric head decline, $\widetilde{LD}$, (c) fair groundwater withdrawals, $\widetilde{FG}$, (d) low cost, $\widetilde{LC}$

When the demand is a discrete stochastic process, each discrete demand is associated with a probability of occurrence, and an expected value of the demand needs to be calculated. The procedure to calculate expected value of demand is explained below.

For a total of q states, let Dem represents the set of possible discrete demand values, $Dem = \{Dem\_1, Dem\_2, ..., Dem\_q\}$. Each element in set Dem, Dem_j, is associated with a probability of occurrence, $p(Dem\_j)$ such that the summation of the probabilities over all the states in the set Dem is equal to 1. The expected value of the demand at demand location i, Dem_i, is calculated using the following equation:

$$Dem_i = \sum_{j=1}^{q} Dem\_j \times p\left(Dem\_j\right) \tag{4.5}$$

where i represents the demand location where demand is provided as a discrete stochastic process. To simplify the analysis, in the application discussed here, only demand of Rincon is modeled as a stochastic process. According to the above described procedure, the expected value of demand at Rincon is calculated as 1.42 MGal/day (see hypothetical case under Section 4.4.2.2). This expected value is used in Equation (4.4) as the Dem_i, $i = \text{Rincon}$. All the other demands are provided as deterministic values, $Dem_{\text{Bloomingdale}} = Dem_{\text{Denmark}} = Dem_{\text{Hinesville}} = 2\ MGal/day$ and $Dem_{\text{Marlow}} = Dem_{\text{Ridgeland}}$ $= 1\ MGal/day$ which can be used directly in Equation (4.4).

ii. Maintain low drawdowns at critical locations other than Hilton Head Island indicator site, $\widetilde{LD} = \widetilde{F_2}$

As discussed earlier, Bull Island and Tybee Island are pointed out as critical locations at which saltwater intrusion may occur (EPD 1997), in addition to identified saltwater intrusion location of Hilton Head Island. The drawdown constraint identified by EPD at the northern end of Hilton Head Island is satisfied as an outcome of our optimization analysis at all times. However, the decision-maker may also prefer a management strategy which provides low drawdowns in the UFA at Bull Island and Tybee Island as well. The additional optimum pumping rates at each one of the six demand locations are entered into the simulation model and drawdowns at Bull Island and Tybee Island are calculated with these additional pumping rates. The resulting drawdown at Bull and Tybee Island for each management scenario, relative to the present conditions, is given in Table 4.8.

As can be seen from Table 4.8, the largest drawdown occurs at Tybee Island and it is 0.45 ft. Thus, the domain for the fuzzy objective for low drawdown is chosen as 0 ft to 0.45 ft and a linearly decreasing function is used as the membership function for $\widetilde{LD}$ (see Figure 4.12 (b)). A drawdown of 0 ft fully belongs to the fuzzy set of $\widetilde{LD}$ while drawdowns greater than 0.45 ft does not belong to the fuzzy set of $\widetilde{LD}$ and the drawdowns in between has a linearly decreasing membership value as shown in Figure 4.12 (b).

Table 4.8 Drawdowns in the UFA at Bull Island and Tybee Island with additional optimum pumping rates at six demand locations

Management Strategy	Drawdown (ft)	
	Bull Island	Tybee Island
MS_1a	0.29	0.41
MS_1b	0.29	0.39
MS_1c	0.29	0.37
MS_1d	0.28	0.37
MS_2a	0.29	0.41
MS_2b	0.29	0.39
MS_2c	0.29	0.37
MS_2d	0.28	0.37
MS_3a	0.30	0.45
MS_3b	0.29	0.42
MS_3c	0.29	0.39
MS_3d	0.29	0.38

iii. Maintain fair groundwater withdrawals for all users in the region, $\widetilde{FG}$

As described earlier, the weighting factor in front of the penalty term (Equation 4.1) can be used to adjust the degree of penalizing for the non-uniform pumping rates. As the magnitude of weighting factor increases the optimized pumping rates get close to each other in magnitude while reducing the "zero pumping zone" near Savannah. Thus, the higher the B value is the more uniform the optimized additional pumping rates are in the region. In this study, this condition (i.e., maintaining optimum additional pumping rates at each demand location as close as possible to each other in magnitude) is identified as being fair to all permit applicants in the region. The impact of choosing various w_i's on the optimum additional pumping rates at six demand locations for the LFA, the UFA, and UFA+LFA are given in Tables 4.9, 4.10, and 4.11, respectively.

Table 4.9 Impact of various $w_i's$ on the additional optimum pumping rates at six demand locations for the LFA

w	Pumping Rate (MGal/day)					
	Rincon	Bloomingdale	Marlow	Ridgeland	Denmark	Hinesville
$w_i = 1;\ i = 1, 2, ..., 6$	0.625	0.884	1.011	0.568	1.39	1.408
$w_i = (2 - d_i / d_{\max})$	0.656	0.825	0.915	0.644	1.287	1.301
$w_i = (4 - d_i / d_{\max})$	0.72	0.794	0.831	0.708	0.958	0.963
$w_i = (6 - d_i / d_{\max})$	0.736	0.783	0.806	0.727	0.882	0.886
$w_i = (8 - d_i / d_{\max})$	0.743	0.777	0.795	0.736	0.849	0.851
$w_i = (10 - d_i / d_{\max})$	0.747	0.774	0.788	0.741	0.83	0.832
$w_i = (25 - d_i / d_{\max})$	0.756	0.767	0.772	0.754	0.788	0.788
$w_i = (300 - d_i / d_{\max})$	0.761	0.762	0.762	0.761	0.764	0.764

Table 4.10 Impact of various $w_i's$ on the additional optimum pumping rates at six demand locations for the UFA

w	Pumping Rate (MGal/day)					
	Rincon	Bloomingdale	Marlow	Ridgeland	Denmark	Hinesville
$w_i = 1;\ i = 1, 2, ..., 6$	0.622	0.882	1.013	0.584	1.393	1.411
$w_i = (2 - d_i / d_{\max})$	0.657	0.826	0.919	0.657	1.291	1.305
$w_i = (4 - d_i / d_{\max})$	0.723	0.797	0.836	0.717	0.962	0.968
$w_i = (6 - d_i / d_{\max})$	0.74	0.787	0.811	0.735	0.887	0.891
$w_i = (8 - d_i / d_{\max})$	0.747	0.782	0.8	0.743	0.854	0.856
$w_i = (10 - d_i / d_{\max})$	0.751	0.779	0.793	0.748	0.835	0.837
$w_i = (25 - d_i / d_{\max})$	0.761	0.772	0.777	0.76	0.793	0.794
$w_i = (300 - d_i / d_{\max})$	0.766	0.768	0.768	0.766	0.77	0.77
$w_i = (300 - d_i / d_{\max})$	0.764	0.766	0.766	0.764	0.768	0.768

Table 4.11 Impact of various w_i's on the additional optimum pumping rates at six demand locations for UFA+LFA

w	Pumping Rate (MGal/day)					
	Rincon	Bloomingdale	Marlow	Ridgeland	Denmark	Hinesville
$w_i = 1;\ i = 1, 2, ..., 6$	0.482	1.002	1.26	0.387	2.019	2.054
$w_i = (2 - d_i / d_{\max})$	0.548	0.887	1.07	0.536	1.814	1.842
$w_i = (4 - d_i / d_{\max})$	0.678	0.826	0.903	0.661	1.156	1.166
$w_i = (6 - d_i / d_{\max})$	0.711	0.806	0.853	0.697	1.006	1.012
$w_i = (8 - d_i / d_{\max})$	0.726	0.794	0.83	0.715	0.938	0.944
$w_i = (10 - d_i / d_{\max})$	0.734	0.788	0.816	0.726	0.9	0.904
$w_i = (25 - d_i / d_{\max})$	0.752	0.774	0.784	0.749	0.816	0.818
$w_i = (300 - d_i / d_{\max})$	0.764	0.766	0.766	0.764	0.768	0.768

The changes in additional optimum pumping rates as a function of B are given in Figures 4.13 (a), (b), and (c) for LFA, UFA, and UFA+LFA, respectively. As can be seen from Figure 4.13, for each management scenario (i.e., LFA, UFA, and UFA+LFA) the additional optimum pumping rates at six demand locations converge rapidly to a common optimal pumping rate as B increases to 10. After this point, the optimal pumping rates stabilize at a value slightly less that 0.8 MGal/day. It should be noted that substrategies with $w_i = 1, \forall i$ (the first substrategy in the table above for the LFA, the UFA, and UFA+LFA cases) do not have weighting factors that include a B value, thus are not plotted in Figure 4.13.

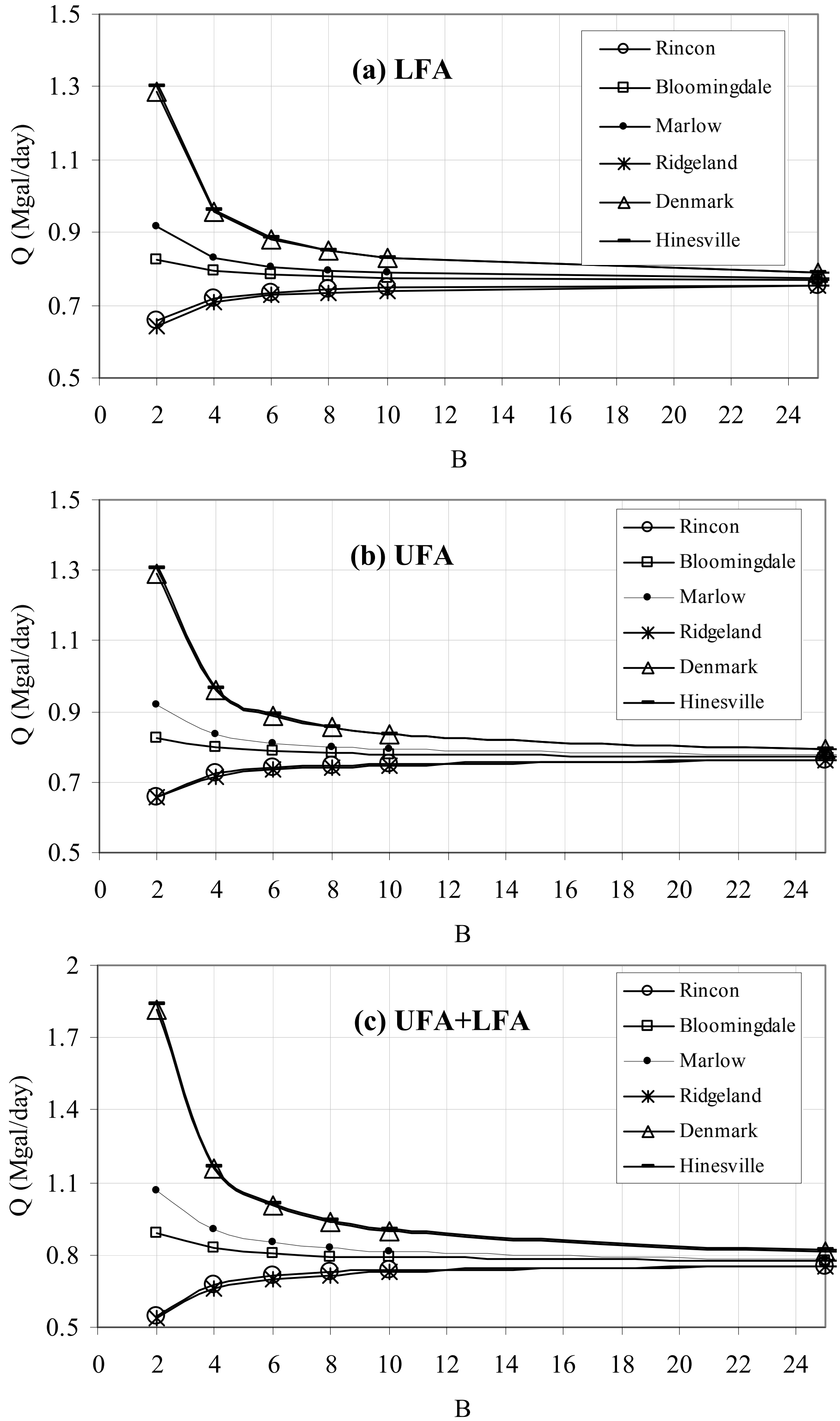
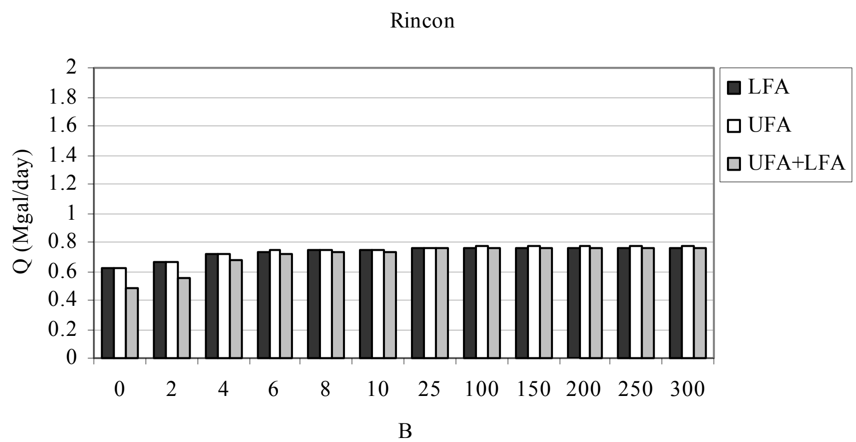
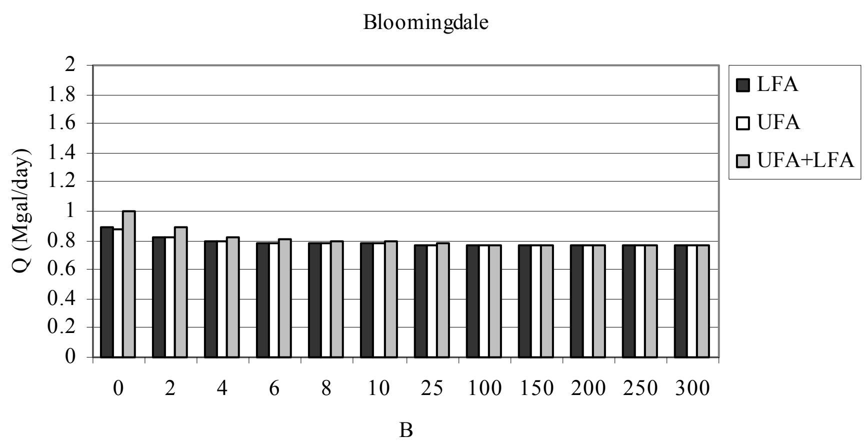


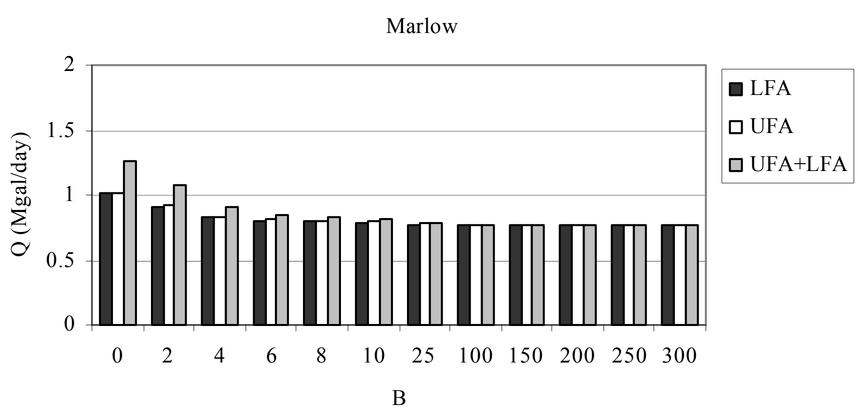
Figure 4.13 Change of additional optimum pumping rates with w_i at each of the six demand locations for (a) LFA, (b) UFA, and (c) UFA+LFA

As can be seen from Tables 4.9, 4.10, and 4.11 choosing $w_i = 1, \forall i$ results in pumping rates that are "not fair" (i.e., additional optimum pumping rates at the demand locations are significantly different from each other in magnitude) when compared to $B = 2, 4, .., 25$ cases as expected. Thus, we will identify the substrategies obtained from $w_i = 1, \forall i$ as the substrategy that does not belong to the fuzzy set of fair groundwater withdrawals (i.e., membership value of substrategies with $w_i = 1, \forall i$ in "fair groundwater withdrawal" is zero). For the rest of the substrategies, the B value is used as an indicator for fairness. The membership function for fair groundwater withdrawal is given in Figure 4.12 (c).

Effect of B on the additional optimum pumping rates at each demand location for three management scenarios (i.e., groundwater withdrawn from the LFA, the UFA, and UFA+LFA) is represented as bar charts in Figure 4.14. As can be seen from Figure 4.14, the additional optimum pumping rates converge to a single value at each one of the demand locations. For Rincon and Ridgeland, which are the closest two cities to the northern end of Hilton Head Island, the optimum pumping rate for UFA+LFA is initially lower than those of the LFA and the UFA. This may be due to the fact that for UFA+LFA case, at each demand location two variables (i.e., one representing the optimum pumping rate from the LFA and the other representing the optimum pumping rate from the UFA) are used in the optimization model. Thus, the variables representing optimum pumping from the LFA and the UFA at Rincon and Ridgeland individually compete with the remaining eight variables (i.e., two variables each at Bloomingdale, Marlow, Denmark, and Hinesville).


Rincon
Q (Mgal/day)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
25
100
150
200
250
300
B
LFA
UFA
UFA+LFA


Bloomingdale
Q (Mgal/day)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
25
100
150
200
250
300
B
LFA
UFA
UFA+LFA


Marlow
Q (Mgal/day)
2
1.5
1
0.5
0
0
2
4
6
8
10
25
100
150
200
250
300
B
LFA
UFA
UFA+LFA

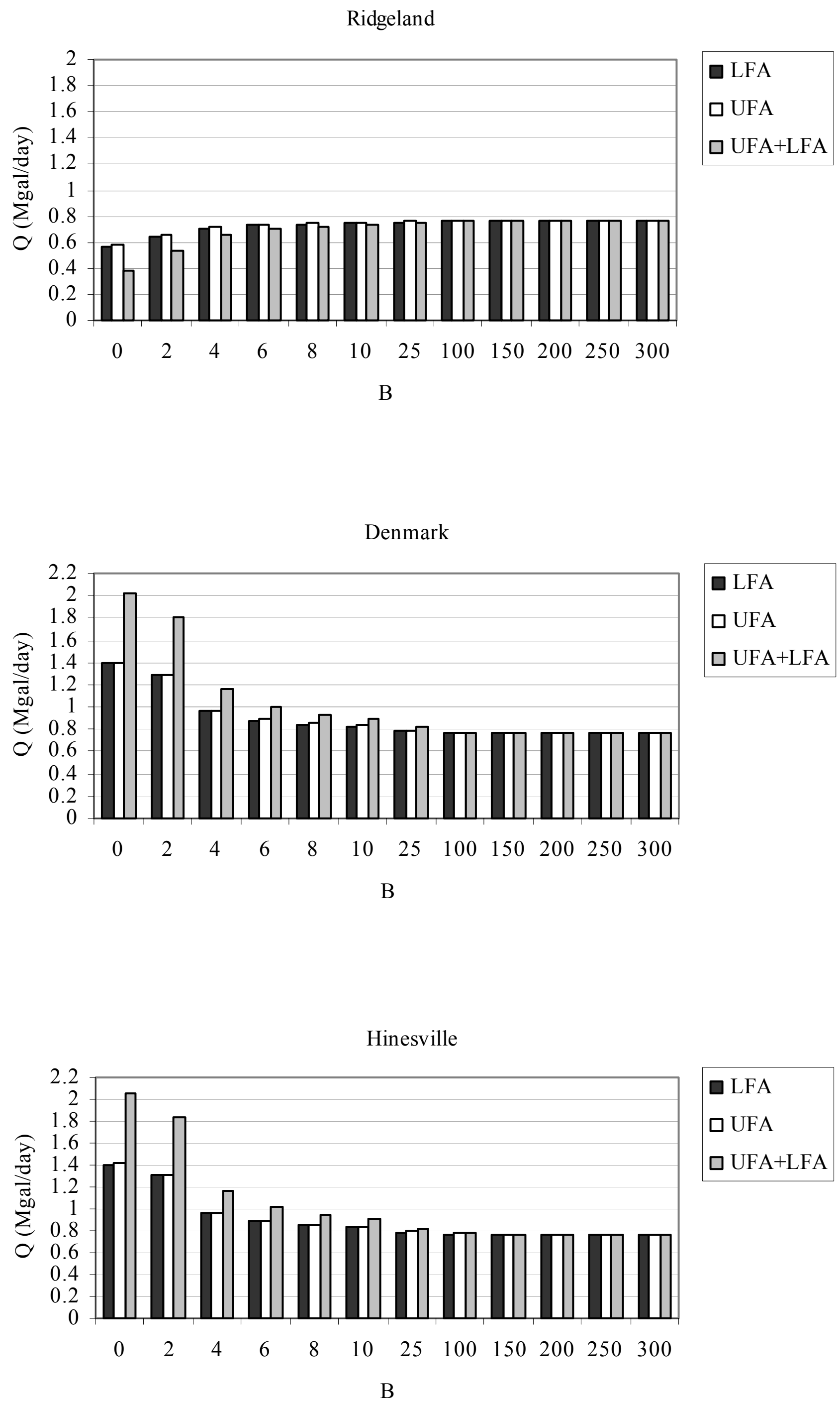


Figure 4.14 Effect of B on additional optimum pumping rates at each demand location

The optimization model favors pumping from the wells that are far away from the northern end of Hilton Head Island. Thus, eight variables representing pumping from the UFA and the LFA at Bloomingdale, Marlow, Denmark, and Hinesville tend to get higher values. As B increases, the high pumping rates from the UFA and the LFA at Bloomingdale, Marlow, Denmark, and Hinesville decreases.

Another important fact that can be observed from the bar charts is that the additional optimum pumping rates at Rincon and Ridgeland increases for all three scenarios, while the pumping rates from the other demand locations decreases as B increases (i.e., fairness increases). However, it should be noticed that the rates of increase at Rincon and Ridgeland is lower than the rates of decrease at Bloomingdale, Marlow, Denmark, and Hinesville. For example, increasing B from 2 to 4 causes a total increase of 0.128 MGal/day at Rincon and Ridgeland when water is pumped from the LFA. However, the combined decrease of pumping rate at the other four demand locations is 0.782 MGal/day. This shows that the impact of changing the pumping rate at locations close to the northern end of Hilton Head Island is more severe (i.e. the effect on the drawdown at the northern end of Hilton Head Island is more) than those at the farther inland demand locations. Thus, in order to keep the drawdown at the northern end of Hilton Head Island below 0.05 ft the slight increases in pumping rates at Rincon and Ridgeland must be balanced by substantially higher decreases in the pumping rates at the other four demand locations.

iv. Maintain low cost, $\widetilde{LC}$

As described earlier the additional optimum pumping rates at each demand location varies for each management scenario. For example, the optimum pumping rate for the first management strategy, MS_1a (i.e., UFA, $w_i = 1, \forall i$) at the city of Denmark in Savannah region is 1.393 MGal/day while at the same location it is only 0.887 MGal/day for the fourth management strategy, MS_1d (i.e., UFA, $w_i = (6 - d_i / d_{\max})$) (see Table 4.10). To generate fair groundwater extraction strategies, higher B values are used in the optimization model, and this results in lower additional optimum pumping rates at Denmark but yield more evenly distributed groundwater extraction rates for the region. Thus, although it is hydraulically possible to grant a pumping rate of 1.393 MGal/day to Denmark in the first management strategy, lower additional optimum pumping rates are assigned to Denmark when other management strategies are considered. The difference between the maximum pumping rate that can be granted to Denmark and the pumping rate granted as a result of one of the other management strategies can be identified as deficit. This deficit may be used as an indication of the additional cost for water supply at Denmark. Since decision-makers have no control on the demand that is requested by the permit applicant, we choose to define the deficit as identified above, and use this definition of deficit to determine the additional cost each management strategy may bear. It should be noted here that the maximum amount of additional pumping that can be granted to a demand location is calculated as a result of the optimization analysis. The maximum amount at each demand location is obtained based on optimal hydraulic control measures, and is the yield at that location given the constraints of the simulation-optimization model.

Lets identify the optimum additional pumping rate at a demand location, i, obtained for the management strategy, j, as "Q_i of MS_j". The maximum additional pumping rate at demand location, i, can be determined by:

$$Q_i^{\max} = \max_{j=1a,1b,\ldots,3d} \{Q_i \text{ of } MS\_j\} \tag{4.6}$$

Then the deficit at demand location, i, can be calculated by:

$$Deficit_i = Q_i^{\max} - Q_i \tag{4.7}$$

The deficit, $Deficit_i$, has to be supplied by some means other than groundwater at the demand location i, and this will cost an extra amount of money for the applicant. These other means may be purchasing water from other municipalities or using surface water with necessary treatment, etc. At each demand location, the alternative source of water may be different, and may cost different amounts. If we call the cost of 1 MGal of water from the alternative source at demand location, i, as c_i, then the total cost in dollars per day for the region for each management strategy can be approximated by:

$$Total\ Cost = \sum_{i=1}^{N} c_i \times Deficit_i \tag{4.8}$$

where $N = 6$. Since we are working with a hypothetical example, the same value of $2 per thousand gallons is used to calculate the cost for all six demand locations. The total annual cost of each management strategy is given in Table 4.12.

Table 4.12 Total annual cost of each management strategy

	Total cost (10^3 $)
MS_1a	1390.65
MS_1b	1573.15
MS_1c	2049.11
MS_1d	2160.07
MS_2a	1404.52
MS_2b	1592.86
MS_2c	2070.28
MS_2d	2182.7
MS_3a	442.38
MS_3b	812.49
MS_3c	1766.6
MS_3d	1989.25

The highest total yearly cost is 2182.7 thousand dollars for MS_2d. Considering the results provided in Table 4.12, a range of 0 to 2200 thousand dollars is used as the domain of the fuzzy set low cost. A linearly decreasing membership function is assigned to the fuzzy set of low cost (see Figure 4.12 (d)). The membership function given in Figure 4.12 (d) is used to determine the satisfaction degree of each management strategy for the fuzzy objective of "maintain low cost." Note that these costs do not represent the real situation by any means. As discussed earlier, the cost of the alternative water source for each demand location may be different in real-world situations. These figures are used to demonstrate the impact of cost on the decision-making process proposed in this study. The second step in the decision-making process is to determine the membership values (i.e., individual satisfaction degrees) of the management strategies for each of the fuzzy objectives. The membership values of the management strategies for each one of the fuzzy objectives are determined using the previously defined membership functions of the fuzzy objectives (Figure 4.12) and they are summarized in Tables 4.13 and 4.14.

Table 4.13 Degrees of satisfaction for management strategies MS_1a, MS_1b, MS_1c, MS_1d, MS_2a, and MS_2b with respect to the fuzzy objectives, $\mu_{s,k}$

Fuzzy Objectives		Membership function, $\mu_{s,k}$, $s = 1,2,..,6$ and $k = 1,2,3,4$					
		MS_1a s = 1	MS_1b s = 2	MS_1c s = 3	MS_1d s = 4	MS_2a S = 5	MS_2b s = 6
$k = 1$ ($\widetilde{HSD}$)		0.65	0.62	0.57	0.56	0.64	0.62
$k = 2$ ($\widetilde{LD}$)	Tybee Island	0.09	0.13	0.18	0.18	0.09	0.13
	Bull Island	0.36	0.36	0.36	0.38	0.36	0.36
$k = 3$ ($\widetilde{FG}$)		0.00	0.11	0.33	0.56	0.00	0.11
$k = 4$ ($\widetilde{LC}$)		0.37	0.28	0.07	0.02	0.36	0.28

Table 4.14 Degrees of satisfaction for management strategies MS_2c, MS_2d, MS_3a, MS_3b, MS_3c, and MS_3d with respect to the fuzzy objectives, $\mu_{s,k}$

Fuzzy objectives		Membership function, $\mu_{s,k}$, $s = 7,8,..,12$ and $k = 1,2,3,4$					
		MS_2c s = 7	MS_2d s = 8	MS_3a s = 9	MS_3b s = 10	MS_3c s = 11	MS_3d s = 12
$k = 1$ ($\widetilde{HSD}$)		0.57	0.55	0.75	0.71	0.60	0.58
$k = 2$ ($\widetilde{LD}$)	Tybee Island	0.18	0.18	0.00	0.07	0.13	0.16
	Bull Island	0.36	0.38	0.33	0.36	0.36	0.36
$k = 3$ ($\widetilde{FG}$)		0.33	0.56	0.00	0.11	0.33	0.56
$k = 4$ ($\widetilde{LC}$)		0.06	0.01	0.80	0.63	0.20	0.10

As can be seen from Table 4.13, the individual satisfactions for management strategies considering groundwater withdrawal from the UFA and from the LFA using various penalty functions are very similar to each other. For example, compare the columns for MS_1a and MS_2a, or MS_1b and MS_2b. Since the individual satisfactions are very similar for these two types of management strategies, the overall satisfaction values will be similar too. The overall satisfaction degrees using various sets of fuzzy objectives are calculated and presented in the following section.

4.6.3 SELECTING THE BEST GROUNDWATER MANAGEMENT STRATEGY FOR VARIOUS SETS OF FUZZY OBJECTIVES

The hypothetical case study for Savannah region problem is analyzed using three different sets of fuzzy objectives, $S_f,\ f = 1,2,3$. Management strategies are evaluated with respect to these three sets of fuzzy objectives using a conjunctive, a disjunctive, and an averaging operator. The three sets of fuzzy objectives considered in this study are:

1. <u>*Case 1*</u>*:* $S_1 = \left\{\widetilde{HSD}\right\}$
2. <u>*Case 2*</u>*:* $S_2 = \left\{\widetilde{HSD}, \widetilde{FG}\right\}$
3. <u>*Case 3*</u>*:* $S_3 = \left\{\widetilde{HSD}, \widetilde{LD}, \widetilde{FG}, \widetilde{LC}\right\}$

4.6.3.1 CONJUNCTIVE OPERATOR, “AND”

<u>Case 1</u>: $S_1 = \left\{ \widetilde{HSD} \right\}$

This case only considers the first fuzzy objective (i.e., maintain high satisfaction of the sum of individual demands) as the decision-making criteria. Since there is only one fuzzy objective, the management strategy which has the highest membership function in the fuzzy set of $\widetilde{HSD}$ is selected as the best management strategy. As can be seen from Table 4.14, MS_3a has the highest membership value, 0.75. If the decision-maker does not have any other objective but to maintain a high satisfaction of the sum of individual demands (i.e., indirectly extracting as much as possible from the aquifer), MS_3a is the best management strategy that satisfies this objective.

<u>Case 2</u>: $S_2 = \left\{ \widetilde{HSD}, \widetilde{FG} \right\}$

This case considers both maintenance of high satisfaction of the sum of individual demands and maintenance of fair groundwater withdrawals for all users in the region as the fuzzy objectives. If these two fuzzy objectives have equal importance, the overall representative performance of each management strategy is calculated as follows:

$$\begin{array}{lll} D_{1a} = 0.65 \cap 0.00 = 0.00 & D_{1b} = 0.62 \cap 0.11 = 0.11 & D_{1c} = 0.57 \cap 0.33 = 0.33 \\ D_{1d} = 0.56 \cap 0.56 = 0.56 & D_{2a} = 0.64 \cap 0.00 = 0.00 & D_{2b} = 0.62 \cap 0.11 = 0.11 \\ D_{2c} = 0.57 \cap 0.33 = 0.33 & D_{2d} = 0.55 \cap 0.56 = 0.55 & D_{3a} = 0.75 \cap 0.00 = 0.00 \\ D_{3b} = 0.71 \cap 0.11 = 0.11 & D_{3c} = 0.60 \cap 0.33 = 0.33 & D_{3d} = 0.58 \cap 0.56 = 0.56 \end{array} \quad (4.9)$$

As can be seen from Equation (4.9), the overall representative performance of MS_3a decreased to zero. MS_3a corresponds to the management strategy which allows pumping from both the upper and the lower aquifers (i.e., a higher total pumping rate is obtained as a result of the optimization model) with $w_i = 1, \forall i$ (i.e., penalizing the non-uniform distribution to the least degree when compared to $B = 2, 4, \mathit{or}\ 6$ cases). Thus, it was selected as the best management strategy for Case 1 above. However, Case 2 also considers the fuzzy objective of maintenance of fair groundwater extraction for all users in the region. Since, MS_3a uses $w_i = 1, \forall i$, the non-uniform pumping rates are not strongly penalized so the optimization model favors the wells that are away from the indicator site (i.e., the northern end of Hilton Head Island), and assigns higher pumping rates to those wells. This causes respectively more unfair groundwater extraction rates compared to the results obtained by using $B = 2, 4, \mathit{or}\ 6$. It can be seen from Tables 4.13 and 4.14 that penalizing non-uniform pumping rates by choosing larger values for B, improves fairness (i.e., membership values in fuzzy objective fair groundwater extraction increases as B increases); however, the total amount of water that can be extracted from the aquifer decreases, causing the degree of maintenance of high satisfaction of the sum of individual demands get smaller. So there is a trade off between these two conflicting fuzzy objectives. Due to this trade off the overall performance of MS_3a decreases to zero.

For Case 2, the best management strategies are MS_1d and MS_3d which both have overall satisfaction degrees of 0.56. Overall satisfaction of MS_2d is very close to those of MS_1d and MS_3d. In these management strategies a value of 6 is assigned to B, thus they have high individual satisfactions for fairness. Although the individual satisfactions of MS_1d and MS_3d in $\widetilde{HSD}$ are not as high as those of MS_3a and MS_3b, the overall satisfactions when both $\widetilde{HSD}$ and $\widetilde{FG}$ are considered are higher.

Now let's consider the same case with unequal importance given to each fuzzy objective. For this case, lets assume that maintenance of high satisfaction of the sum of individual demands is more important then maintenance of fair groundwater extraction rates: $w_{\widetilde{HSD}} = 2$, and $w_{\widetilde{FG}} = 1$. The overall performances are calculated as follows:

$$\begin{array}{lll}
D_{1a} = 0.65^2 \cap 0.00 = 0.00 & D_{1b} = 0.62^2 \cap 0.11 = 0.11 & D_{1c} = 0.57^2 \cap 0.33 = 0.32 \\
D_{1d} = 0.56^2 \cap 0.56 = 0.31 & D_{2a} = 0.64^2 \cap 0.00 = 0.00 & D_{2b} = 0.62^2 \cap 0.11 = 0.11 \\
D_{2c} = 0.57^2 \cap 0.33 = 0.32 & D_{2d} = 0.55^2 \cap 0.56 = 0.30 & D_{3a} = 0.75^2 \cap 0.00 = 0.00 \\
D_{3b} = 0.71^2 \cap 0.11 = 0.11 & D_{3c} = 0.60^2 \cap 0.33 = 0.33 & D_{3d} = 0.58^2 \cap 0.56 = 0.34
\end{array} \tag{4.10}$$

As can be seen from Equation (4.10), MS_3d is still the best management strategy. However, notice that overall satisfactions of MS_1c, MS_1d, MS_2c, MS_2d, MS_3c, and MS_3d are all very close to each other. Thus when higher importance is assigned to $\widetilde{HSD}$, management strategies that use $B = 4 \ or \ 6$ perform equally well. While overall performances of MS_1a, MS_1b, MS_2a, MS_2b, MS_3a, MS_3b, and MS_3c are determined by maintenance of fair groundwater extractions objective, maintenance of high satisfaction of the sum of individual demands determines the overall performances of the rest of the management strategies. As can be seen from the comparison of Equations (4.9) and (4.10), the overall satisfaction of MS_1d and MS_3d decreased significantly when maintenance of high satisfaction of the sum of individual demands is considered to be twice as important as maintenance of fair groundwater extractions. However, MS_3d still has the highest overall satisfaction.

Case 3: $S_3 = \left\{ \widetilde{HSD}, \widetilde{LD}, \widetilde{FG}, \widetilde{LC} \right\}$

For Case 3, all fuzzy objectives are considered. Assuming each fuzzy objective has equal importance, the overall performances using the conjunctive operator "and" are calculated as follows:

$$\begin{aligned}
D_{1a} &= 0.65 \cap 0.09 \cap 0.36 \cap 0.00 \cap 0.37 = 0.00 \\
D_{1b} &= 0.62 \cap 0.13 \cap 0.36 \cap 0.11 \cap 0.28 = 0.11 \\
D_{1c} &= 0.57 \cap 0.18 \cap 0.36 \cap 0.33 \cap 0.07 = 0.07 \\
D_{3d} &= 0.56 \cap 0.18 \cap 0.38 \cap 0.56 \cap 0.02 = 0.02 \\
D_{2a} &= 0.64 \cap 0.09 \cap 0.36 \cap 0.00 \cap 0.36 = 0.00 \\
D_{2b} &= 0.62 \cap 0.13 \cap 0.36 \cap 0.11 \cap 0.28 = 0.11 \\
D_{2c} &= 0.57 \cap 0.18 \cap 0.36 \cap 0.33 \cap 0.06 = 0.06 \\
D_{2d} &= 0.55 \cap 0.18 \cap 0.38 \cap 0.56 \cap 0.01 = 0.01 \\
D_{3a} &= 0.75 \cap 0.00 \cap 0.33 \cap 0.00 \cap 0.80 = 0.00 \\
D_{3b} &= 0.71 \cap 0.07 \cap 0.36 \cap 0.11 \cap 0.63 = 0.11 \\
D_{3c} &= 0.60 \cap 0.13 \cap 0.36 \cap 0.33 \cap 0.20 = 0.13 \\
D_{3d} &= 0.58 \cap 0.16 \cap 0.36 \cap 0.56 \cap 0.10 = 0.10
\end{aligned} \tag{4.11}$$

As can be seen from Equation (4.11), all the management strategies perform very poorly when all the fuzzy objectives are considered using the conjunctive operator "and" for aggregation. The highest overall satisfaction is 0.13 and it is attained by MS_3c. However, overall satisfactions of MS_1b, MS_2b, and MS_3b are 0.11 which is very close to that of MS_3c. Using the conjunctive operator "and" to aggregate fuzzy objectives results in a low overall performance value whenever one of the individual satisfaction degrees is low. For example for MS_3a, the individual satisfactions of $\widetilde{HSD}$ and $\widetilde{LC}$ are very high, but since the individual satisfactions of $\widetilde{LD}$ and $\widetilde{FG}$ are zero the overall satisfaction of MS_3a is zero. Aggregation with "and" does not have the compensation property, and it generates a high value only if all the individual satisfaction degrees are high.

This result indicates that aggregating all the fuzzy objectives with an "and" does not help the decision-maker to make a preference among the possible management scenarios. One alternative may be using a different aggregation operator which has compensation property (i.e., lower values are compensated by higher values by using an averaging algorithm). An averaging operator like OWA to calculate the overall performances of the management strategies may result in more reasonable overall satisfaction degrees. Impact of using OWA as the aggregator operator on the results is presented in Section 4.6.3.3.

Since all the overall satisfaction degrees are already very low and making a decision using the results provided in Equation (4.11) is not possible, assigning higher importance values and trying to conduct the analysis does not make sense. Because assigning higher importance values will only decrease the overall satisfaction degrees which are already very low.

4.6.3.2 Disjunctive operator, “or”

In this study we chose to use “or” as the disjunctive operator. The analysis for three different cases is provided below.

<u>Case 1</u>: $S_1 = \left\{ \widetilde{HSD} \right\}$

This case only considers the first fuzzy objective (i.e., maintenance of high satisfaction of the sum of individual demands) as the decision-making criteria. Since there is only one fuzzy objective, the best management strategy is not affected by the choice of the aggregation operator. Thus the best management strategy is MS_3a.

<u>Case 2</u>: $S_2 = \left\{ \widetilde{HSD}, \widetilde{FG} \right\}$

Maintenance of high satisfaction of the sum of individual demands and maintenance of fair groundwater withdrawals for all users in the region as the fuzzy objectives are considered for this case. The overall performance of each management strategy is calculated as follows:

$$\begin{array}{lll}
D_{1a} = 0.65 \cup 0.00 = 0.65 & D_{1b} = 0.62 \cup 0.11 = 0.62 & D_{1c} = 0.57 \cup 0.33 = 0.57 \\
D_{1d} = 0.56 \cup 0.56 = 0.56 & D_{2a} = 0.64 \cup 0.00 = 0.64 & D_{2b} = 0.62 \cup 0.11 = 0.62 \\
D_{2c} = 0.57 \cup 0.33 = 0.57 & D_{2d} = 0.55 \cup 0.56 = 0.56 & D_{3a} = 0.75 \cup 0.00 = 0.75 \\
D_{3b} = 0.71 \cup 0.11 = 0.71 & D_{3c} = 0.60 \cup 0.33 = 0.60 & D_{3d} = 0.58 \cup 0.56 = 0.58
\end{array} \qquad (4.12)$$

The best management strategy with two fuzzy objectives (i.e., $\widetilde{HSD}$ and $\widetilde{FG}$) is MS_3a. However, all the management strategies have overall satisfactions of greater than 0.5. When Equation (4.9) and Equation (4.12) are compared, it can be observed that overall performances of all the management strategies are higher for the disjunctive aggregator case when compared to the conjunctive aggregator case. This is due to the fact that the disjunctive operator (i.e., "or" in this study), assigns the highest individual satisfaction degree as the overall satisfaction. This behavior of disjunctive operator is not appropriate for evaluating management alternatives when decision-makers' goal is to find the best strategy which satisfies all the objectives. For example, MS_3a has an overall performance of 0.75. This result only guarantees that MS_3a performs good with one of the objectives (i.e., $\widetilde{HSD}$). The performance of the other objectives can be anything. Actually, MS_3a performs very poorly for $\widetilde{FG}$ objective.

<u>Case 3</u>: $S_3 = \left\{\widetilde{HSD}, \widetilde{LD}, \widetilde{FG}, \widetilde{LC}\right\}$

In this case, all the potential fuzzy objectives are included in the decision. The overall representative performances are calculated as follows:

$$
\begin{aligned}
D_{1a} &= 0.65 \cup 0.09 \cup 0.36 \cup 0.00 \cup 0.37 = 0.65 \\
D_{1b} &= 0.62 \cup 0.13 \cup 0.36 \cup 0.11 \cup 0.28 = 0.62 \\
D_{1c} &= 0.57 \cup 0.18 \cup 0.36 \cup 0.33 \cup 0.07 = 0.57 \\
D_{1d} &= 0.56 \cup 0.18 \cup 0.38 \cup 0.56 \cup 0.02 = 0.56 \\
D_{2a} &= 0.64 \cup 0.09 \cup 0.36 \cup 0.00 \cup 0.36 = 0.64 \\
D_{2b} &= 0.62 \cup 0.13 \cup 0.36 \cup 0.11 \cup 0.28 = 0.62 \\
D_{2c} &= 0.57 \cup 0.18 \cup 0.36 \cup 0.33 \cup 0.06 = 0.57 \\
D_{2d} &= 0.55 \cup 0.18 \cup 0.38 \cup 0.56 \cup 0.01 = 0.56 \\
D_{3a} &= 0.75 \cup 0.00 \cup 0.33 \cup 0.00 \cup 0.80 = 0.80 \\
D_{3b} &= 0.71 \cup 0.07 \cup 0.36 \cup 0.11 \cup 0.63 = 0.71 \\
D_{3c} &= 0.60 \cup 0.13 \cup 0.36 \cup 0.33 \cup 0.20 = 0.60 \\
D_{3d} &= 0.58 \cup 0.16 \cup 0.36 \cup 0.56 \cup 0.10 = 0.58
\end{aligned}
\tag{4.13}
$$

As in Case 2, MS_3a is again the best management strategy; however it has an overall satisfaction degree of 0.8 for Case 3. When all the fuzzy objectives are considered, the overall performance of each management strategy gets the highest overall satisfaction value it can get. One useful information, the disjunctive operator provides is that a low overall performance for a specific alternative indicates that the performance of that specific alternative is poor for all the objectives. For example for MS_2d and MS_1d, we can conclude that these management strategies do not perform very well with any of the fuzzy objectives.

4.6.3.3 AVERAGING OPERATOR, "OWA"

Between conjunctive and disjunctive operators, there exists a third category, namely averaging operators. Here, we choose ordered weight averaging (OWA) as an example application of averaging operators.

Now lets assume the quantifier guiding the aggregation is "most" and it is defined by $Q(r) = r^2$. This translates into "the decision-maker desires to satisfy most of the fuzzy objectives." Evaluation of management strategies for all three cases is provided below.

<u>Case 1</u>: $S_1 = \left\{ \widetilde{HSD} \right\}$

Again the best management strategy is not affected by the choice of aggregation operator since there is only one fuzzy objective to be satisfied. So MS_3a is the best management strategy.

<u>Case 2</u>: $S_2 = \left\{ \widetilde{HSD}, \widetilde{FG} \right\}$

When maintenance of high satisfaction of the sum of individual demands and maintenance of fair groundwater withdrawals for all users in the region are the fuzzy objectives with equal importance, the weights associated with these two criteria using "most" as the quantifier are calculated as follows:

$$\begin{aligned} v_1 &= Q\left(\frac{1}{2}\right) - Q\left(\frac{1-1}{2}\right) = Q(0.5) - Q(0) = 0.25 \\ v_2 &= Q\left(\frac{2}{2}\right) - Q\left(\frac{2-1}{2}\right) = Q(1) - Q(0.5) = 0.75 \end{aligned} \tag{4.14}$$

The second step is to aggregate the fuzzy objectives using Equation (G.5) given in Appendix G:

$$D_{1a} = \sum_{j=1}^{2} v_j b_j = (0.25 \times 0.65) + (0.75 \times 0.0) = 0.16$$

$$\begin{array}{llll} \textit{similarly} & & & \\ D_{1b} = 0.24 & D_{1c} = 0.39 & D_{1d} = 0.56 & D_{2a} = 0.16 \\ D_{2b} = 0.24 & D_{2c} = 0.39 & D_{2d} = 0.56 & D_{3a} = 0.19 \\ D_{3b} = 0.26 & D_{3c} = 0.40 & D_{3d} = 0.57 & \end{array} \tag{4.15}$$

As can be seen from Equation (4.15), the best management strategy is MS_3d which has an overall satisfaction of 0.57. However, MS_1d and MS_2d have overall satisfactions of 0.56, which is very close to that of MS_3d. Thus, these three management strategies perform better than the rest when both maintenance of high satisfaction of the sum of individual demands and maintenance of fair groundwater withdrawals for all users in the region are considered.

The results obtained for MS_1a, MS_2a, and MS_3a are very different than the ones obtained by the conjunctive operator (see Equation (4.9)). For example, MS_1a had an overall satisfaction of 0.0 with the conjunctive operator while it has an overall satisfaction of 0.16 with OWA operator. This is because the OWA operator uses some sort of averaging. It calculates a combined score instead of just using the smallest individual satisfaction like conjunctive operator “and” does. The OWA operator compensates an individual satisfaction degree of zero when it is evaluated together with another individual satisfaction degree of 0.65 (see Table 4.13 for MS_1a). Thus, it is more reasonable to rank the alternatives according to the overall performances obtained by using the OWA operator rather than the conjunctive operator.

Now lets consider a case in which maintenance of high satisfaction of the sum of individual demands is more important then maintenance of fair groundwater extraction rates, i.e., $w_{\widetilde{HSD}} = 2$, and $w_{\widetilde{FG}} = 1$. The procedure to calculate overall satisfaction for the management strategy, MS_1a:

	b_j	u_j
$\widetilde{HSD}$	0.65	2.0
$\widetilde{FG}$	0.00	1.0

and $T = \sum_{j=1}^{2} u_j = 3$

$$
\begin{aligned}
v_1 &= Q\left(\frac{2}{3}\right) - Q\left(\frac{0}{3}\right) = Q\left(0.\bar{6}\right) - Q\left(0\right) = 0.44 \\
v_2 &= Q\left(\frac{3}{3}\right) - Q\left(\frac{2}{3}\right) = Q(1) - Q\left(0.\bar{6}\right) = 0.56 \\
D_{1a} &= (0.65 \times 0.44) + (0.0 \times 0.56) = 0.29
\end{aligned}
\tag{4.16}
$$

The procedure to calculate overall satisfaction for the management strategy, MS_2d:

	b_j	u_j
$\widetilde{FG}$	0.56	1.0
$\widetilde{HSD}$	0.55	2.0

and $T = \sum_{j=1}^{2} u_j = 3$

$$
\begin{aligned}
v_1 &= Q\left(\frac{1}{3}\right) - Q\left(\frac{0}{3}\right) = Q\left(0.\bar{3}\right) - Q\left(0\right) = 0.11 \\
v_2 &= Q\left(\frac{3}{3}\right) - Q\left(\frac{1}{3}\right) = Q(1) - Q\left(0.\bar{3}\right) = 0.89 \\
D_{2d} &= (0.56 \times 0.11) + (0.55 \times 0.89) = 0.55
\end{aligned}
\tag{4.17}
$$

Similarly, overall performances for the rest of the management strategies are calculated and provided below:

$$D_{1b}=0.33 \quad D_{1c}=0.44 \quad D_{1d}=0.56 \quad D_{2a}=0.28$$
$$D_{2b}=0.33 \quad D_{2c}=0.44 \quad D_{3a}=0.33 \quad D_{3b}=0.37 \tag{4.18}$$
$$D_{3c}=0.45 \quad D_{3d}=0.57$$

Assigning different importance values to fuzzy objectives either increased or did not significantly change the overall satisfaction degrees of each management strategy when compared to the equal importance case (see Equation (4.15)). However, the best strategy is still MS_3d. Since individual satisfactions for $\widetilde{HSD}$ are higher than individual satisfactions of $\widetilde{FG}$ for all of the management strategies other than MS_2d, the overall satisfactions increased when higher importance is assigned to maintenance of high satisfaction of the sum of individual demands.

Overall satisfaction degrees obtained by using three different aggregation operators (i.e., "and", "or", and "OWA") for Case 2 are plotted in Figure 4.15. When the individual satisfactions of two fuzzy objectives (i.e., maintenance of high satisfaction of the sum of individual demands and maintenance of fair groundwater withdrawals for all users) are aggregated using the disjunctive operator "or" the overall satisfactions decrease as B increases for the LFA, the UFA, and UFA+LFA. For example, when groundwater is extracted from the LFA, the overall satisfaction decreases from 0.62 to 0.56 for $B=2$ (i.e., MS_1b) and $B=6$ (i.e., MS_1d), respectively. The overall satisfaction for LFA when $w_i=1;\ i=1,2,...,6$ (i.e., MS_1a) is 0.65 which is higher than $B=2$ case. This is due to the fact that penalizing non-uniform withdrawal rates by increasing B causes withdrawal rates from each demand location close to Hilton Head Island to increase. However, at the same time the withdrawal rates from demand locations further away from Hilton Head Island decreases. As explained earlier, the relative decrease is higher than the increase thus the individual satisfaction for $\widetilde{HSD}$ decreases as B increases. Although increasing B causes the individual satisfaction for $\widetilde{FG}$ to increase, the resulting individual satisfaction degrees for $\widetilde{FG}$ are not greater than the individual satisfaction degrees for $\widetilde{HSD}$ (note that only for MS_2d the individual satisfaction of $\widetilde{FG}$ is greater than that of $\widetilde{HSD}$). Thus $\widetilde{HSD}$ is still the controlling objective (i.e., the overall satisfaction degree is the individual satisfaction degree of $\widetilde{HSD}$). As a result, the overall satisfaction degrees when the fuzzy objectives are aggregated using "or" decreases as non-uniform withdrawal rates penalized more (i.e., B increases).

When the conjunctive operator "and" and averaging operator "OWA" are considered, just the opposite trend can be observed in Figure 4.15. As B increase the overall performance of the management strategies increases. For the conjunctive operator case when $w_i = 1;\ i = 1, 2, \ldots, 6$ is used as the weighting factor (i.e., MS_1a, MS_2a, and MS_3a) the individual satisfactions for the fuzzy objective $\widetilde{FG}$ are zero. The overall satisfactions for MS_1a, MS_2a, and MS_3a are also zero when the conjunctive operator "and" is used since it assigns the minimum individual satisfaction as the overall satisfaction. As B increases from 2 to 4 to 6, the individual satisfaction values for $\widetilde{FG}$ increases to 0.11 to 0.33 to 0.56, respectively. As explained earlier for all the management strategies other than MS_2d, $\widetilde{HSD}$ has higher individual satisfaction degrees than those for $\widetilde{FG}$. Hence, for the conjunctive operator case $\widetilde{FG}$ is the controlling objective and the overall satisfactions increase as individual satisfaction of $\widetilde{FG}$ increases (i.e., B increases). The

“OWA” operator compensates for the lower individual satisfaction degrees with higher ones. As can be seen from Figure 4.15, “OWA” always assigns overall satisfaction degrees that fall in between those of “and” and “or” cases.

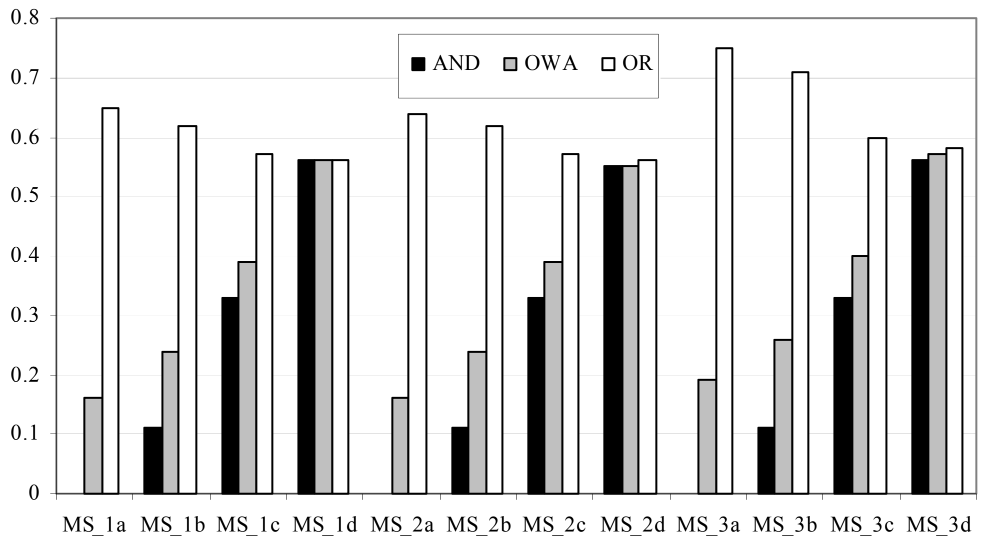


Figure 4.15 Comparison of overall satisfaction degrees for Case 2 using “and”, “or”, and “OWA” as the aggregation operators

<u>Case 3</u>: $S_3 = \left\{ \widetilde{HSD}, \widetilde{LD}, \widetilde{FG}, \widetilde{LC} \right\}$

When all four fuzzy objectives are equally important, the weights associated with these objectives are calculated as follows (maintenance of low drawdown at Tybee and Bull Islands are considered as two separate fuzzy objectives):

$$
\begin{aligned}
&v_1 = Q\left(\frac{1}{5}\right) - Q\left(\frac{1-1}{5}\right) = Q(0.2) - Q(0) = 0.04 \\
&v_2 = Q\left(\frac{2}{5}\right) - Q\left(\frac{2-1}{5}\right) = Q(0.4) - Q(0.2) = 0.12 \\
&v_3 = Q(0.6) - Q(0.4) = 0.2 \\
&v_4 = Q(0.8) - Q(0.6) = 0.28 \\
&v_5 = Q(1.0) - Q(0.8) = 0.36
\end{aligned}
\tag{4.19}
$$

The overall performances when all the fuzzy objectives are considered are as follows:

$$
\begin{aligned}
&D_{1a} = 0.04 \times 0.65 + 0.12 \times 0.37 + 0.2 \times 0.36 + 0.28 \times 0.09 + 0.36 \times 0.0 = 0.17 \\
&\textit{similarly} \\
&D_{1b} = 0.20 \qquad D_{1c} = 0.21 \qquad D_{1d} = 0.22 \qquad D_{2a} = 0.17 \\
&D_{2b} = 0.20 \qquad D_{2c} = 0.20 \qquad D_{2d} = 0.22 \qquad D_{3a} = 0.19 \\
&D_{3b} = 0.23 \qquad D_{3c} = 0.24 \qquad D_{3d} = 0.24
\end{aligned}
\tag{4.20}
$$

As can be seen from Equation (4.20), when all the fuzzy objectives are considered the overall satisfaction degrees of the management strategies decrease when compared to Case 2 (see Equations (4.16) to (4.18)). The best management strategies are MS_3c and MS_3d, however MS_3b perform almost as good as MS_3c and MS_3d. Actually, all the management strategies have overall satisfactions around 0.2. It should be noticed that when the number of fuzzy objectives to be satisfied increases the overall performances of the alternatives decrease. This is due to the fact that as more relatively small individual satisfaction degrees are included in the averaging process, the overall satisfaction degrees decrease.

The overall satisfaction degrees for Case 3 in which all the fuzzy objectives are considered to have equal importance are plotted for “and,” “or,” and “OWA” (see Figure 4.16). The overall satisfaction degrees obtained by using “and” as the aggregation operator are respectively low while those obtained by using “or” are much higher. Since “OWA” compensates the low individual satisfactions with the higher ones while aggregating the fuzzy objective, the overall satisfaction degrees obtained by “OWA” lie in between the results obtained by “and” and “or” aggregation operators. We observed from the results that in evaluating groundwater resources management alternatives “OWA” that utilize some sort of averaging mechanism produces more reasonable results when compared to “and” and “or” as aggregation operators.

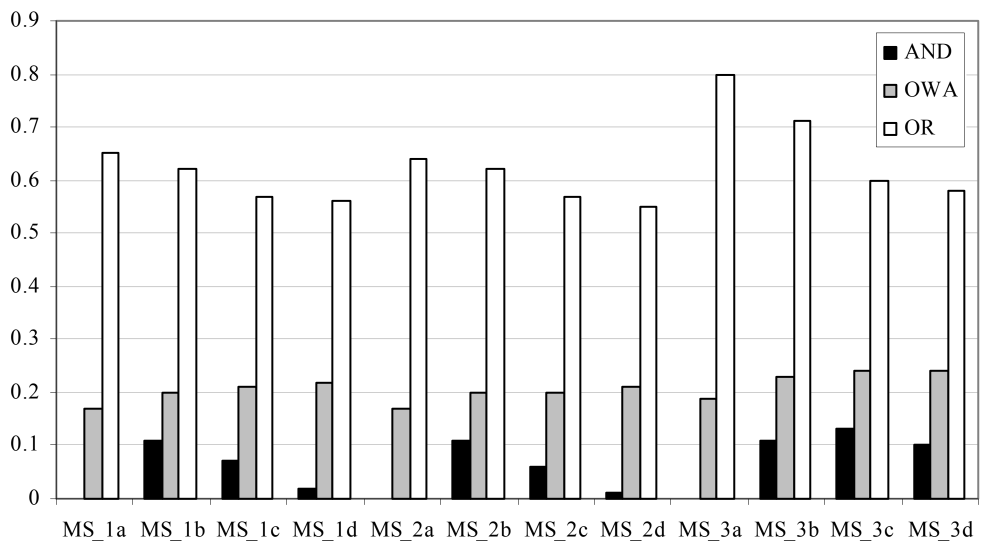


Figure 4.16 Comparison of overall satisfaction degrees for Case 3 using “and”, “or”, and “OWA” as the aggregation operators

As can be seen from Equation (4.20), although satisfaction degrees are higher than those calculated for the conjunctive aggregator case (Equation (4.11)), the overall satisfaction degrees are low. Lets now examine how will these results be effected if we assign higher importance values for $\widetilde{HSD}$ and $\widetilde{FG}$. Consider, maintenance of high satisfaction of the sum of individual demands and maintenance of fair groundwater withdrawals are more important than the rest of the fuzzy objectives: $w_{\widetilde{HSD}} = 2$, $w_{\widetilde{LD}\_Tybee} = 1$, $w_{\widetilde{LD}\_Bull} = 1$, $w_{\widetilde{FG}} = 2$, $w_{\widetilde{LC}} = 1$. The details of the calculations and the results are provided below. Note that maintenance of low drawdown at Tybee and Bull Islands are again considered as two separate fuzzy objectives.

For MS_1a, MS_1b, MS_2a and MS_2b the overall satisfactions are calculated as:

	b_{MS_1a}	u_j
$\widetilde{HSD}$	0.65	2
$\widetilde{LC}$	0.37	1
$\widetilde{LD}\_B$	0.36	1
$\widetilde{LD}\_T$	0.09	1
$\widetilde{FG}$	0.00	2

	b_{MS_1b}	u_j
$\widetilde{HSD}$	0.62	2
$\widetilde{LD}\_B$	0.36	1
$\widetilde{LC}$	0.28	1
$\widetilde{LD}\_T$	0.13	1
$\widetilde{FG}$	0.11	2

	b_{MS_2a}	u_j
$\widetilde{HSD}$	0.64	2
$\widetilde{LC}$	0.36	1
$\widetilde{LD}\_B$	0.36	1
$\widetilde{LD}\_T$	0.09	1
$\widetilde{FG}$	0.00	2

	b_{MS_2b}	u_j
$\widetilde{HSD}$	0.62	2
$\widetilde{LD}\_B$	0.36	1
$\widetilde{LC}$	0.28	1
$\widetilde{LD}\_T$	0.13	1
$\widetilde{FG}$	0.11	2

$$and\ \ T = \sum_{j=1}^{5} u_j = 7$$

$$v_1 = Q\left(\frac{2}{7}\right) - Q\left(\frac{0}{7}\right) = 0.0816 \qquad v_2 = Q\left(\frac{3}{7}\right) - Q\left(\frac{2}{7}\right) = 0.1020$$

$$v_3 = Q\left(\frac{4}{7}\right) - Q\left(\frac{3}{7}\right) = 0.1429 \qquad v_4 = Q\left(\frac{5}{7}\right) - Q\left(\frac{4}{7}\right) = 0.1837$$

$$v_5 = Q\left(\frac{7}{7}\right) - Q\left(\frac{5}{7}\right) = 0.4898$$

$$\begin{aligned} D_{1a} &= (0.65\times0.0816) + (0.37\times0.102) + (0.36\times0.1429) \\ &\quad + (0.09\times0.1837) + (0.00\times0.4898) = 0.16 \\ D_{1b} &= (0.62\times0.0816) + (0.38\times0.102) + (0.28\times0.1429) \\ &\quad + (0.13\times0.1837) + (0.11\times0.4898) = 0.21 \\ D_{2a} &= (0.64\times0.0816) + (0.36\times0.102) + (0.36\times0.1429) \\ &\quad + (0.09\times0.1837) + (0.00\times0.4898) = 0.16 \\ D_{2b} &= (0.62\times0.0816) + (0.36\times0.102) + (0.28\times0.1429) \\ &\quad + (0.13\times0.1837) + (0.11\times0.4898) = 0.21 \end{aligned} \tag{4.21}$$

For MS_1c, MS_2c, and MS_3c the overall satisfactions are calculated as follows:

	b_{MS_1c}	u_j
$\widetilde{HSD}$	0.57	2
$\widetilde{LD\_B}$	0.36	1
$\widetilde{FG}$	0.33	2
$\widetilde{LD\_T}$	0.18	1
$\widetilde{LG}$	0.07	1

	b_{MS_2c}	u_j
$\widetilde{HSD}$	0.57	2
$\widetilde{LD\_B}$	0.36	1
$\widetilde{FG}$	0.33	2
$\widetilde{LD\_T}$	0.18	1
$\widetilde{LC}$	0.06	1

	b_{MS_3c}	u_j
$\widetilde{HSD}$	0.60	2
$\widetilde{LD\_B}$	0.36	1
$\widetilde{FG}$	0.33	2
$\widetilde{LC}$	0.20	1
$\widetilde{LD\_T}$	0.13	1

$$v_1 = Q\left(\frac{2}{7}\right) - Q\left(\frac{0}{7}\right) = 0.0816 \qquad v_2 = Q\left(\frac{3}{7}\right) - Q\left(\frac{2}{7}\right) = 0.1020$$

$$v_3 = Q\left(\frac{5}{7}\right) - Q\left(\frac{3}{7}\right) = 0.3265 \qquad v_4 = Q\left(\frac{6}{7}\right) - Q\left(\frac{5}{7}\right) = 0.2245$$

$$v_5 = Q\left(\frac{7}{7}\right) - Q\left(\frac{6}{7}\right) = 0.2653$$

$$\begin{aligned} D_{1c} &= (0.57\times0.0816) + (0.36\times0.102) + (0.33\times0.3265) \\ &\quad + (0.18\times0.2245) + (0.07\times0.2653) = 0.25 \\ D_{2c} &= (0.57\times0.0816) + (0.36\times0.102) + (0.33\times0.3265) \\ &\quad + (0.18\times0.2245) + (0.06\times0.2653) = 0.25 \\ D_{3c} &= (0.60\times0.0816) + (0.36\times0.102) + (0.33\times0.3265) \\ &\quad + (0.20\times0.2245) + (0.13\times0.2653) = 0.27 \end{aligned} \tag{4.22}$$

For MS_1d, MS_2d, and MS_3d, the overall satisfactions are calculated as follows:

	b_{MS_1d}	u_j
$\widetilde{HSD}$	0.56	2
$\widetilde{FG}$	0.56	2
$\widetilde{LD}\_B$	0.38	1
$\widetilde{LD}\_T$	0.18	1
$\widetilde{LC}$	0.02	1

	b_{MS_2d}	u_j
$\widetilde{FG}$	0.56	2
$\widetilde{HSD}$	0.55	2
$\widetilde{LD}\_B$	0.38	1
$\widetilde{LD}\_T$	0.18	1
$\widetilde{LC}$	0.01	1

	b_{MS_3d}	u_j
$\widetilde{HSD}$	0.58	2
$\widetilde{FG}$	0.56	2
$\widetilde{LD}\_B$	0.36	1
$\widetilde{LD}\_T$	0.16	1
$\widetilde{LC}$	0.10	1

$v_1 = 0.0816 \quad v_2 = 0.2449 \quad v_3 = 0.1837 \quad v_4 = 0.2245 \quad v_5 = 0.2653$

$$\begin{aligned} D_{1d} &= (0.56\times 0.0816)+(0.56\times 0.2449)+(0.38\times 0.1837) \\ &\qquad +(0.18\times 0.2245)+(0.02\times 0.2653)=0.30 \\ D_{2d} &= (0.56\times 0.0816)+(0.55\times 0.2449)+(0.38\times 0.1837) \\ &\qquad +(0.18\times 0.2245)+(0.01\times 0.2653)=0.29 \\ D_{3d} &= (0.58\times 0.0816)+(0.56\times 0.2449)+(0.36\times 0.1864) \\ &\qquad +(0.16\times 0.2245)+(0.10\times 0.2653)=0.31 \end{aligned} \tag{4.23}$$

For MS_3a the overall satisfaction is calculated as follows:

	b_{MS_3a}	u_j
$\widetilde{LC}$	0.80	1
$\widetilde{HSD}$	0.75	2
$\widetilde{LD}\_B$	0.33	1
$\widetilde{LD}\_T$	0.00	1
$\widetilde{FG}$	0.00	2

$$\begin{aligned} v_1 &= 0.0204 \\ v_2 &= 0.1633 \\ v_3 &= 0.1429 \\ v_4 &= 0.1837 \\ v_5 &= 0.4897 \end{aligned} \tag{4.24}$$

$$\begin{aligned} D_{3a} &= (0.80\times 0.0204)+(0.75\times 0.1633)+(0.33\times 0.1429) \\ &\qquad +(0.00\times 0.1837)+(0.00\times 0.4897)=0.19 \end{aligned}$$

Finally, for MS_3b, the overall satisfaction degree is calculated as follows:

$$
\begin{array}{|lcc|}
 & b_{MS\_3b} & u_j \\
\widetilde{HSD} & 0.71 & 2 \\
\widetilde{LC} & 0.63 & 1 \\
\widetilde{LD}\_B & 0.36 & 1 \\
\widetilde{FG} & 0.11 & 2 \\
\widetilde{LD}\_T & 0.07 & 1
\end{array}
\qquad
\begin{array}{l}
v_1 = 0.0816 \\
v_2 = 0.1020 \\
v_3 = 0.1429 \\
v_4 = 0.4082 \\
v_5 = 0.2653
\end{array}
\tag{4.25}
$$

$$
\begin{aligned}
D_{3b} = & (0.71 \times 0.0816) + (0.63 \times 0.102) + (0.36 \times 0.1429) \\
& + (0.11 \times 0.4082) + (0.07 \times 0.2653) = 0.24
\end{aligned}
$$

As can be seen from Equations (4.21) through (4.25), MS_3d has the highest overall satisfaction degree of 0.31. MS_1d and MS_2d have overall satisfaction degrees of 0.3 and 0.29, respectively which are very close to that of MS_3d. All the other management strategies have respectively lower overall satisfactions. Thus, for this unequal importance case MS_3d, MS_2d, and MS_1d out perform all the other alternatives. This indicates that when all the fuzzy objectives are considered with unequal objectives, the management strategies with $B = 6$ perform better than the others. With the selected membership functions, our solution favors the management strategies which forces uniform groundwater withdrawal rates throughout the model domain. This analysis indicates that pumping from the LFA, the UFA, or UFA+LFA has minor significance when compared to maintenance of a fair groundwater withdrawal strategy.

4.7 Conclusions for Groundwater Resources Management in Savannah Region

The UFA is the main source of drinking and industrial process water in the Savannah region. Long-term pumping from the UFA has significantly lowered the groundwater piezometeric heads in the area. Seawater encroachment into the aquifer at the northern end of Hilton Head Island has been identified (Clarke and Krause 2000; Clarke and Krause 2001; Garza and Krause 1996). The saltwater contamination has constrained further development of the UFA in the region. Future economic development in the region depends on appropriate management of the water supply sources. Earlier studies conducted to aid these management decisions seem to emphasize only the localized hydraulic management of the aquifer, and do not consider the effect of multiple additional extraction wells on the decision-making process, nor do they consider the optimal solution to the problem. In this study, we evaluated the optimum additional groundwater supply potential in the Savannah region due to simultaneous pumping rates at the demand locations and proposed a robust, objective, and systematic approach to determine the best groundwater management strategy among many alternatives. The proposed approach also allows various forms of uncertainties to be included in the decision-making process using a heuristic approach that employs “fuzzy” algebra.

The groundwater resources supply potential in the Savannah region is evaluated with respect to two separate goals: (i) determining the spatial distribution of the additional groundwater supply potential within the model domain; and, (ii) evaluating multiple groundwater withdrawal permit applications. To achieve the first goal, we developed a coupled simulation-optimization model. The coupled simulation-optimization model utilizes the USGS Savannah Area Model without alteration. Further, the optimization model developed employs the management constraint (i.e., drawdown at the northern end of Hilton Head Island should not exceed 0.05 ft) as identified by EPD (1997; 2001; 2003; 2004) without alteration as well. Our use of this constraint in this study does not imply that we agree with the scientific basis of this criterion. We selected this constraint, as identified by EPD, in order to not introduce further variability into an already very complex problem. The primary purpose of this study is the development of a methodology which combines “optimal management options” with “heuristic management objectives (i.e., objectives identified in linguistic terms)” and identification of the “best” overall management decision that is suitable for the utilization of the resource under current management constraints. In doing so, a robust and objective methodology for solving the multi-objective decision-making problem that is identified for Savannah, GA is developed.

The first part of our analysis is directed to evaluate the first goal identified above. For this purpose, a total of 70 potential well locations are used to represent the subareas in the Savannah region. Equal pumping rate contours are plotted for three cases in which groundwater is withdrawn from: (i) the UFA; (ii) the LFA; and, (iii) UFA+LFA. The objective function of the optimization model includes a penalty term which can be adjusted to obtain fair groundwater withdrawal rates throughout the model domain. By “fair,” we mean the optimum additional pumping rates are forced to be as close to each other as possible in magnitude, thus decreasing the size of the “zero pumping zone.” As the analysis shows, there is a trade-off between obtaining a "fair" spatial distribution throughout the model domain and the total amount of additional groundwater that can be withdrawn from the Savannah region. More fair pumping rates (i.e., additional optimum pumping rates are closer in magnitude to each other) yield less total additional groundwater withdrawal. The spatial distribution of the additional groundwater withdrawal potential in the Savannah region may provide preliminary guidance in long-term management and planning goals. For example, these results may provide guidance in the site selection processes of the future industries which are planning to apply for groundwater withdrawal permits.

The second part of the analysis considers a hypothetical case in which a total of six groundwater permit applications are placed for the next planning period. Our goal here is to develop an approach which can be used in evaluating these multiple groundwater permit applications objectively. For this case, the coupled simulation-optimization model developed earlier is used to calculate the additional amount of groundwater withdrawals that can be granted to each of these six demand locations. As an extension to the coupled simulation-optimization model, a decision-making framework is also proposed to evaluate various groundwater management alternatives based on their potential for satisfying additional management criteria that may be important for the region as a whole. Groundwater withdrawals from the UFA, the LFA, and UFA+LFA using various weighting factors are chosen as alternative management strategies. These management strategies are evaluated with respect to a set of additional heuristic objectives. Groundwater resources management problems in coastal areas may involve various conflicting objectives. We identified four heuristic objectives such that they may represent the real-world situation in the Savannah region to the best of our knowledge. However, the approach proposed in this study is general, and it can be used with any other set of objectives with relative ease. The selection of these objectives and their appropriate parameters is the task of the managers of the resource.

The coupled simulation-optimization model proposed in this study provides the optimum additional pumping rates from the potential wells, and the decision-making framework allows evaluation of each management strategy with respect to a pre-selected set of heuristic objectives. The proposed fuzzy multi-objective decision-making approach provides an avenue for including fuzzy information together with probabilistic information into the decision-making process. These are the benefits of the proposed approach when compared to the other studies conducted to evaluate the groundwater resources potential in the Savannah region.

In this study, we identified four heuristic objectives considering the current needs and constraints of the Savannah region. Twelve management strategies are evaluated with respect to three different cases (i.e. each case considers a different combination of these four heuristic objectives). Case 1 considers only one objective, maintenance of high satisfaction of the sum of individual demands, $\widetilde{HSD}$. This case is simple, because there is only one objective to be fulfilled, and no aggregation operation is required. MS_3a has the highest overall satisfaction degree of 0.75. Thus, when a decision-makers only goal is to satisfy the demand, MS_3a (i.e., groundwater is withdrawn from UFA+LFA, and $w_i = 1;\ i = 1, 2, ..., 6$) is chosen as the best management strategy.

The second case considers both maintenance of high satisfaction of the sum of individual demands and maintenance of fair groundwater withdrawals for all users in the region, $\widetilde{FG}$. When decision-makers' goal is to satisfy most of the fuzzy objectives, OWA operator can be used to aggregate the individual satisfaction degrees into a single overall performance. Considering two fuzzy objectives, $\widetilde{HSD}\ and\ \widetilde{FG}$, and using OWA operator the best groundwater management strategy is determined to be MS_3d. The overall satisfaction of MS_3d is 0.57. However, MS_1d and MS_2d have overall satisfactions of 0.56. All of these management strategies use $w_i = \left(6 - d_i / d_{\max}\right)$ as the weighting factor which penalizes unfair groundwater withdrawal rates more severely compared to the other three choices of w_i's (i.e., $w_i = 1, \forall i$ and $w_i = \left(B - d_i / d_{\max}\right)$, $B = 2\ or\ 4$). Thus, when maintenance of fair groundwater withdrawals for all users in the region is added to the set of fuzzy objectives, the best management strategies are the ones that use high penalty factors for non-uniform optimum additional pumping rates.

The last case considers all four fuzzy objectives: (i) maintenance of high satisfaction of the sum of individual demands; (ii) maintenance of fair groundwater withdrawals for all users in the region; (iii) maintenance of low drawdowns at Tybee and Bull Islands; and, (iv) maintenance of low cost. When the decision-makers' goal is to select the best groundwater management strategy which satisfies most of these four fuzzy objectives, the

OWA operator which allows compensation should be utilized to aggregate individual satisfaction degrees into an overall performance value. When all the fuzzy objectives have equal importance, and OWA is used as the aggregation operator, MS_3c and MS_3d cases where groundwater is withdrawn from UFA+LFA using $B = 4$ *and* $B = 6$, respectively, are selected as the best management strategies. However, as can be seen from the results presented in Equations (4.19) and (4.20) all the management strategies perform close to one another for this case (i.e. all overall performance values are around 0.2). This indicates that with the selected four fuzzy objectives, the individual satisfaction degrees for each one of the alternative management strategies are compensated in a way to result in similar overall satisfaction degrees. Each management strategy performs well with respect to some of the fuzzy objectives and poorly with respect to the others, and for each management strategy, the high and low individual satisfactions are attained for different fuzzy objectives. However, the overall performances which are obtained as a result of counter balancing the individual satisfactions are similar to each other.

In real-world situations, the decision-maker may want to assign higher importance values to some of the fuzzy objectives. For example, in this study, we considered a situation in which maintenance of high satisfaction of the sum of individual demands and maintenance of fair groundwater withdrawals are considered to be more important than maintenance of low drawdowns at Tybee and Bull Islands and maintenance of low cost. MS_3d, which has an overall satisfaction of 0.31, is selected as the best management strategy when the OWA operator is used to aggregate the individual satisfaction degrees such that most of the fuzzy objectives are satisfied. MS_1d and MS_2d have overall satisfaction degrees of 0.3 and 0.29, respectively, which are very close to that of MS_3d. All the other management strategies have respectively lower overall satisfactions thus drop out of further consideration.

The proposed approach is useful for those cases in which the decision-maker has crisp goals and constraints as well as non-crisp ones. The crisp goals are used in the optimization model and the fuzzy ones are used during the decision-making process. The decision problem we are solving here has a collection of alternatives and a collection of fuzzy objectives. For each alternative we can evaluate the degree to which it satisfies each of the fuzzy objectives (i.e., individual satisfaction degrees). In order to determine the overall performance of an alternative, its individual satisfaction degrees need to be aggregated into a single overall performance value. Evaluation of the overall performance of multiple fuzzy objectives can be performed using various aggregation operators, and they result in different "best" solutions. In this study, satisfaction degrees which define to what extent a given management strategy is satisfactory with respect to a selected fuzzy objective are aggregated using a conjunctive (i.e., intersection operator), a disjunctive (i.e., union operator), and an averaging (i.e., OWA) operator.

Conjunctive operator aggregates the criteria by a logical "and." Thus, the overall performance is high if and only if all the individual performances are high. However, the conjunctive operator does not have any compensation mechanism. If one of the individual performances is low, then the overall performance of the management strategy is low as well. Thus, the conjunctive aggregator does not provide an average overall performance, but rather it provides the worst degree a management strategy will respond to a set of fuzzy objectives. On the other hand, disjunctive operators perform aggregation where criteria are combined by a logical "or." In this case, the overall performance is high when at least one of the individual performances is high. The overall performance for aggregation with the disjunctive operator is low if and only if all the individual performances are low. Thus, disjunctive operators are useful when ruling out alternatives.

Unlike the conjunctive operator, the disjunctive operator does not punish the management strategy for low individual satisfaction as long as one of the fuzzy objectives has high individual satisfaction. The disjunctive operator assigns the best individual satisfaction degree as the overall satisfaction. Thus, adding new fuzzy objectives to the decision-making problem can only increase the overall satisfaction degree. This causes loss of some useful information. To summarize, the conjunctive operator is useful for situations in which all the criteria should be satisfied, while the disjunctive operator is useful for the cases in which the satisfaction of any of the criteria is required.

The OWA operator is a general case. It is possible to utilize OWA operator like a conjunctive or a disjunctive operator by choosing the appropriate V vector (see Equation (G.4) in Appendix G). OWA operator also allows decision-makers' preferences to be included in the selection process. Using quantifier guided aggregation it is possible to implement preferences like "most/many/few of the objectives should be satisfied." Although the best management strategies determined by conjunctive and OWA operators are the same for some of the scenarios in this study, we observed that results obtained by using OWA operators deliver more information to the decision-maker. Thus, utilization of an OWA operator to aggregate the fuzzy objectives of the groundwater management problem considered in this study seems beneficial and leads to more informed decisions.

The proposed approach can also treat unequal objectives for conjunctive and OWA operator cases. This treatment forces the objectives with higher importance values to become more conclusive in the decision process. For the Savannah problem, when all four fuzzy objectives with equal importance are aggregated using OWA operator all the management strategies perform almost equally (i.e., all the overall satisfactions are close to 0.2). Although, each management strategy have very different individual satisfactions with respect to various fuzzy objectives, the compensative aggregation used by the OWA operator results in similar overall performances for each strategy. However, when different importance values are assigned to the fuzzy objectives such as maintenance of high satisfaction of the sum of individual demands and maintenance of fair groundwater withdrawals for all users in the region are more important than the other fuzzy objectives (i.e., $w_{\widetilde{HSD}}=2$, $w_{\widetilde{LD}\_Tybee}=1$, $w_{\widetilde{LD}\_Bull}=1$, $w_{\widetilde{FG}}=2, w_{\widetilde{LC}}=1$), management strategies with $B=6$ perform better than the others. With the selected membership functions, our solution favors the management strategies which force uniform groundwater withdrawal rates throughout the domain. Thus, when maintenance of high satisfaction of the sum of individual demands and maintenance of fair groundwater withdrawals for all users in the region are more important then the other two fuzzy objectives, withdrawing groundwater from which aquifers (i.e., the UFA, the LFA, or UFA+LFA) does not effect the decision significantly.

The proposed decision-making framework will be useful in evaluating groundwater management alternatives in coastal regions if the objectives and the membership functions of these objectives are specifically determined using the needs and requirements of the region and goals of the decision-makers. When combined with expert knowledge and real-world data (i.e., real groundwater withdrawal permit applications, both locations and demands, cost of alternative water supply sources, allowable drawdowns at specific critical locations, etc.) the proposed approach may provide objective guidance in evaluating alternative groundwater management strategies in coastal areas.

In conclusion, we provided here some insight to a number of methodologies that may be useful in coastal aquifer management. Based on our findings and our understanding of saltwater intrusion problems in coastal areas, the next step of analysis should at a minimum include the following components in order to provide a rigorous scientific solution of the problem identified in Savannah region:

i. Three-dimensional density dependent flow and transport analysis of the saltwater intrusion problem in the region.
ii. Optimal solutions based on multiple extraction wells during a planning period which utilizes the density dependent flow and transport models developed in the first stage above as the predictor.

iii. A heuristic decision-making process which incorporates several other objectives into the decision-making process. A multi-objective analysis framework described in this study will be suitable for this purpose.

iv. A groundwater flow simulation model which allows integration of uncertainties in model parameters, such as transmissivities, into the solution.

v. Finally, a framework which integrates degree of satisfaction of the constraints, in the presence of these hydrogeologic uncertainties (i.e., transmissivity), into the decision-making process. The groundwater simulation model with uncertain parameters and the decision-making framework which accomplishes these goals is proposed in Chapter 5.

We are of the opinion that a scientific study which includes all components identified above will yield a robust management tool for the region. Such a study, when fully documented, will also provide a sound direction to follow in other coastal area management applications as well. Given the state of current literature, the analytic decision-making processes, such as the one suggested here, should be an integral component of sound aquifer management in coastal areas.

5 GROUNDWATER FLOW SIMULATION WITH IMPRECISE PARAMETERS AND SUCCESSIVE DECISION-MAKING FRAMEWORK FOR THE SAVANNAH REGION

In Chapter 4, a coupled simulation-optimization model and a decision-making framework is developed. Within this framework, aggregated satisfaction of various fuzzy objectives for alternative management scenarios are used to select the best groundwater management strategy. In the coupled simulation-optimization model, for groundwater flow simulations, we used the Savannah Area Model developed by USGS without any alteration. The Savannah Area Model is a deterministic model. Thus, no uncertainty associated with the groundwater flow parameters is treated within this model. However, parameters such as transmissivities used in groundwater flow simulations are usually imprecise. In this chapter, it is our goal to apply the groundwater model operator method proposed by Dou et al. (1995) to groundwater simulation in the Savannah region in the presence of fuzzy transmissivities.

The groundwater model operator method allows incorporation of imprecise parameters characterized by membership distributions into the groundwater flow model. As a result of this method fuzzy hydraulic heads may be generated at specific locations within the study domain based on fuzzy uncertainty in some parameters of the groundwater flow model. In this application coupled simulation-optimization approach proposed in Chapter 4 is not utilized and a direct uncertainty solution is employed since our focus here is on the treatment of groundwater flow model uncertainty. This chapter also includes a decision-making procedure which may be used to evaluate performance of various groundwater management scenarios in the presence of these imprecise parameters. The proposed decision-making framework makes use of the risk tolerance measure developed in Section 3.5.2. In the following section various simplifications applied to the groundwater simulation model for the Savannah region is reviewed.

5.1 STEADY STATE GROUNDWATER FLOW SIMULATION FOR THE UFA

The Savannah Area Model developed by USGS is a multi-layer model. This model is designed to actively simulate flow in the UFA and the LFA (Clarke and Krause 2000). In this model a finite difference approach is used to simulate groundwater flow in a multilayer aquifer system. The total area within the model boundaries is about 6,700 mi^2 (Garza and Krause 1996) and a 76x88 finite difference grid with square elements is used to idealize the region. Each square element has an area of 1 mi^2. Transmissivities and vertical leakance values at each node are input to the model. The steady state simulations are conducted to represent the conditions for long-term response of the aquifer system.

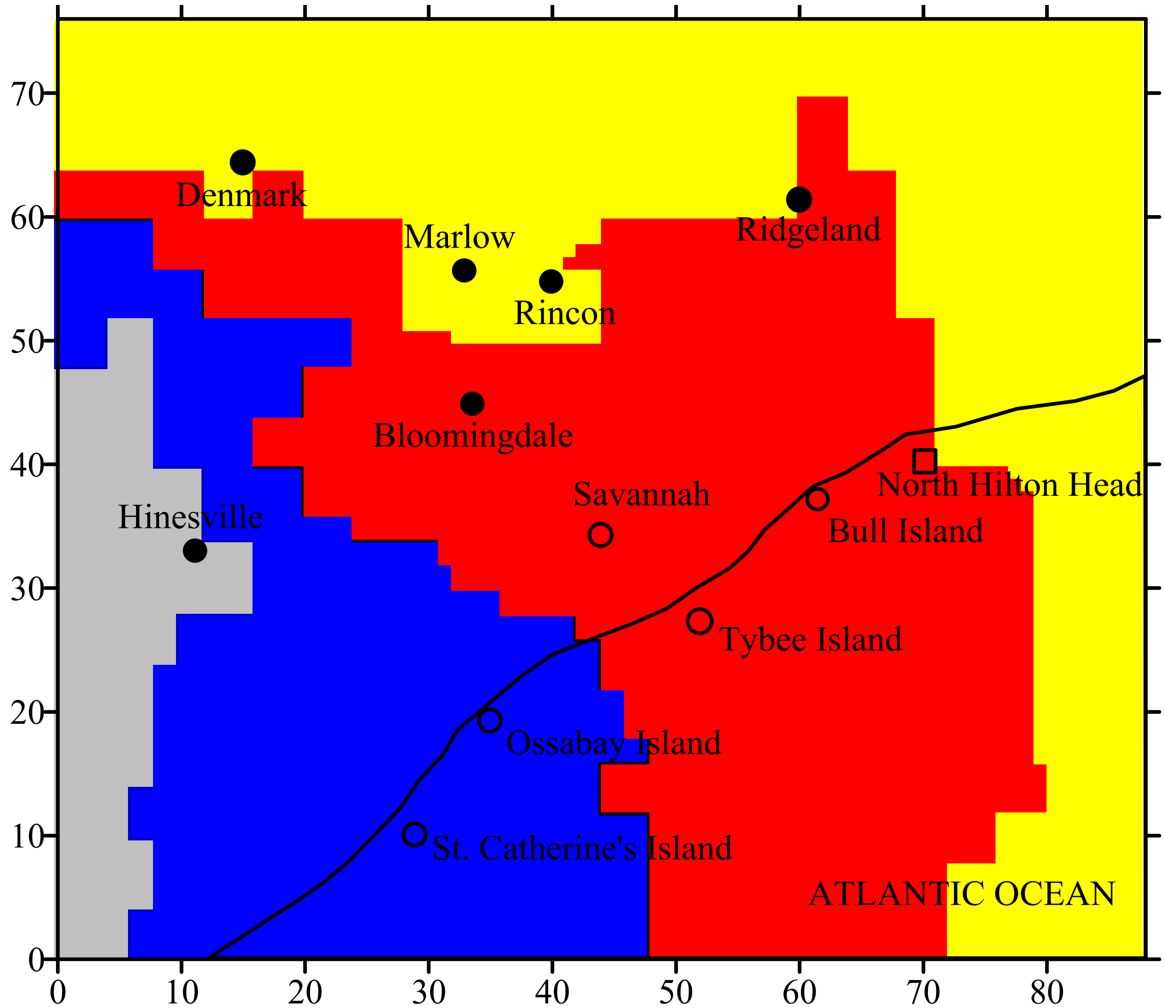
In this chapter, our goal is to conduct steady state groundwater flow simulation with imprecise parameters (i.e., transmissivities) in the UFA. Thus, we developed a two-dimensional finite difference model for a 19x22 grid with 16 mi^2 square elements as a simplified idealization based on the Savannah Area Model described above. Another simplification we implemented in our model is that we divided the model domain into four zones in which transmissivities are represented by four distinct ranges again based on the Savannah Area Model. From hereafter we will refer to this simplified model as the

Single Layer Model. Four zones of transmissivities in layer 2 of the Savannah Area Model (i.e., layer 2 represents the Upper Floridan Aquifer in the Savannah Area Model) are depicted in Figure 5.1. We used the mid values of these ranges, 0.14 ft^2/sec, 0.435 ft^2/sec, 0.895 ft^2/sec, and 1.805 ft^2/sec in the Single Layer Model for zones 1, 2, 3, and 4, respectively.

The governing equation for a two-dimensional, steady state, heterogenous, isotropic, and confined groundwater flow system, provided that the principal axes of transmissivity are aligned with the coordinate directions may be represented by (Frind 2003):

$$\frac{\partial}{\partial x}\left(T(x,y)\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T(x,y)\frac{\partial h}{\partial y}\right)+L_q(x,y)=\sum_{w=1}^{k} Q\delta(x-x_w,y-y_w) \qquad (5.1)$$

where $T\left(x,y\right)$ is the transmissivity (L^2T^{-1}), h is the piezometric head (L), $L_q(x,y)$ is the leakage flux into or out of the aquifer (LT^{-1}), Q is the recharge or pumping rate (L^3T^{-1}) of a well located at $\left(x_w,y_w\right)$, $\delta(x-x_w,y-y_w)$ is the dirac-delta function (L^{-2}), and k is the total number of wells. L is positive if the leakage flux is into the aquifer and negative if it is out of the aquifer. Similarly, Q is positive if the well is a discharge well and negative if it is a recharge well.



Transmissivities in square ft per second

(yellow)	0.00-0.28	zone 1
(red)	0.29-0.58	zone 2
(blue)	0.59-1.20	zone 3
(gray)	1.21-2.40	zone 4

Figure 5.1 Four zones of transmissivity in the UFA

The leakage flux term in Equation (5.1) accounts for leakage from both the top and bottom of the UFA. The leakage term for various cases (i.e., additional pumping wells with various pumping rates are considered) are approximated by using the leakance values that are provided in the databases of the Savannah Area Model and the final head distributions in the layers at the top and bottom (from hereafter we will refer to the aquifers which are located at the top of the UFA and underneath the UFA as the top and bottom layers, respectively) of the UFA:

$$
\begin{aligned}
&L_q(x,y) = L_{q\_top}(x,y) + L_{q\_bottom}(x,y) \\
&L_{q\_top}(x,y) = -K_{top}\frac{\left(h-h_{top}\right)}{b} = Leakance_{top}\left(h-h_{top}\right) \\
&L_{q\_bottom}(x,y) = -K_{bottom}\frac{\left(h-h_{bottom}\right)}{b} = Leakance_{bottom}\left(h-h_{bottom}\right)
\end{aligned}
\tag{5.2}
$$

where $L_{q\_top}(x,y)$ and $L_{q\_bottom}(x,y)$ are leakance values from the top and bottom layers, respectively (LT^{-1}), K_{top} and K_{bottom} are hydraulic conductivities of the top and the bottom layers (LT^{-1}), b_{top} and b_{bottom} are thicknesses of the top and bottom layers (LT^{-1}), h_{top} and h_{bottom} are hydraulic heads at the top and the bottom layers (L).

The terms $-K_{top}/b_{top}$ and $-K_{bottom}/b_{bottom}$ are referred as $Leakance_{top}$ and $Leakance_{bottom}$ and can be directly obtained from the databases of the Savannah Area Model for each node. Hydraulic heads at the top and the bottom layers for each node can be extracted from the outputs of the Savannah Area Model. This allows us to represent the original multi-layer model with a single layer model which only considers the UFA. The simplifications we used in the Single Layer Model compared to the Savannah Area Model are given in Table 5.1. The Nonlinear Model referred to in Table 5.1 is explained in Section 5.2.

Table 5.1 Simplifications used in the Single Layer Model compared to the Savannah Area Model

	The Savannah Area Model	The Single Layer Model and The Nonlinear Model
Number of layers modeled	Multiple	Single
Grid	76x88	19x22
Transmissivity	Nodal values	Four zones
Leakage	Dynamically calculated	Estimated from leakance values and final hydraulic head distribution

Using a mesh-centered and fully implicit finite difference approximation with equal space increments in x and y directions (i.e., $\Delta x = \Delta y$), the steady state groundwater flow equation can be described by:

$$
\begin{aligned}
&L_{i,j}h_{i+1,j} + R_{i,j}h_{i-1,j} + U_{i,j}h_{i,j+1} + D_{i,j}h_{i,j-1} - \\
&\qquad \left(L_{i,j} + R_{i,j} + U_{i,j} + D_{i,j} + v_{i,j}\right)h_{i,j} = -Q_{i,j} - W_{i,j}
\end{aligned}
$$

$$
\begin{aligned}
&L_{i,j} = \frac{1}{2}\left(T_{i+1,j} + T_{i,j}\right) \qquad R_{i,j} = \frac{1}{2}\left(T_{i,j} + T_{i-1,j}\right) \\
&U_{i,j} = \frac{1}{2}\left(T_{i,j+1} + T_{i,j}\right) \qquad D_{i,j} = \frac{1}{2}\left(T_{i,j} + T_{i,j-1}\right) \\
&v_{i,j} = \left(Leakance_{i,j\_top} + Leakance_{i,j\_bottom}\right)\left(\Delta x^2\right) \\
&W_{i,j} = \left(Leakance_{i,j\_top} \times h_{i,j\_top} + Leakance_{i,j\_bottom} \times h_{i,j\_bottom}\right)\left(\Delta x^2\right)
\end{aligned}
\tag{5.3}
$$

where subscripts i, j used in $Leakance_{top}$, $Leakance_{bottom}$, h_{top}, and h_{bottom} terms refer to nodal values of these terms. The terms on the right hand side of Equation (5.3) are all know values. Writing Equation (5.3) for each node in the model domain we obtain the following system of equations:

$$\mathbf{A}(T) \times h = \mathbf{b}(T) \tag{5.4}$$

where $\mathbf{A}(T)$ is a matrix of head coefficients and it is a function of transmissivity, h is a vector of unknown heads, and $\mathbf{b}(T)$ is a vector containing boundary head conditions and source/sink terms (i.e., pumping/recharge wells, contribution from leakage terms) collected at the right hand side of the equation.

Assuming transmissivities are deterministic values (i.e., 0.14 ft^2/sec, 0.435 ft^2/sec, 0.895 ft^2/sec, and 1.805 ft^2/sec for zones 1, 2, 3, and 4, respectively) steady state groundwater flow in the Savannah region is simulated by using the Single Layer Model. Comparison of the hydraulic heads calculated by the Single Layer Model and the Savannah Area Model are presented in Figure 5.2. As can be seen from the hydraulic head contours given in Figure 5.2, the results of the Savannah Area Model and the Single Layer Model reasonably agree with each other. Figure 5.2 includes the results of The Nonlinear Model which is explained in Section 5.2.

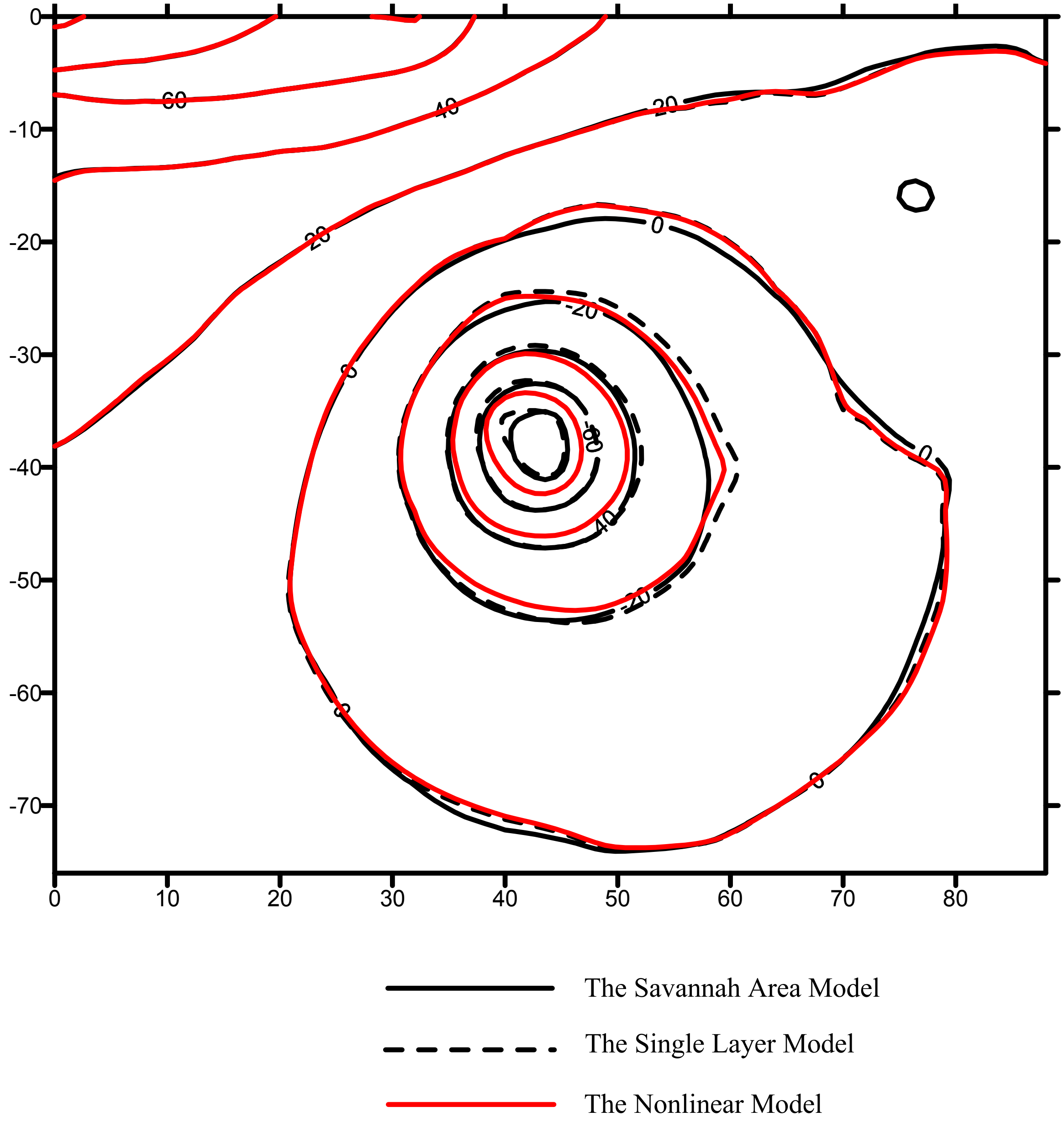


Figure 5.2 Hydraulic heads (ft) calculated by the Savannah Area Model, Single Layer Model and the Nonlinear Model

The next step is to use the groundwater model operator model proposed by Dou et al. (1995) to simulate steady state groundwater flow in the UFA in the presence of parameter imprecision. To achieve this goal we characterize transmissivities in each zone as fuzzy numbers. In this process, transmissivities at all nodes which are located inside zone 1, are represented by the same fuzzy number. Details of the groundwater model operator method are explained in the following section.

5.2 GROUNDWATER MODEL OPERATOR METHOD TO CALCULATE FUZZY HYDRAULIC HEADS IN THE UPPER FLORIDAN AQUIFER

Traditionally, aquifer parameters have been treated deterministically. However, the information available about these parameters may be imprecise. The groundwater model operator method uses fuzzy set theory concepts to capture the uncertainty associated with the available data (Dou et al. 1995). This method allows characterization of the model parameters as fuzzy numbers.

In this study transmissivities at each node are represented as fuzzy numbers. Thus, the head coefficients in matrix $\mathbf{A}(T)$ and vector $\mathbf{b}(T)$ as given in Equation (5.4) are also fuzzy numbers since they are functions of transmissivity. As a result, the dependent variable piezometric head, h will be a fuzzy variable as well. Equation (5.4) with imprecise transmissivities can be represented as follows:

$$\widetilde{\mathbf{A}} \times \tilde{h} = \tilde{\mathbf{b}} \tag{5.5}$$

where the tilde represents the presence of fuzzy numbers with in the matrices and vectors.

The groundwater model operator method uses alpha-cut concept to transform a system of fuzzy equations as given in Equation (5.5) to a system of interval equations at a specified membership level. These interval equations are solved by a constrained nonlinear optimization algorithm. The upper and lower bounds of the model inputs (i.e., transmissivities in this case) are used to calculate upper and lower bounds of the output (i.e., heads in this case). The algorithm used by the groundwater model operator method proposed by Dou et al. (1995) can be summarized as follows:

1. Write finite difference representation of Equation (5.1) for each node as given by Equation (5.3).
2. Divide membership domain into alpha-cut levels. For example, intervals of 0.1: $\alpha = 0, 0.1, 0.2, ..., 1.0$
3. Determine the lower and upper bounds of transmissivities at each node for each alpha-cut level. For example, $T_{i.j}^{\alpha} = \left[lb_{i,j}^{\alpha}, ub_{i,j}^{\alpha} \right] \quad \alpha = 0, 0.1, ..., 1.0$
4. Calculate lower and upper bounds of head at each node for each alpha-cut, $\alpha = 0, 0.1, ..., 1$ by the following procedure:
   i. Solve the following nonlinear optimization problem to calculate the lower bound of head for each node:

$$\begin{aligned} &Min \quad && h_{i,j}^{\alpha} \\ &s.t. \quad && \mathbf{A}(T^{\alpha}) \times h^{\alpha} = \mathbf{b}^{\alpha} \\ & && lb_{i,j}^{\alpha} \leq T^{\alpha} \leq ub_{i,j}^{\alpha} \end{aligned} \tag{5.6}$$

ii. Solve the following nonlinear optimization problem to calculate the upper bound of head for each node:

$$\begin{aligned} &Max \quad h_{i,j}^{\alpha} \\ &s.t. \quad \mathbf{A}(T^{\alpha}) \times h^{\alpha} = \mathbf{b}^{\alpha} \\ &\qquad\quad lb_{i,j}^{\alpha} \le T^{\alpha} \le ub_{i,j}^{\alpha} \end{aligned} \tag{5.7}$$

where T^{α} is the vector of transmissivities at the specified alpha-cut level, $\mathbf{A}\left(T^{\alpha}\right)$ is the matrix of head coefficients which is a function of T^{α}, $\mathbf{b}^{\alpha}$ is the right hand side vector containing the boundary conditions and source/sink terms, h^{α} is the vector of unknown heads at the specified alpha-cut level.

Thus, to calculate fuzzy head at a specific node two nonlinear optimization problems (i.e., one minimization and one maximization to calculate the lower and upper bound of the unknown head, respectively) need to be solved for each alpha-cut level. These nonlinear optimization problems are again solved by GAMS software (see Appendix E). From here after we will refer to the groundwater model operator model developed for the Savannah region as the Nonlinear Model. The Nonlinear Model has the same simplifications that are applied to the Single Layer Model (see Table 5.1).

Same system of equations is used in the Single Layer Model and the Nonlinear Model. The system of equations is directly solved by finite difference scheme in the Single Layer Model. The Nonlinear Model is composed of a minimization and a maximization problem and both of these optimization problems have the system of equations as constraints. Head contours that are generated by the Single Layer Model with the following crisp transmissivities for each of the four zones (i.e., 0.14 ft^2/sec, 0.435 ft^2/sec, 0.895 ft^2/sec, and 1.805 ft^2/sec for zones 1, 2, 3, and 4, respectively) are given in Figure 5.2. Four fuzzy transmissivities are required by the Nonlinear Model. Choosing the above given crisp transmissivities as the values corresponding to a membership function value of 1.0, the Nonlinear Model is used to simulate the groundwater flow in the model domain. The head contours obtained by the Nonlinear Model are in good agreement with the results of the Single Layer Model (see Figure 5.2).

The next step is to use fuzzy transmissivities and calculate fuzzy hydraulic heads at specific locations, such as Hilton Head Island (i.e., indicator site), Bull Island, and Tybee Island by using the Nonlinear Model. Fuzzy hydraulic heads are calculated for three cases. The first case considers only the existing pumping wells, the second case considers an additional pumping well at Rincon, and third case considers six additional pumping wells at the six locations identified as the hypothetical case in Section 4.4.2.2. Pumping rates used for these three cases are summarized in Table 5.2.

Table 5.2 Additional pumping rates for Cases 1, 2, and 3

Case	Location	Pumping Rate (ft^3/sec)
1	-	-
2	Rincon	6.5
3	Rincon	2
	Bloomingdale	2
	Marlow	3
	Ridgeland	2
	Denmark	3
	Hinesville	3

As explained before four zones of transmissivities are used in the model domain. The fuzzy transmissivities assigned to each one of these zones are given in Figure 5.3. The mid values of the transmissivity ranges are used as the peak transmissivities (i.e., transmissivity with a membership function value of one) and symmetric triangular membership functions are assigned to the fuzzy transmissivities to represent uncertainty. The methods used in this chapter are not restricted to symmetric triangular membership functions; these distributions are used for the sake of simplicity. The support of each one of the fuzzy transmissivities has a variability of 50 percent.

The results for Cases 1, 2, and 3 are provided in the following section. First, the fuzzy hydraulic heads at three locations, Hilton Head, Bull, and Tybee Islands are presented. Then these fuzzy hydraulic heads are used to calculate acceptability of a decision criteria by utilizing the risk tolerance measure proposed in Section 3.5.2. This acceptability value may be used as individual satisfaction of a management alternative with respect to the decision criteria, and can be aggregated into an overall performance value by using the OWA aggregator as explained in Appendix G.

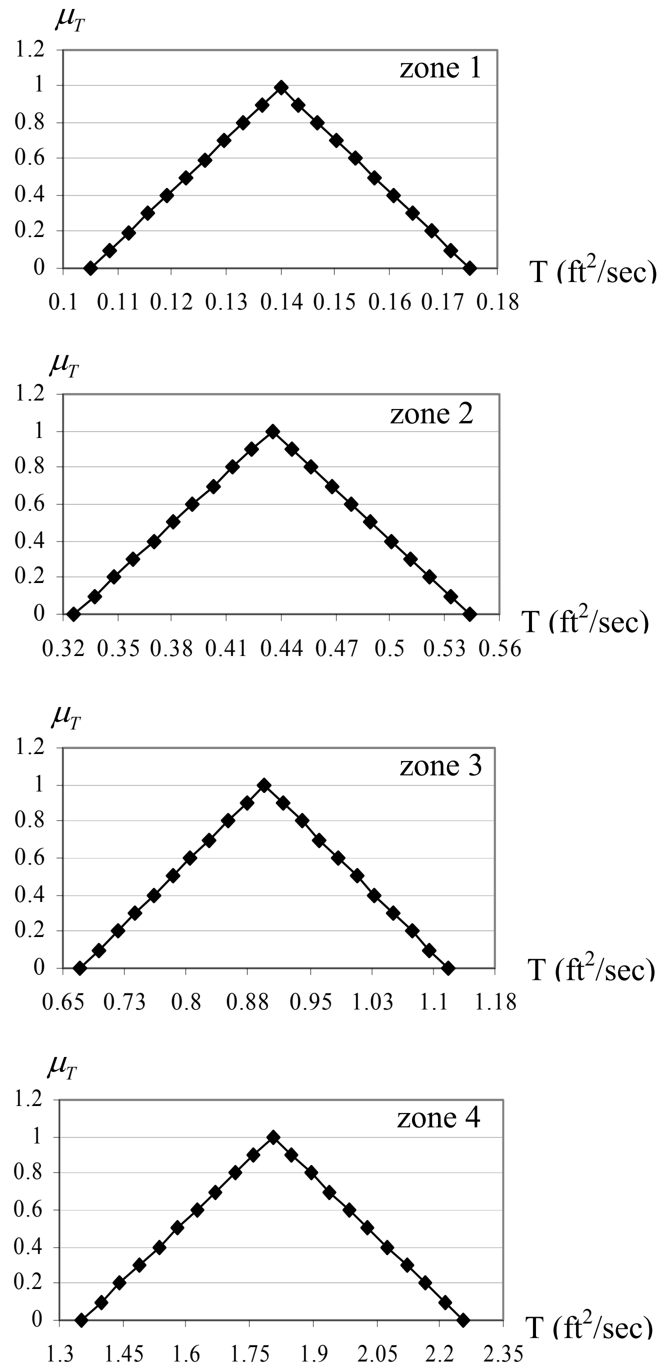


Figure 5.3 Fuzzy transmissivities in zones 1, 2, 3, and 4

5.3 FUZZY HYDRAULIC HEADS AND DRAWDOWNS AT HILTON HEAD ISLAND, BULL ISLAND, AND TYBEE ISLAND

Fuzzy transmissivities are assigned to all the nodes located inside each one of the four zones (see Figure 5.1). Then for each alpha-cut level, two nonlinear optimization problems given in Equations (5.6) and (5.7) are solved to calculate the lower and upper values of the fuzzy head, respectively at a specific node (for example, node corresponding to Hilton Head Island). Fuzzy heads calculated at Hilton Head Island, Bull Island, Tybee Island are given in Figures 5.4, 5.5, and 5.6 for Cases 1, 2, and 3, respectively (see Table 5.2).

As can be seen from Figure 5.4, with only the existing pumping wells the hydraulic head at Hilton Head Island is "about -0.14 ft." At Tybee Island and Bull Island, the hydraulic heads are "about -19.42 ft" and "about -24.84 ft," respectively. Although the transmissivities had symmetric triangular distributions, the resulting fuzzy hydraulic heads at these three locations do not have symmetric triangular membership functions.

For Case 2, when a new well at Rincon with a pumping rate of 6.5 ft^3/sec is added to the already existing pumping wells, fuzzy hydraulic drawdowns at Hilton Head, Bull, and Tybee Islands are "about -0.20 ft," "about -20.16 ft," and "about -25.58 ft." Since there is additional pumping from the model domain, the hydraulic heads at all of the three locations decreased. Fuzzy hydraulic head distributions are again not symmetric triangular distributions indicating that the response is not linear in the presence of fuzzy transmissivities.

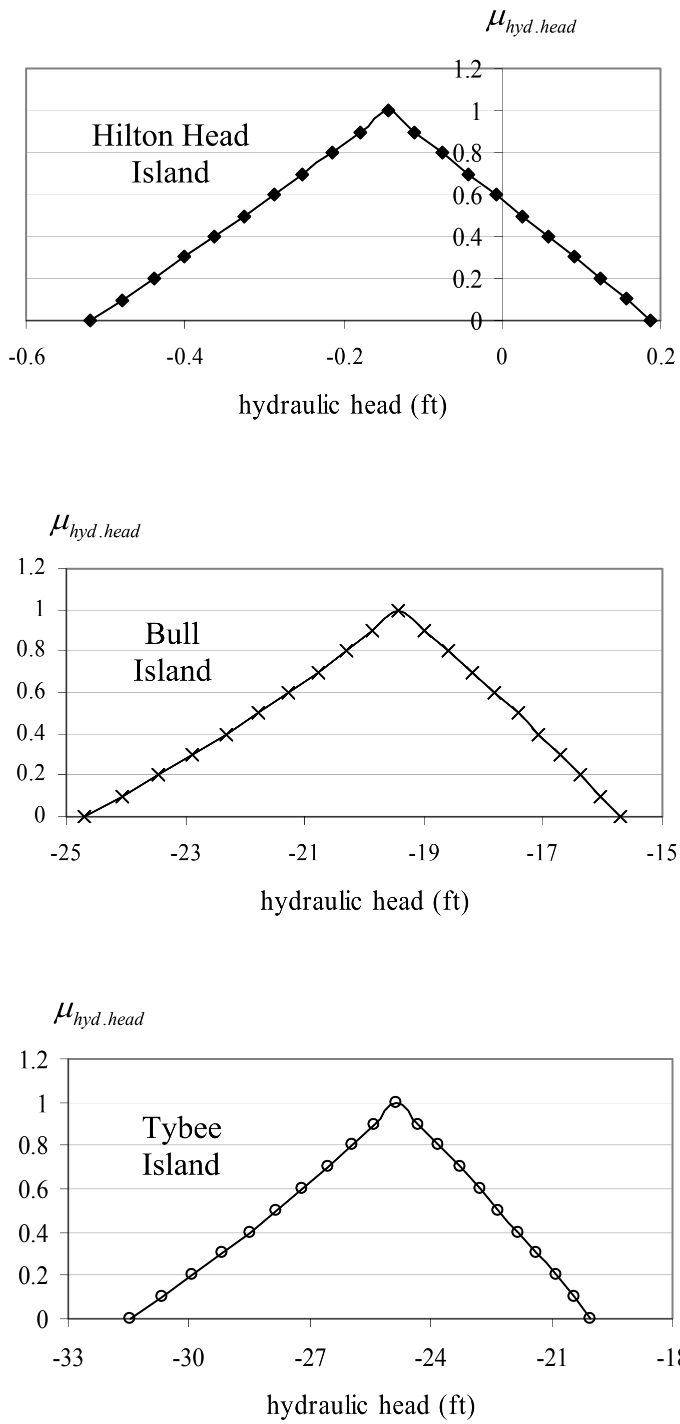


Figure 5.4 Fuzzy hydraulic heads at Hilton Head Island, Bull Island, and Tybee Island for Case 1

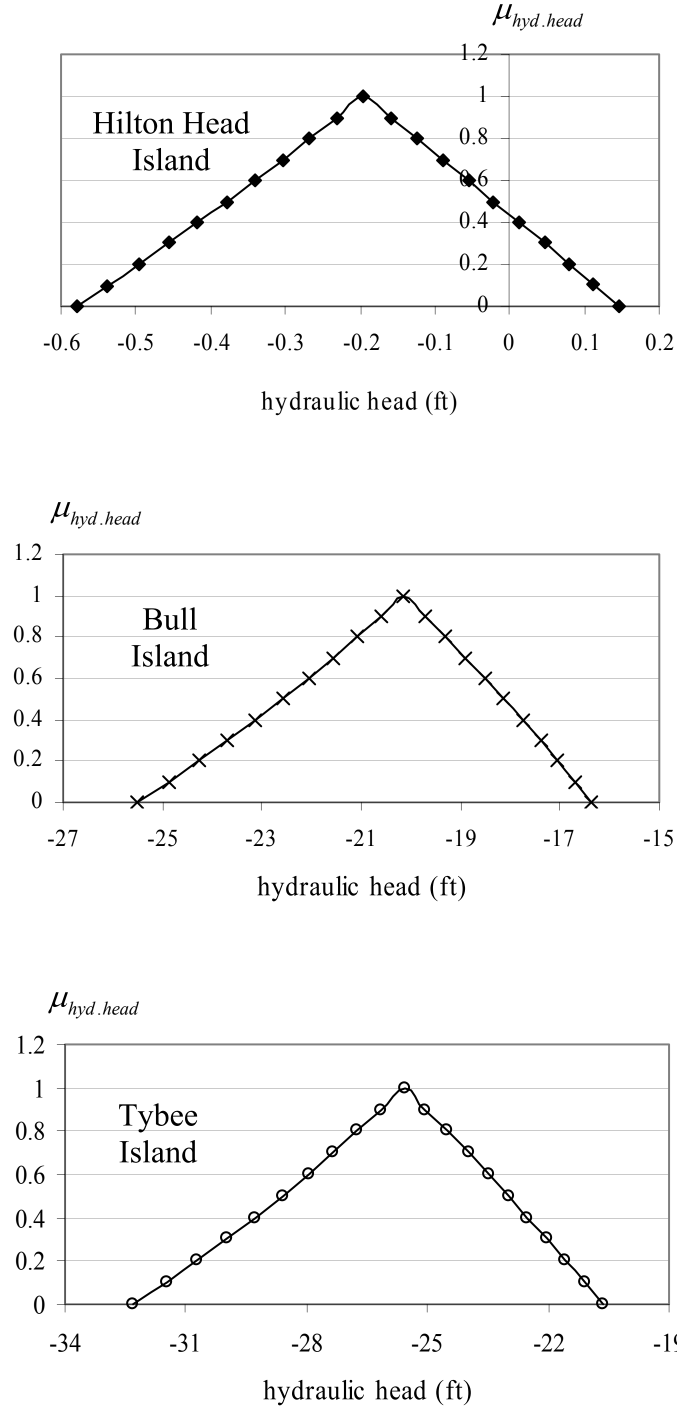


Figure 5.5 Fuzzy hydraulic heads at Hilton Head Island, Bull Island, and Tybee Island for Case 2

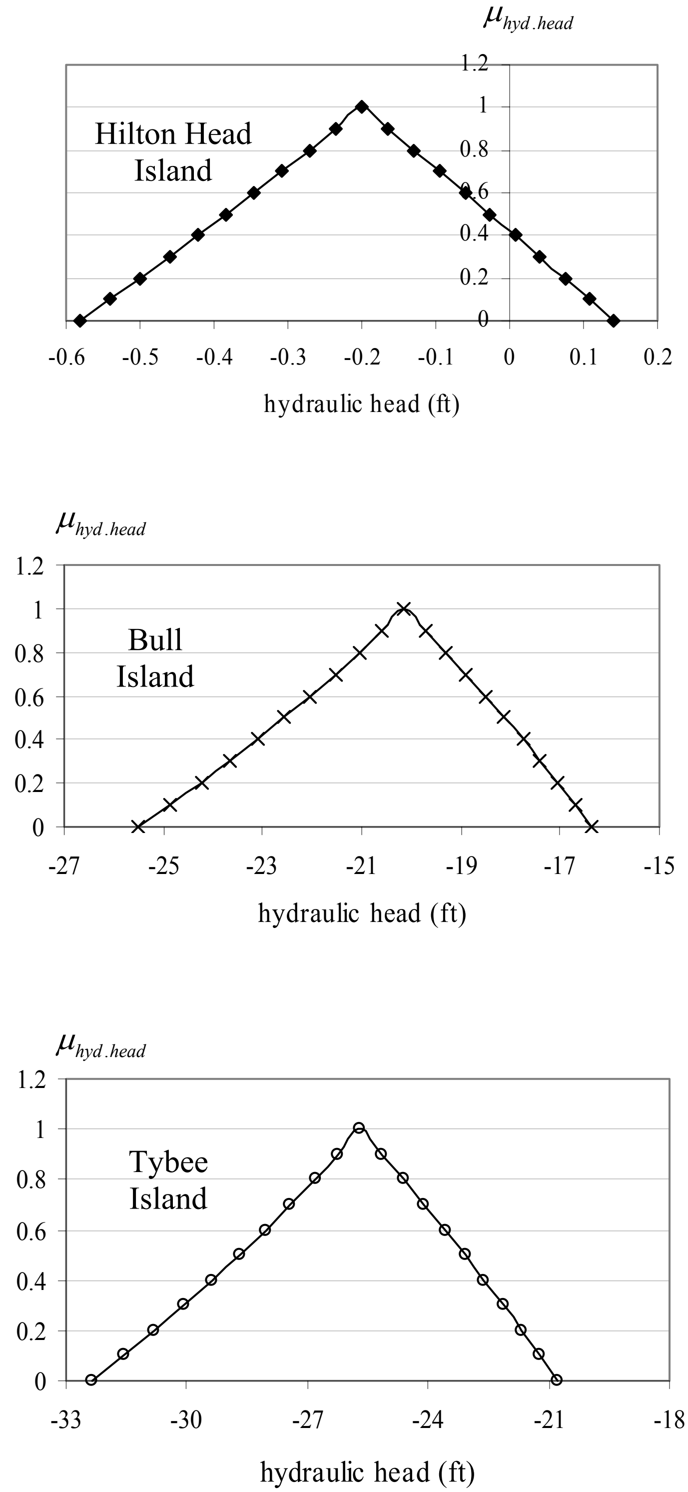


Figure 5.6 Fuzzy hydraulic heads at Hilton Head Island, Bull Island, and Tybee Island for Case 3

In Case 3, we considered six additional pumping wells at Rincon, Bloomingdale, Marlow, Ridgeland, Denmark, and Hinesville. The pumping rates at each one of these locations are given in Table 5.2. The total additional pumping rate added to the already existing withdrawal from the model domain is 15 ft^3/sec. As can be seen from Figure 5.6, the resulting hydraulic heads are similar to those of Case 2. The fuzzy hydraulic heads at Hilton Head, Bull, and Tybee Islands are "about -0.20 ft," "about -20.15 ft," and "about -25.68 ft" respectively.

Cases 2 and 3 involve additional pumping wells and cause drawdowns at each node in the model domain. The fuzzy drawdowns at Hilton Head Island, Bull Island, and Tybee Island are calculated for Cases 2 and 3, and provided in Tables 5.3 and 5.4, respectively for each alpha-cut level (i.e., $\alpha = 0, 0.1, ..., 1.0$). Fuzzy drawdowns for Cases 2 and 3 are plotted in Figures 5.7 and 5.8, respectively. As can be seen from Figure 5.7, the fuzzy drawdown at Hilton Head Island is "about 0.051 ft" when an additional well is pumping at a rate of 6.5 ft^3/sec at Rincon. The associated fuzzy drawdowns at Bull and Tybee Islands for this case are both "about 0.74 ft." This indicates that Bull and Tybee Islands are more responsive to pumping at Rincon. When six additional pumping wells are used (Case 3), the resulting fuzzy drawdowns are "about 0.055 ft," "about 0.73 ft," and "about 0.83 ft" at Hilton Head, Bull, and Tybee Islands, respectively. As can be seen from Figures 5.7 and 5.8, the membership functions of fuzzy drawdowns are closer to symmetric triangular distributions than those of fuzzy hydraulic heads.

Table 5.3 Fuzzy drawdowns at Hilton Head Island, Bull Island, and Tybee Island for Case 2

$\alpha - cut$	Hydraulic head for Case 1 (ft)			Hydraulic head for Case 2 (ft)			Drawdown for Case 2 (ft)		
0	-0.520	-24.696	-31.443	-0.579	-25.529	-32.289	0.060	0.833	0.846
0.1	-0.479	-24.062	-30.653	-0.538	-24.886	-31.487	0.059	0.824	0.834
0.2	-0.439	-23.456	-29.899	-0.497	-24.271	-30.721	0.058	0.815	0.822
0.3	-0.400	-22.876	-29.176	-0.457	-23.682	-29.987	0.057	0.805	0.811
0.4	-0.362	-22.321	-28.483	-0.418	-23.117	-29.282	0.056	0.796	0.800
0.5	-0.324	-21.789	-27.817	-0.379	-22.575	-28.606	0.055	0.787	0.789
0.6	-0.287	-21.278	-27.177	-0.341	-22.055	-27.955	0.054	0.777	0.778
0.7	-0.251	-20.787	-26.561	-0.304	-21.555	-27.329	0.053	0.767	0.768
0.8	-0.215	-20.315	-25.968	-0.267	-21.073	-26.725	0.052	0.758	0.758
0.9	-0.179	-19.859	-25.395	-0.231	-20.608	-26.143	0.052	0.749	0.748
1	-0.144	-19.420	-24.843	-0.195	-20.159	-25.580	0.051	0.739	0.738
0.9	-0.110	-18.996	-24.308	-0.159	-19.725	-25.036	0.050	0.730	0.729
0.8	-0.075	-18.586	-23.789	-0.124	-19.306	-24.508	0.049	0.720	0.719
0.7	-0.041	-18.189	-23.280	-0.090	-18.899	-23.990	0.048	0.711	0.710
0.6	-0.008	-17.804	-22.784	-0.055	-18.505	-23.485	0.047	0.701	0.701
0.5	0.026	-17.431	-22.298	-0.021	-18.123	-22.990	0.047	0.692	0.691
0.4	0.058	-17.069	-21.823	0.013	-17.752	-22.505	0.046	0.683	0.682
0.3	0.091	-16.717	-21.357	0.047	-17.392	-22.032	0.045	0.675	0.675
0.2	0.124	-16.375	-20.900	0.080	-17.041	-21.566	0.044	0.667	0.666
0.1	0.156	-16.042	-20.452	0.113	-16.700	-21.108	0.043	0.659	0.657
0	0.188	-15.717	-20.012	0.146	-16.369	-20.659	0.042	0.651	0.647

Table 5.4 Fuzzy drawdowns at Hilton Head Island, Bull Island, and Tybee Island for Case 3

$\alpha - cut$	Hydraulic head for Case 1 (ft)			Hydraulic head for Case 3 (ft)			Drawdown for Case 3 (ft)		
0	-0.520	-24.696	-31.443	-0.582	-25.492	-32.358	0.062	0.795	0.915
0.1	-0.479	-24.062	-30.653	-0.541	-24.850	-31.560	0.062	0.789	0.906
0.2	-0.439	-23.456	-29.899	-0.500	-24.239	-30.797	0.061	0.783	0.898
0.3	-0.400	-22.876	-29.176	-0.460	-23.653	-30.065	0.060	0.777	0.889
0.4	-0.362	-22.321	-28.483	-0.421	-23.092	-29.363	0.060	0.771	0.881
0.5	-0.324	-21.789	-27.817	-0.383	-22.553	-28.689	0.059	0.765	0.872
0.6	-0.287	-21.278	-27.177	-0.345	-22.036	-28.041	0.058	0.757	0.864
0.7	-0.251	-20.787	-26.561	-0.308	-21.538	-27.417	0.057	0.750	0.856
0.8	-0.215	-20.315	-25.968	-0.271	-21.058	-26.816	0.057	0.743	0.848
0.9	-0.179	-19.859	-25.395	-0.235	-20.596	-26.235	0.056	0.737	0.840
1	-0.144	-19.420	-24.843	-0.199	-20.150	-25.675	0.055	0.730	0.832
0.9	-0.110	-18.996	-24.308	-0.164	-19.719	-25.132	0.054	0.723	0.824
0.8	-0.075	-18.586	-23.789	-0.129	-19.302	-24.605	0.054	0.716	0.817
0.7	-0.041	-18.189	-23.280	-0.094	-18.897	-24.089	0.053	0.709	0.809
0.6	-0.008	-17.804	-22.784	-0.060	-18.506	-23.585	0.052	0.702	0.801
0.5	0.026	-17.431	-22.298	-0.026	-18.126	-23.091	0.052	0.695	0.793
0.4	0.058	-17.069	-21.823	0.008	-17.757	-22.608	0.051	0.688	0.785
0.3	0.091	-16.717	-21.357	0.041	-17.399	-22.135	0.050	0.682	0.778
0.2	0.124	-16.375	-20.900	0.075	-17.050	-21.670	0.049	0.675	0.770
0.1	0.156	-16.042	-20.452	0.108	-16.711	-21.213	0.048	0.669	0.762
0	0.188	-15.717	-20.012	0.141	-16.380	-20.765	0.048	0.662	0.753

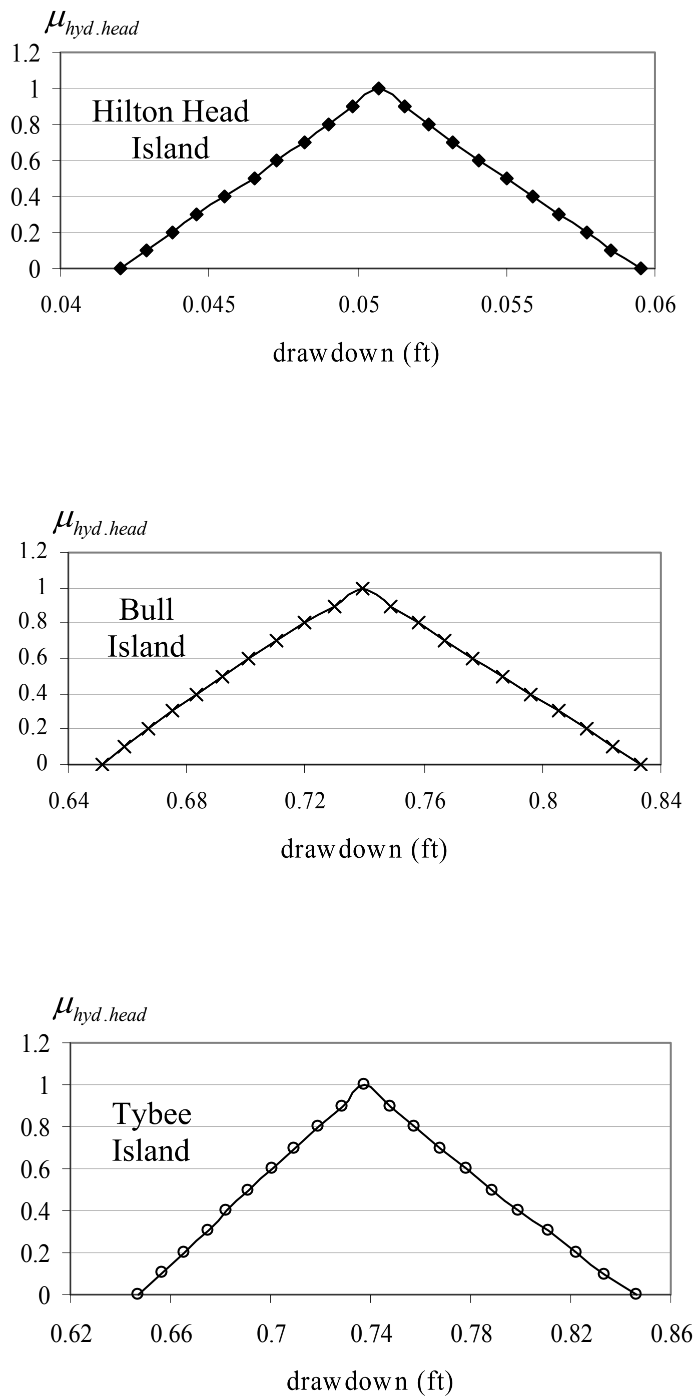


Figure 5.7 Fuzzy drawdown at Hilton Head Island, Bull Island, and Tybee Island for Case 2

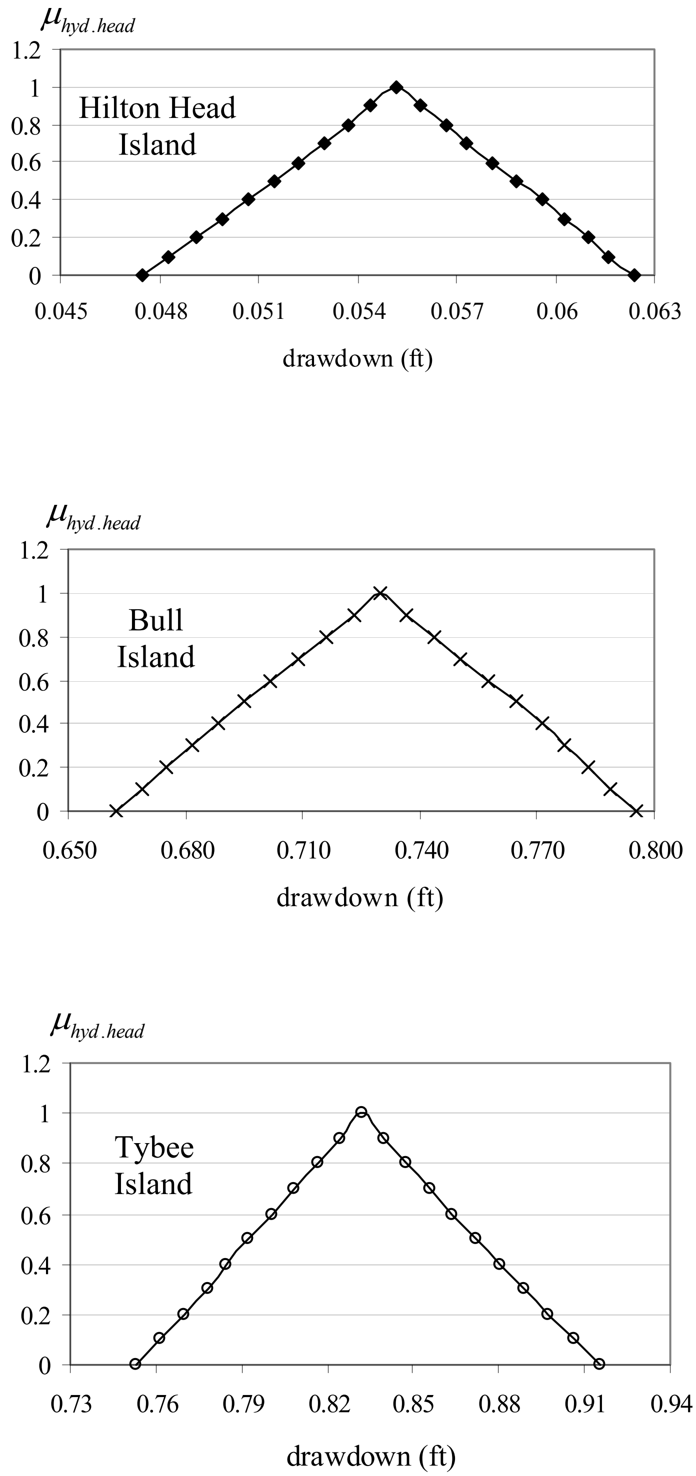


Figure 5.8 Fuzzy drawdown at Hilton Head Island, Bull Island, and Tybee Island for Case 3

Fuzzy hydraulic heads generated by using only the existing wells (i.e., Case 1) are used to calculate the fuzzy drawdowns due to additional pumping (i.e., Case 2 or Case 3). For example, when an additional well is pumping at Rincon with a pumping rate of 6.5 ft^3/sec, the fuzzy drawdown at Hilton Head Island is calculated by subtracting the fuzzy hydraulic head for Case 2 from the fuzzy hydraulic head for Case 1 at Hilton Head Island. A similar procedure is used to calculate the fuzzy drawdown at Bull and Tybee Islands. As can be seen from the comparison of Figure 5.5 with Figure 5.4, and Figure 5.6 with Figure 5.4, the fuzzy hydraulic heads are skewed in a similar fashion in all three locations. Thus, the resulting fuzzy drawdowns have membership functions closer to symmetric triangular distributions (see Figures 5.7 and 5.8).

As can be seen from Figures 5.7 and 5.8 the fuzzy drawdowns are similar for each case at Hilton Head, Bull, and Tybee Islands. This is because we intentionally chose appropriate pumping rates (i.e., at Rincon for Case 2 and at six demand locations for Case 3) such that they will result in drawdowns “around 0.05 ft” at Hilton Head Island. Since the Nonlinear Model has several simplifications (see Table 5.1) compared to the Savannah Area Model, we could not directly use the additional optimum pumping rates that are calculated by the Savannah Area Model in the previous chapter. Our next goal is to utilize the resulting fuzzy drawdowns in a decision-making framework. A decision-making procedure which utilizes the risk tolerance measure proposed in Section 3.5.2 to generate individual satisfaction of an objective is outlined in the following section.

5.4 DECISION-MAKING FRAMEWORK IN THE PRESENCE OF IMPRECISE PARAMETERS

A decision-making framework was proposed in Section 4.6 to select the best groundwater resources management strategy among alternatives. In Section 4.6 individual satisfactions of each management strategy for multiple fuzzy objectives are aggregated into a single overall performance value and the management strategy with the highest overall performance value is selected as the best alternative. Crisp results of the coupled simulation-optimization model are used to determine the individual satisfaction degrees of each management scenario with respect to various fuzzy objectives. For example, crisp drawdown values at Tybee and Bull Island are used to evaluate individual satisfaction degrees of various management alternatives with respect to the fuzzy objective of "maintain low drawdowns at critical locations other than Hilton Head Island indicator site" (see Section 4.6.2). In Section 4.6.2, "low drawdown" is characterized by a fuzzy number (Figure 4.12) and the crisp drawdown obtained from the coupled simulation-optimization model is used to determine the individual satisfaction degree with respect to this fuzzy "low drawdown." Since groundwater simulation in the presence of imprecise transmissivities results in fuzzy drawdowns, we need a procedure to carry out the analysis with fuzzy drawdowns at Tybee and Bull Islands.

5.4.1 Determination of Individual Satisfaction Degrees Using Risk Tolerance Measure

In Chapter 5, we used the Nonlinear Model to estimate fuzzy heads in the Savannah region. The Nonlinear Model allows characterization of imprecise parameters (i.e., transmissivities) as fuzzy numbers and integration of these fuzzy parameters into the solution of the groundwater flow model. As a result, fuzzy hydraulic heads and drawdowns may be calculated at specific locations within the flow domain (see Section 5.3). One way to determine an individual satisfaction degree for an objective is utilizing the risk tolerance measure. The risk tolerance measure evaluates acceptability of a fuzzy result with respect to a crisp objective. Here, in order to demonstrate the analysis we assume crisp objectives such as "drawdown should not exceed 0.8 ft." More specifically, three crisp objectives are considered: (i) "drawdown at Bull Island should not exceed 0.8 ft;" (ii) "drawdown at Tybee Island should not exceed 0.8 ft;" and, (iii) "drawdown at Hilton Head Island should not exceed 0.05 ft."

Fuzzy drawdowns for Bull and Tybee Islands for Cases 2 and 3 are given in Figures 5.7 and 5.8, respectively. The risk tolerance measure (Equation (3.29)) is used to evaluate the validity of the objective, O_{dd}: "the fuzzy drawdown $\tilde{d}$ is smaller than or equal to the design criteria $d_{critical} = 0.8\ ft$." As an example, the fuzzy drawdown at Bull Island due to an additional well at Rincon with a pumping rate of 6.5 ft^3/sec (i.e., Case 2) and the design criteria $d_{critical}$ are shown in Figure 5.9. Risk tolerance measures together with the β values (Equation (3.30)) for acceptability of fuzzy drawdowns at Hilton Head, Bull, and Tybee Islands for Cases 2 and 3 are calculated and given in Table 5.5.

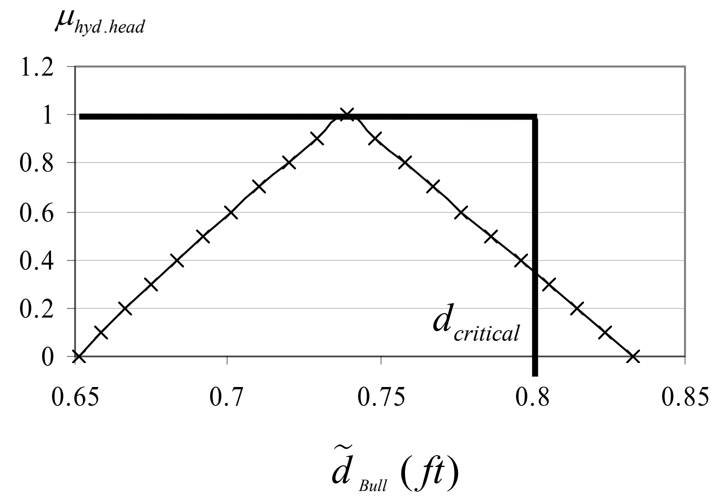


Figure 5.9 Fuzzy drawdown at Bull Island for Case 2

Table 5.5 The risk tolerance measures for acceptability of fuzzy drawdowns at Bull and Tybee Islands for Cases 2 and 3

	β	γ	$T(\boldsymbol{P})$
Case 2			
Hilton Head	0.40	1.0	0.18
Bull Island	0.92	1.0	0.79
Tybee Island	0.89	1.0	0.75
Case 3			
Hilton Head	0.07	1.0	0.01
Bull Island	1.00	1.0	1.00
Tybee Island	0.16	1.0	0.05

Risk tolerance values for the validity of the fuzzy drawdowns being less than 0.8 ft for Bull and Tybee Islands are 0.79 and 0.75, respectively for Case 2. However, the risk tolerance value for Hilton Head Island is much lower, 0.18. For Case 3, as can be seen from Figure 5.8, the membership function of the fuzzy drawdown at Hilton Head Island is located almost totally on the right of the critical drawdown, 0.05 ft. Thus, the risk tolerance value for the validity of the fuzzy drawdown at Hilton Head Island being less than 0.05 ft is very small, 0.01 (Table 5.5). Similarly, for Case 3, the risk tolerance value for Tybee Island is very small as well (i.e., 0.05). However, risk tolerance value for Bull Island is 1.0. This indicates that both the possibility and the necessity measures are 1.0 and the membership function of the fuzzy drawdown at Bull Island is completely located on the left of the design criteria, 0.8 ft.

The risk tolerance value is calculated to evaluate the validity of the objective, O_{dd} : "the fuzzy drawdown $\tilde{d}$ is smaller than or equal to the design criteria $d_{critical}$." In order to evaluate the validity of the objective we need to identify a minimum risk tolerance value that needs to be satisfied by the objectives. For example, a risk tolerance measure of at least 0.5 is required for the objective to be valid. Determination of the minimum risk tolerance value is a case-specific problem. The range and shape of the membership function and relative location of the design criteria impact the risk tolerance measure. Effect of relative location of the design criteria with respect to the membership function is investigated in the following section. This analysis provides some guidance in selecting a reasonable minimum risk tolerance value.

5.4.2 Effect of Relative Location of the Membership Function with respect to the Design Criteria on Risk Tolerance Measure

Effect of the shape of the membership function on the risk tolerance measure is investigated in Section 3.5.3. Our goal here is to further study the impact of the relative location of the membership function with respect to the design criteria. For example, if a minimum risk tolerance value of 1.0 is enforced for the objective to be valid, then the membership function has to be completely on the left of the design criteria. Here, we are trying to answer the following question: If a minimum risk tolerance measure of 0.5 or 0.75 is required what will be the relative location of the membership function with respect to the design criteria? In other words, is 0.5 a reasonable minimum risk tolerance measure, or at least 0.75 has to be chosen?

The fuzzy drawdowns for Cases 2 and 3 are given in Figures 5.7 and 5.8 respectively. As can be seen from these figures all of the membership functions are close to symmetric triangular distributions. The effect of relative location of the membership function with respect to the design criteria on risk tolerance measure is a function of the shape of the membership function. However, since all the membership functions of the fuzzy drawdowns resemble a symmetric triangular distribution, to simplify our analysis we will only work with the fuzzy drawdown at Hilton Head Island for Case 2. The results we derive for the fuzzy drawdown at Hilton Head Island for Case 2 will be essentially valid for the fuzzy drawdowns at Tybee and Bull Islands for Cases 2 and 3 and fuzzy drawdown at Hilton Head Island for Case 3.

In order to investigate how the relative location of the membership function with respect to the design criteria impacts the risk tolerance value, risk tolerance values are calculated for various design criteria. The support of the membership function for the fuzzy drawdown at Hilton Head Island for Case 2 ranges from 0.042 ft to 0.0595 ft. Risk tolerance values corresponding to a number of different design criteria within this range are calculated and plotted together with the membership function of the fuzzy drawdown at Hilton Head Island for Case 2 in Figure 5.10.

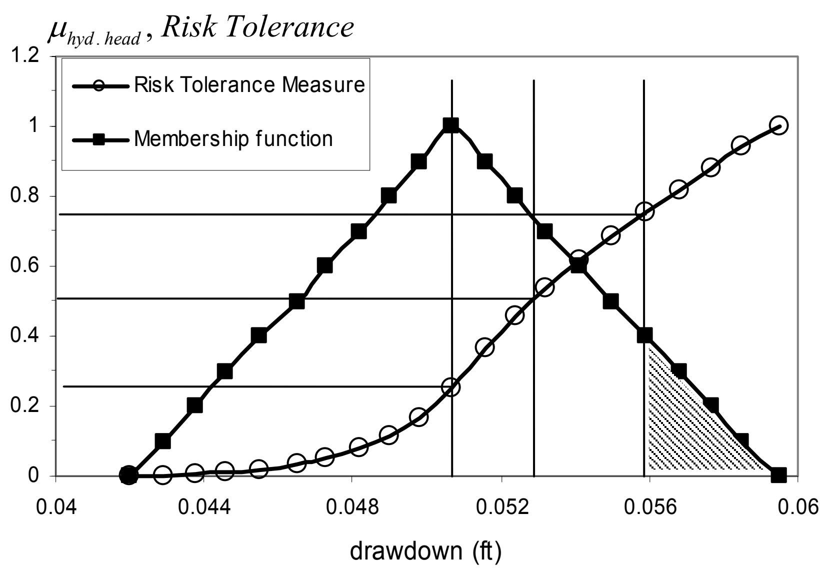


Figure 5.10 Membership function of the drawdown at Hilton Head Island for Case 2 and change of risk tolerance measure with design criteria

As can be seen from Figure 5.10, the change of risk tolerance value with respect to the design criteria is characterized by an S-shaped curve. For example, for a design criterion of 0.0507 ft (i.e., hydraulic head corresponding to a membership function of 1), the risk tolerance measure is 0.25. This indicates that for a symmetric triangular membership function, a minimum risk tolerance measure of 0.25 guarantees half of the membership function to lie on the left and the other half on the right of the design criteria. This also indicates a possibility measure of 1.0 and a necessity measure of 0.0. Thus, for symmetric triangular distributions, a minimum risk tolerance measure of 0.25 may be a reasonable choice for applications in which guaranteeing a fuzzy result "around the design criteria" is sufficient for assuring the validity of the objective.

If a higher risk tolerance measure is required, then clearly the peak of the membership function will be located on the left of the design criteria. For example, as can be seen from Figure 5.10, design criteria of 0.0528 ft and 0.056 ft will yield risk tolerance values of 0.5 and 0.75, respectively. In order to attain a risk tolerance measure of 0.75, nearly all of the membership function has to be located on the left of the design criteria. The shaded area in Figure 5.10 identifies the area that is allowed to be on the right of the design criteria. It can be concluded that a minimum risk tolerance value of 0.75 yields rather conservative results. In other words, if the decision-maker requires a minimum risk tolerance value of 0.75 to accept the validity of an objective, then he or she is expecting a relatively larger fraction of the membership function to be located on the left of the design criteria (i.e., in Figure 5.10, only the shaded region of the whole symmetric triangular distribution is allowed to lie on the right of the design criteria). For the groundwater resources management analysis a minimum risk tolerance measure of 0.5 seems reasonable. A minimum value of 0.75 or higher may be required depending on the context and tolerance to risk. For example, if the fuzzy drawdown is an indication of a hazardous contaminant then a more conservative requirement of minimum risk tolerance measure may be enforced.

5.4.3 AGGREGATING INDIVIDUAL SATISFACTIONS INTO AN OVERALL PERFORMANCE VALUE

The risk tolerance measure which indicates the acceptability degree of a fuzzy result with respect to a crisp criteria always ranges between zero and one (see Section 3.5.2), thus it can be used as the individual satisfaction degree of a management scenario with respect to a crisp objective. For example, individual satisfaction of Case 3 as an alternative management scenario with respect to the objective, O_{dd} : "the fuzzy drawdown at Bull Island is smaller than or equal to $0.8\ ft$" is 1.0. In the presence of multiple objectives, aggregation operators can be used to calculate a single overall performance value for each management alternative as demonstrated in Section 4.6.3. Overall performances for Case 2 and Case 3 with three objectives (i.e., fuzzy drawdown at Bull and Tybee Islands being less than 0.8 ft, and fuzzy drawdown at Hilton Head Islands being less than 0.05 ft) are calculated using OWA operator.

Case 2: $Q_{Rincon} = 6.5\ ft^3 / \sec$

Considering all three objectives have equal importance, the weights associated with these three criteria using "most" as the quantifier are calculated as follows:

$$
\begin{aligned}
v_1 &= Q\left(\frac{1}{3}\right) - Q\left(\frac{1-1}{3}\right) = 0.11 - 0.0 = 0.11 \\
v_2 &= Q\left(\frac{2}{3}\right) - Q\left(\frac{2-1}{3}\right) = 0.44 - 0.11 = 0.33 \\
v_3 &= Q\left(\frac{3}{3}\right) - Q\left(\frac{3-1}{3}\right) = 1 - 0.44 = 0.56
\end{aligned}
\tag{5.8}
$$

The second step is to aggregate the fuzzy objectives using Equation (G.5) given in Appendix G:

$$
D_{Case\ 2} = \sum_{j=1}^{3} v_j b_j = (0.79 \times 0.11) + (0.75 \times 0.44) + (0.18 \times 0.56) = 0.52 \tag{5.9}
$$

As can be seen from Equation (5.9), when OWA is used as the aggregation operator, a single overall performance value of 0.52 is calculated for Case 2. If "and" was used as the aggregation operator, the overall performance would be 0.11 and if "or" was used it would be 0.79. As concluded in Chapter 4, the OWA operator is an averaging operator and generates an overall satisfaction in between those estimated by "and" and "or" operators.

Case 3: $Q_{Rincon} = 2.0\ ft^3/\sec, Q_{Bloomingdale} = 2.0\ ft^3/\sec, Q_{Marlow} = 3.0\ ft^3/\sec,$

$Q_{Ridgeland} = 2.0\ ft^3/\sec,\ Q_{Denmark} = 3.0\ ft^3/\sec, Q_{Hinesville} = 3.0\ ft^3/\sec$

The weights associated with three objectives are the same for Case 3 (see Equation (5.8)). However, the individual satisfaction degrees are different as given in Table 5.5. The overall performance value for Case 3 is calculated as follows:

$$D_{Case\ 3} = \sum_{j=1}^{3} v_j b_j = (1.0 \times 0.11) + (0.05 \times 0.33) + (0.01 \times 0.56) = 0.13 \tag{5.10}$$

An overall performance value of 0.13 is calculated for Case 3. Although one of the individual satisfactions (i.e., drawdown being less than 0.8 ft at Bull Island) is 1.0, the overall performance of Case 3 is very low due to low individual satisfactions of the other two objectives.

The overall performance value calculated by aggregating risk tolerance values for each objective may be considered as an average risk tolerance value. As explained in Section 3.5, the risk tolerance measure generates more conservative results when compared to the possibility measure. However, it yields less conservative results than those of the necessity measure.

The calculated overall performances can be used to make decisions. For example, if an overall performance of at least 0.5 is required for a management strategy to be acceptable, Case 2 (i.e., an additional well at Rincon with a pumping rate of 6.5 ft^3sec) will be an acceptable scenario while Case 3 (i.e., six additional wells at Rincon, Bloomingdale, Marlow, Ridgeland, Denmark, and Hinesville) will not satisfy this requirement.

5.5 CONCLUSIONS FOR GROUNDWATER FLOW SIMULATION WITH IMPRECISE PARAMETERS AND SUCCESSIVE DECISION-MAKING FRAMEWORK

In this chapter we used the fuzzy set theory concepts to treat imprecise parameters (i.e., transmissivities) in a two-dimensional steady state groundwater flow model. Characterizing the uncertainties in transmissivities allowed us to estimate uncertainties associated with piezometric heads due to parameter imprecision. The imprecise input parameters may come from indirect measurements, subjective interpretations, and expert judgment of available information (Dou et al. 1995).

The groundwater model operator method proposed by Dou et al. (1995) is used to integrate uncertainties associated with transmissivities into the groundwater flow simulations. Transmissivities in the model domain are represented by fuzzy numbers and the resulting piezometric heads within the flow domain are calculated as fuzzy numbers. Two nonlinear optimization problems are solved at each alpha-cut level to determine the lower and upper bounds of the piezometric head at a specific location. Then these lower and upper bounds at each alpha-cut level are utilized to construct the membership function of the piezometric head at that location. The groundwater model operator method is used to simulate groundwater flow in the UFA in the Savannah region. A simplified version of the Savannah Area Model is developed and solved in the presence of imprecise transmissivities. We refer to this model as the Nonlinear Model.

The UFA is represented by a 19x22 finite difference grid and the transmissivities in the model domain are grouped into four zones. Transmissivities are represented as fuzzy numbers and transmissivities at all the nodes within the same zone are characterized by the same membership function. We used the Nonlinear Model to calculate fuzzy piezometric heads at specific locations such as Hilton Head Island, Bull Island, and Tybee Island for three different cases: (i) with the existing pumping wells, Case 1; (ii) an additional pumping well at Rincon, Case 2; and, (iii) six additional pumping wells at Rincon, Bloomingdale, Marlow, Ridgeland, Denmark, and Hinesville, Case 3 (see Table 5.2). Fuzzy drawdowns at Hilton Head, Bull, and Tybee Islands are calculated. Although symmetric triangular membership functions are used to represent fuzzy transmissivities in each one of the four zones, the membership functions of the resulting piezometric heads are not symmetric triangular distributions. Since the fuzzy piezometric heads without any additional pumping wells and with additional pumping wells have similar skewed membership functions, the fuzzy drawdowns have membership functions close to symmetric triangular distributions.

The Nonlinear Model taking into account the uncertainties associated with transmissivities in the model domain allows us to calculate fuzzy piezometric heads in the Savannah region. The fuzzy piezometric heads may be used to calculate the fuzzy drawdowns at critical locations, such as Tybee Island and Bull Island. In Section 4.6.2, maintaining low drawdowns at these two locations is considered as one of the fuzzy objectives in evaluating the best groundwater management strategy. The coupled simulation-optimization model utilized in Chapter 4, resulted in crisp drawdown values at Bull and Tybee Islands and these crisp values are used in the decision-making framework. Since, the groundwater model operator method allows us to calculate fuzzy drawdowns at these two critical locations, the decision-making procedure proposed in Chapter 4 need to be revised in a way to allow the fuzzy drawdowns to be utilized as decision criteria as well.

The risk tolerance measure proposed in Section 3.5.2 is used to calculate the acceptability of a proposition such as “the fuzzy drawdown is smaller than or equal to the criteria, $d_{critical} = 0.8\ ft$” for drawdowns at Bull and Tybee Islands. Since the risk tolerance measure always results in a value in the range zero and one, it can be used as the individual satisfaction of a design criterion. Different than Section 4.6.2, here, we are utilizing a crisp design criteria and evaluating the validity of a fuzzy result (i.e., drawdown) with respect to this design criteria. The individual satisfaction values which are estimated by using the risk tolerance measure are aggregated into an overall performance value for each case (i.e., Cases 2 and 3). If the overall performance value is higher than the required minimum risk tolerance then that case is considered as acceptable.

When it is possible to characterize the imprecision in the parameters of the groundwater simulation model as fuzzy numbers, the groundwater model operator method may be used to predict the uncertainty in the piezometric heads in the flow domain. These fuzzy piezometric heads may be used to determine the fuzzy drawdowns which may in turn be used as decision criteria in groundwater resources management frameworks. Combined utilization of the Nonlinear Model together with the risk tolerance measure allows the decision-maker to characterize uncertainty in piezometric heads in the study area and integrate them into the decision-making process. This may allow decision-makers to better recognize the associated uncertainties and yield in more informed decisions.

6 CONCLUSIONS

In the first part of this thesis, we proposed two hybrid-models which allow combined utilization of probabilistic (i.e., Monte Carlo Analysis) and non-probabilistic (i.e., fuzzy set theory and possibility theory) approaches for treating model parameter uncertainties in human health risk assessment context. Added carcinogenic risk is calculated by an analytical formula. Parameters such as ingestion rate, contaminant concentration, exposure frequency and duration, body weight, averaging time, and cancer slope factor are used to estimate the added risk. Traditionally, health risk is calculated characterizing these parameters by either deterministic values or probability density functions. Recently, hybrid-models which utilize mathematical tools of both fuzzy set theory and probability theory are developed.

Depending on the form of the available information parameters of the risk equation can be characterized by probability density functions, membership functions, or deterministic values. In the first hybrid-model, we developed a methodology to propagate both uncertainty and variability associated with the parameters of the risk equation into resulting risk by combined utilization of Monte Carlo Analysis, fuzzy arithmetic, and interval analysis. The numerical results and specific conclusions are presented in Sections 3.3.4 and 3.3.5, respectively. The proposed hybrid-method provides a mean to propagate all available information into resulting risk by allowing the available information about a parameter be characterized by a membership function when the available information is not sufficient to establish the appropriate probability density function for the parameter.

When sufficient data is available model parameters can be adequately represented by probability density functions. The hybrid-method combines probability density functions and membership functions in carcinogenic risk assessment. As a result membership functions of risk to individuals at certain risk fractiles of risk are calculated. When the number of parameters characterized by membership functions increases the uncertainty associated with the resulting risk increases. In such cases the hybrid-method may yield risk membership functions with a large support base which may imply less informative results. This is a good indication of the need for additional data or information collection and better characterization of the parameters.

In the second study an alternative to 2D Monte Carlo Analysis (2D MCA) is developed. 2D MCA is one of the advanced probabilistic risk assessment techniques in which one or more of the parameters of the risk equation are characterized by second order random variables (i.e., parameters of a probability density functions are characterized by probability density functions as well). Even in recent risk assessment studies, although the need for 2D MCA is justified, it cannot be conducted due to data limitations. Thus, 1D MCA or multiple 1D MCA are conducted instead. In the hybrid-model, the 2D Fuzzy Monte Carlo Analysis (2D FMCA), the variability in the random variable of the risk equation is characterized by a probability density function while the uncertainty associated with it is characterized by a membership function. For example, the exposure frequency is represented by a normal distribution whose mean and standard deviation are represented by membership functions. 2D FMCA uses a combination of probability theory and possibility theory concepts to propagate probabilistic and imprecise information into the resulting risk. The numerical results and associated conclusions are presented in detail in Sections 3.4.4 and 3.4.5, respectively. As a result of the 2D FMCA two cumulative distribution functions (cdfs) of risk at each alpha-cut level are obtained. These cdfs are used to generate fuzzy risks corresponding to each percentile. Fuzzy risks may only be used in decision making after health authorities identify acceptable level of possibility of risk violation. Acceptability of a fuzzy risk with respect to a compliance criterion can be evaluated by using the possibility measure, the necessity measure, or the risk tolerance measures which is proposed in this thesis.

As a result of the hybrid methods fuzzy health risks are calculated. Evaluation of acceptability of the resulting fuzzy risk with respect to a compliance guideline is another challenge. To streamline this task, a new measure, the risk tolerance measure which is a combination of the possibility and necessity measures is proposed in the first part of this thesis as well. The risk tolerance measure can be used to make decisions in the presence of incomplete information. Utilization of the risk tolerance measure in human health risk assessment context is presented. Effect of various membership functions of fuzzy risks on decision-making utilizing the possibility, the necessity, or the risk tolerance measure are evaluated as well. Cases in which the risk tolerance measure provides more plausible and intuitive results compared to the necessity and possibility measures are demonstrated. Utilization of the risk tolerance measure is not restricted to the human health risk assessment context; it can be used for decision-making in other areas which involve incomplete information. As an example, the risk tolerance measure is used to evaluate the acceptability of various management scenarios for selecting the best groundwater resources management strategy in the Savannah region.

In the second part of this thesis various methods which may be used sequentially in solving the groundwater resources management problem in the Savannah region are proposed. The goal is to select the best management strategy among alternatives in the presence of multiple conflicting objectives that may involve uncertainty.

The first framework we proposed is composed of a coupled simulation-optimization model followed by a fuzzy multi-objective decision-making framework. First, a crisp simulation-optimization model is used to calculate additional optimum pumping rates and corresponding drawdowns within the model domain for each alternative management scenario. The objective function of the optimization model includes a penalty term which can be adjusted by the user to control degree of "fairness" with respect to the distribution of limited groundwater resources in the region. The crisp results obtained from the coupled simulation-optimization model are used to evaluate overall performance of each alternative with respect to various objectives. Multiple objectives of the decision-maker are characterized by fuzzy sets. For example, satisfaction degrees of objectives such as "maintaining low drawdowns at critical locations" or "maintaining fair groundwater withdrawals for all users in the region" are used as criteria to evaluate each alternative. The individual satisfaction degrees are aggregated into an overall performance value by using various aggregators such as "and," "or," and "OWA." The management alternative with the highest degree of overall performance is selected as the best management strategy. The proposed decision making framework allows evaluation of hydrologically, economically, and politically motivated conflicting objectives in selecting the best groundwater management strategy. The results show that the proposed methodology might be implemented at the state or government level for groundwater resources management in coastal areas. Application of the proposed methodology to Savannah problem, numerical results and conclusions are provided in detail in Chapter 4.

As a result of the analysis conducted, it is found that the LFA is another reliable fresh water source in the region. The total amount of groundwater withdrawal potential for the LFA is similar to that of the UFA for each of the investigated management strategies (i.e., management strategies using various penalty terms). It is also demonstrated that additional pumping from the areas further away from Hilton Head Island have minimal impact on the saltwater intrusion problem at Hilton Head Island. However, there is a trade-off between maximizing the total amount of groundwater withdrawal from the aquifer and obtaining uniform withdrawal from the model domain. This is due to the fact that a unit decrease in pumping rate from a well located further away from Hilton Head Island results in a lower than unit increase in pumping from a well that is close to Hilton Head Island. Thus, it is possible to obtain more uniform withdrawal throughout the model domain by shifting some of the additional available water from inland areas towards Hilton Head Island; however this results in a decrease in the total amount of withdrawal from the aquifer.

The second approach in this area of research includes a groundwater flow model which uses the groundwater model operator method proposed by Dou et al. (1995) to simulate flow in the Savannah region in the presence of fuzzy aquifer parameters. For situations in which sufficient data does not exist to represent transmissivities by probability density functions, these parameters may be characterized by fuzzy sets using expert opinion or available imprecise information. The output of the groundwater model operator method is the fuzzy head distribution within the model domain. The fuzzy heads are used to calculate fuzzy drawdowns at critical locations in the region and acceptability of these fuzzy drawdowns with respect to crisp constraints are evaluated by using the risk tolerance measure. The resulting risk tolerance measures for each fuzzy drawdown at critical locations are then aggregated into an average risk tolerance value which in turn may be used in selecting the best management strategy. Two different management scenarios, first with a single additional well at Rincon and second with six additional wells at Rincon, Bloomingdale, Marlow, Ridgeland, Denmark, and Hinesville are evaluated with respect to three crisp hydrological objectives (i.e., drawdown at Tybee and Bull Islands should not exceed 0.8 ft and drawdown at Hilton Head Island should not exceed 0.05 ft). Individual risk tolerance measures associated with each one of the three objectives are aggregated into a single overall performance. The single additional well scenario resulted in an overall performance value of 0.52 while the six additional wells scenario resulted in a value of 0.13. Assuming a minimum overall performance of 0.5 is required for a management strategy to be acceptable; one additional well case is identified as an acceptable management scenario while six well case is not. This second decision making framework, different than the first, allows evaluation of fuzzy results with respect to crisp objectives. Although only hydrological objectives are used to demonstrate the proposed decision making approach, various other environmentally or economically motivated objectives may be included into the decision making process easily.

The second part of this thesis provides two decision-making frameworks for a site specific application (i.e., groundwater resources management problem in the Savannah region, GA). Several methods which allow treatment of uncertainties by non-probabilistic approaches are proposed and used sequentially to select the best groundwater resources management scenario for the Savannah region. Appropriate management of the water supply sources significantly impacts the future economic development in the region. Thus, methodologies proposed in the second part of this thesis may provide better living standards for the residents and improved opportunities for the existing and future industries in the region. Moreover, the analysis may provide guidance for groundwater resources management problems at other locations especially in coastal regions after necessary modifications are realized. For example, the decision-maker may need to identify appropriate objectives and fuzzy sets to characterize these objectives. However, the methodologies proposed in this thesis are general and can be used in other applications as well.

Application of non-probabilistic methods is relatively new in environmental engineering problems. We only explored a limited portion of this wide application range. Uncertainties associated with many other components of health risk assessment and groundwater resources management problems may require non-probabilistic uncertainty treatments. Most engineering problems, especially those that involve natural systems, provide excellent grounds for combined utilization of probabilistic and non-probabilistic techniques in uncertainty modeling. Many new theories are emerging in the area of uncertainty modeling with non-probabilistic and hybrid methods (Carlsson and Fullér 2001; Ferson and Ginzburg 1996; Fortemps and Roubens 1996; Guyonnet et al. 2005; Helton 2004; Moens and Vandepitte 2005; Oberkampf et al. 2004; Tonon et al. 2001). When the source and nature of available information is appropriate using these emerging methods in modeling natural systems is the future goal of our research

APPENDIX A - POSSIBILITY THEORY IN RELATION TO FUZZY SET THEORY

Possibility theory is a measure-theoretic counterpart of fuzzy set theory based upon the standard fuzzy operations. It provides us with appropriate tools for processing incomplete information expressed in terms of fuzzy propositions; consequently, it plays a major role in fuzzy logic and approximate reasoning (Klir and Yuan 1995).

The theory of possibility is related to the theory of fuzzy sets by defining the concept of a possibility distribution as a fuzzy restriction which acts as an elastic constraint on the values that may be assigned to a variable. More specifically (Zadeh 1978):

∴ Let F be a fuzzy subset of a universe of discourse U which is characterized by its membership function μ_F, with the grade of membership $\mu_F(u)$, interpreted as the compatibility of u with the concept labeled F.

∴ Let X be a variable taking values in U, and let F act as a fuzzy restriction, $R(X)$, associated with X. Then the proposition "X is F" which translates into

$$R(X) = F \tag{A.1}$$

which associates a possibility distribution, Π_X, with X which is postulated to be equal to $R(X)$:

$$\Pi_X = R(X) \tag{A.2}$$

Correspondingly, the possibility distribution function associated with X (or the possibility distribution function of Π_X) is denoted by π_X and is defined to be numerically equal to the membership function of F :

$$\pi_X \stackrel{\Delta}{=} \mu_F \tag{A.3}$$

Thus, $\pi_X(u)$, the possibility that $X = u$, is postulated to be equal to $\mu_F(u)$. In this way, X become a fuzzy variable which is associated with the possibility distribution, Π_X in much the same way as a random variable is associated with a probability distribution.

APPENDIX B - A GENERALIZED VERSION OF EXTENSION PRINCIPLE

The process of performing addition, subtraction, multiplication, etc. with fuzzy numbers is identified as fuzzy arithmetic. The extension principle, which is one of the most important concepts of fuzzy set theory, is used to conduct arithmetic operations on fuzzy numbers. In general, it enables us to extend any point operations to operations involving fuzzy sets. The extension principle can be generalized as follows (Yager and Filev 1994):

∴ Let A be a fuzzy subset of X and $\mu_{A(x)}$ the degree of membership of x in A.

∴ Let $X_1, X_2, \ldots, X_n$ and Y be a family of sets. Assume f is a mapping from the Cartesian product $X_1 \times X_2 \times \ldots \times X_n$ into Y, that is, for each n-tuple $(x_1, x_2, \ldots, x_n)$ such that $x_i \in X_i$, we have $f(x_1, x_2, \ldots, x_n) = y \in Y$. Let $A_1, A_2, \ldots, A_n$ be fuzzy subsets of $X_1, X_2, \ldots, X_n$ respectively; then, the extension principle allows for the evaluation of $f(A_1, A_2, \ldots, A_n)$. In particular, $f(A_1, A_2, \ldots, A_n) = B$, where B is a fuzzy subset of Y such that

$$B(y) = \underset{\substack{over\ all \\ (x_1 \ldots x_n) \in X_1 x..xX_n \\ such\ that \\ f(x_1,\ldots,x_n)=y}}{Max} [\mu_{A_1(x)} \wedge \mu_{A_2(x)} \wedge \ldots \wedge \mu_{A_n(x)}] \tag{B.1}$$

where $\wedge$ is used as the Min operator. Note that if there exists no tuple $(x_1, x_2, \ldots, x_n)$ such that $f(x_1, x_2, \ldots, x_n) = y$, then $B(y) = 0$.

APPENDIX C - RISK TOLERANCE MEASURE

When $L_{\widetilde{R}}\left(\alpha\right)$ and $U_{\widetilde{R}}\left(\alpha\right)$ (see Equation (3.33) in Chapter 3) are represented by general functions such as $L_{\widetilde{R}}\left(\alpha\right)=(p-b)\alpha^{m}+b$ and $U_{\widetilde{R}}\left(\alpha\right)=c-(c-p)\alpha^{n}$ respectively, β in Equation (3.30) decreases as $b=L_{\widetilde{R}}\left(0\right)$ goes from zero to $g=C_{comp}$ for all m and n (see Figure C.1). Proof is given below.

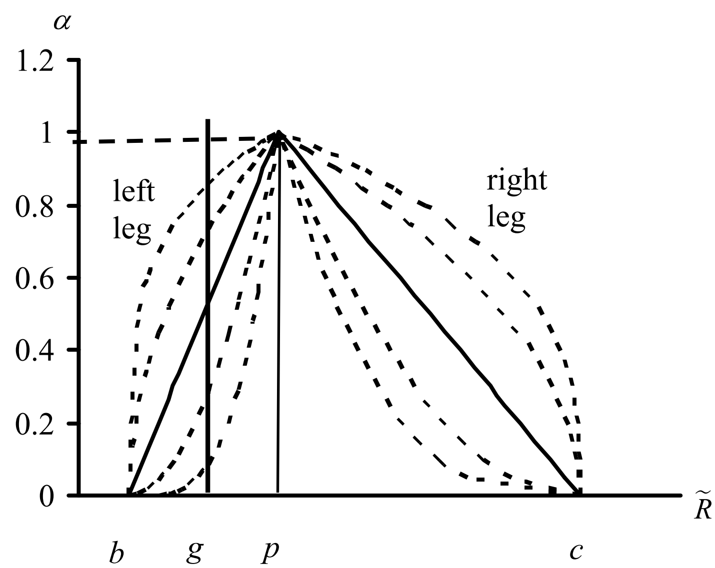


Figure C.1 Various membership functions for fuzzy risk, $\widetilde{R}$

The left and right legs of the membership functions shown in Figure C.1 can be represented by:

$$\text{left leg:} \quad \alpha = \left(\frac{x-b}{p-b}\right)^{1/m} \qquad L_{\tilde{R}}(\alpha) = (p-b)\alpha^m + b$$
$$\text{right leg:} \quad \alpha = \left(\frac{c-x}{c-p}\right)^{1/n} \qquad U_{\tilde{R}}(\alpha) = c-(c-p)\alpha^n \tag{C.1}$$

In order to show that β in Equation (3.30) in Chapter 3 increases as $b = L_{\tilde{R}}(0)$ goes from $g = C_{comp}$ to zero for all m and n we need to show that β decreases as b increases.

Let's first calculate A_{poss_l} and A_{poss_T}

$$\begin{aligned}
A_{poss_l} &= \int_0^{\left(\frac{g-b}{p-b}\right)^{1/m}} \alpha\left[g-(p-b)\alpha^m - b\right]d\alpha \\
&= \frac{1}{2}(g-b)\alpha^2 - \frac{1}{2+m}(p-b)\alpha^{2+m} \\
&= \frac{1}{2}(g-b)\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}} - \frac{1}{2+m}(p-b)\left(\frac{g-b}{p-b}\right)^{\frac{(2+m)}{m}} \\
&= \frac{1}{2}(g-b)\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}} - \frac{1}{2+m}(p-b)\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}}\left(\frac{g-b}{p-b}\right) \\
&= \frac{1}{2}(g-b)\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}} - \frac{1}{2+m}\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}}(g-b) \\
&= (g-b)\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}}\left(\frac{1}{2} - \frac{1}{2+m}\right) \\
&= (g-b)\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}}\left(\frac{m}{2(2+m)}\right)
\end{aligned} \tag{C.2}$$

$$
\begin{aligned}
A_{poss\_T} &= A_l + \int_0^{\left(\frac{g-b}{p-b}\right)^{1/m}} \alpha\left[c-(c-p)\alpha^n - g\right]d\alpha \\
&= A_l + \frac{1}{2}(c-g)\alpha^2 - \frac{1}{2+n}(c-p)\alpha^{2+n} \\
&= A_l + \frac{1}{2}(c-g)\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}} - \frac{1}{2+n}(c-p)\left(\frac{g-b}{p-b}\right)^{\frac{(2+n)}{m}} \\
&= A_l + \frac{1}{2}(c-g)\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}} - \frac{1}{2+n}(c-p)\left(\frac{g-b}{p-b}\right)^{\frac{n}{m}}\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}}
\end{aligned}
\tag{C.3}
$$

Now, lets calculate β:

$$
\begin{aligned}
\beta &= \frac{A_{poss\_l}}{A_{poss\_T}} \\
&= \frac{1}{1+\dfrac{\frac{1}{2}(c-g)\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}} - \frac{1}{2+n}(c-p)\left(\frac{g-b}{p-b}\right)^{\frac{n}{m}}\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}}}{(g-b)\left(\frac{g-b}{p-b}\right)^{2/m}\left(\frac{m}{2(2+m)}\right)}}
\end{aligned}
\tag{C.4}
$$

Let's call the second term in the denominator of β as D:

$$
\begin{aligned}
D &= \frac{\frac{1}{2}(c-g)\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}} - \frac{1}{2+n}(c-p)\left(\frac{g-b}{p-b}\right)^{\frac{n}{m}}\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}}}{(g-b)\left(\frac{g-b}{p-b}\right)^{\frac{2}{m}}\left(\frac{m}{2(2+m)}\right)} \\
&= \frac{1}{2}(c-g)\frac{2(2+m))}{m}\frac{1}{(g-b)} - \frac{1}{2+n}(c-p)\frac{2(2+m))}{m}\frac{1}{(g-b)}\left(\frac{g-b}{p-b}\right)^{\frac{n}{m}}
\end{aligned}
\tag{C.5}
$$

Remember the following derivatives:

$$\frac{\partial}{\partial b}\left(\frac{1}{(g-b)}\right)=\frac{1}{(g-b)^2}$$

$$\frac{\partial}{\partial b}\left(\left(\frac{g-b}{p-b}\right)\right)=\frac{-(p-g)}{(p-b)^2}$$

$$\frac{\partial}{\partial b}\left(\left(\frac{g-b}{p-b}\right)^{n/m}\right)=\frac{n}{m}\left(\frac{g-b}{p-b}\right)^{\frac{n}{m}-1}\left(\frac{-(p-g)}{(p-b)^2}\right)=\frac{-n}{m}\left(\frac{g-b}{p-b}\right)^{\frac{n}{m}}\frac{(p-b)}{(g-b)}\frac{(p-g)}{(p-b)^2}$$
$$=\frac{-n}{m}\left(\frac{g-b}{p-b}\right)^{\frac{n}{m}}\frac{(p-g)}{(g-b)}\frac{1}{(p-b)} \tag{C.6}$$

If the rate of change in D with respect to b is always positive, then β decreases as b increases (see Equation (C.4)). Let's take the derivative of D with respect to b:

$$\frac{\partial D}{\partial b}=\frac{1}{2}(c-g)\frac{2(2+m)}{m}\frac{1}{(g-b)^2}-\frac{1}{2+n}(c-p)\frac{2(2+m))}{m}$$
$$\left[\frac{1}{(g-b)^2}\left(\frac{g-b}{p-b}\right)^{\frac{n}{m}}-\frac{-n}{m}\left(\frac{g-b}{p-b}\right)^{\frac{n}{m}}\frac{(p-g)}{(g-b)}\frac{1}{(p-b)}\frac{1}{(g-b)}\right]$$
$$=\frac{1}{(g-b)^2}\frac{2(2+m))}{m}$$
$$\left[\frac{1}{2}(c-g)-\frac{1}{2+n}(c-p)\left(\frac{g-b}{p-b}\right)^{\frac{n}{m}}+\frac{n}{(2+n)m}(c-p)\frac{(p-g)}{(p-b)}\left(\frac{g-b}{p-b}\right)^{\frac{n}{m}}\right] \tag{C.7}$$

As can be seen from the above equation $\frac{\partial D}{\partial b}$ is always positive. Thus, as b increases D increases as well and $\beta=\frac{1}{1+\mathrm{D}}$ will decrease.

APPENDIX D - MODFLOW COMPUTER CODE AND THREE DIMENSIONAL GROUNDWATER FLOW EQUATION

MODFLOW is a computer program that solves the three-dimensional groundwater flow equation for a porous medium by using finite difference method (Harbough et al. 2000). The three-dimensional groundwater flow equation and finite difference formulation used to solve this partial differential equation in MODFLOW is explained in the following paragraphs.

The three-dimensional movement of groundwater of constant density through porous earth material in a heterogeneous and anisotropic medium, provided that the principal axes of hydraulic conductivity are aligned with the coordinate directions, may be described by the partial differential equation (McDonald and Harbaugh 1988):

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)-W=S_s\frac{\partial h}{\partial t} \qquad \text{(D.1)}$$

K_{xx}, K_{yy}, K_{zz} are values of hydraulic conductivity along the x, y, and z coordinate axes, which are assumed to be parallel to the major axes of hydraulic conductivity (Lt^{-1}); h is the piezometric head (L); W is a volumetric flux per unit volume and represents sources and/or sinks of water (t^{-1}); S_s is the specific storage of the porous material (L^{-1}); and t is time (t). In general, $S_s, K_{xx}, K_{yy}, and\ K_{zz}$ may be functions of space

($S_s = S_s\left(x, y, z\right)$, $K_{xx} = K_{xx}\left(x, y, z\right)$, etc.) and W may be a function of space and time $\left(W = W\left(x, y, z, t\right)\right)$.

Equation (D.1) together with initial and boundary conditions constitutes a mathematical representation of the groundwater flow. The solution of Equation (D.1) will yield time-varying piezometric head distribution, $h = h\left(x, y, z, t\right)$. Except for very simple systems, analytical solutions of Equation (D.1) are rarely possible, so various numerical methods must be employed to obtain approximate solutions (McDonald and Harbaugh 1988). One such approach is the finite difference method (FDM). Information about other numerical methods such as method of finite elements, relaxation methods, or Scmidt's graphic method can be found in Bear (1972). Since MODFLOW uses a block centered finite difference approach, brief explanation of FDM method is provided below.

In the FDM method the continuous model domain is represented by a finite set of discrete points (i.e., grid points) in space and time. Partial derivatives in Equation (D.1) are replaced by algebraic finite difference equations which are relationships among values of piezometric head, h at neighboring grid points of the $\left(x, y, z, t\right)$ space. As result, series of system of simultaneous linear equations are generated, (i.e., for each time step a system of simultaneous linear equations is formed). Solution of these systems of equations yields values of piezometric head at specific points and times. These values constitute an approximation to the time-varying head distribution throughout the flow domain.

Finite Difference Equation

Development of the groundwater flow equation in finite difference form follows from the application of the continuity equation: sum of all flows into and out of the cell must be equal to the rate of change in storage within the cell (McDonald and Harbaugh 1988). A block-centered formulation is used to discretize the model domain. A cell i, j, k, and six adjacent cells $i-1, j, k$; $i+1, j, k$; $i, j-1, k$; $i, j+1, k$; $i, j, k-1$; and $i, j, k+1$ are depicted in Figure D.1.

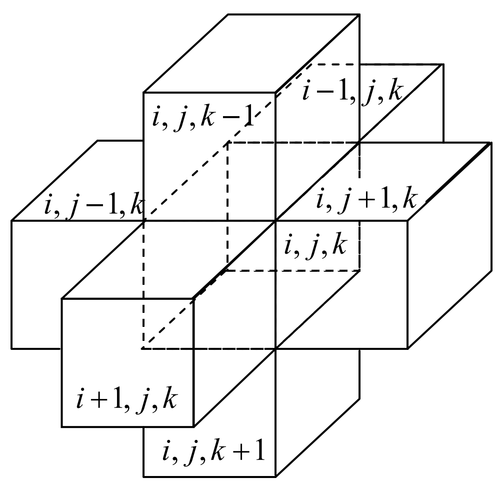


Figure D.1 Cell i, j, k and six adjacent cells (McDonald and Harbaugh 1988)

Using a backward difference approach for the time derivative finite difference equation for a cell becomes (McDonald and Harbaugh 1988):

$$
\begin{aligned}
& CR_{i,j-\frac{1}{2},k}\left(h^m_{i,j-1,k}-h^m_{i,j,k}\right)+CR_{i,j+\frac{1}{2},k}\left(h^m_{i,j+1,k}-h^m_{i,j,k}\right) \\
&\quad +CC_{i-\frac{1}{2},j,k}\left(h^m_{i-1,j,k}-h^m_{i,j,k}\right)+CC_{i+\frac{1}{2},j,k}\left(h^m_{i+1,j,k}-h^m_{i,j,k}\right) \\
&\quad +CV_{i,j,k-\frac{1}{2}}\left(h^m_{i,j,k-1}-h^m_{i,j,k}\right)+CV_{i,j,k+\frac{1}{2}}\left(h^m_{i,j,k+1}-h^m_{i,j,k}\right) \\
&\quad +P_{i,j,k}h^m_{i,j,k}+Q_{i,j,k}=SS_{i,j,k}\left(\Delta r_j \Delta c_i \Delta v_k\right)\frac{\left(h^m_{i,j,k}-h^{m-1}_{i,j,k}\right)}{t_m-t_{m-1}}
\end{aligned}
\tag{D.2}
$$

where $CR, CC, and\ CV$ are hydraulic conductances between node i, j, k and a neighboring node (L^2/t); $h^m_{i,j,k}$ is the head at cell i, j, k at time step m (L); $P_{i,j,k}$ is the sum of coefficients of head from source and sink terms (L^2/t); $Q_{i,j,k}$ is the sum of constants from source and sink terms, with $Q_{i,j,k} < 0.0$ for flow out of the groundwater system, and $Q_{i,j,k} > 0.0$ for flow in (L^3/t); $SS_{i,j,k}$ is the specific storage (L^{-1}); Δr_j is the cell width of column j in all rows (L); Δc_i is the cell width of row i in all columns (L); Δv_k is the vertical thickness of cell i, j, k (L); and t_m is the time at time step m (t). For example,

$$
CR_{i,j-\frac{1}{2},k}=KR_{i,j-\frac{1}{2},k}\times\frac{\Delta c_i \Delta v_k}{\Delta r_{j-\frac{1}{2}}}
\tag{D.3}
$$

where $KR_{i,j-\frac{1}{2},k}$ is the hydraulic conductivity along the row between nodes i, j, k and $i, j-1, k$ (Lt^{-1}); $\triangle c_i \triangle v_k$ is the area of the cell faces normal to the row direction; and $\triangle r_{j-\frac{1}{2}}$ is the distance between i, j, k and $i, j-1, k$ (L). Thus, $CR_{i,j-\frac{1}{2},k}$ is the conductance in row i and layer k between nodes i, j, k and $i, j-1, k$ (L^{2}t^{-1}), $CC_{i-\frac{1}{2},j,k}$ is the conductance in column j and layer k between nodes i, j, k and $i-1, j, k$ (L^{2}t^{-1}), $CV_{i,j,k-\frac{1}{2}}$ is the conductance in row i column j and between nodes i, j, k and $i, j, k-1$ (L^{2}t^{-1}).

For steady state case, the storage term is set to zero, and only one piezometric head value is calculated at each node. The objective of transient simulation is generally to predict piezometric head distribution at successive times, given the initial head distribution, the boundary conditions, the hydraulic parameters, and the external stresses (McDonald and Harbaugh 1988). The initial head distribution provides a piezometric head value at each point in the model domain, $h^1_{i,j,k}, \forall i, j, k$. Application of Equation (D.2) to each cell in the model domain for the next time step t_2 yields a system of equations and this system of equations is solved to estimate piezometric head at each node for time, $h^2_{i,j,k}$. For the next time step, the set of finite difference equations are reformulated and they yield a new system of equations. Solution of this new system of equations generates $h^3_{i,j,k}$. This process is continued for as many time steps as necessary to reach the time of interest.

Few key issues about the finite difference formulation used in MODFLOW are summarized below. Information below is mostly extracted from McDonald and Harbough (1988):

- Status of certain cells within the model domain is specified in advance to simulate boundary conditions. Thus, it is not generally necessary to write one finite difference equation as given in Equation (D.2), for each cell. Such cells are grouped into two categories, “constant-head” and “inactive” (or “no-flow”) cells. Constant-head cells are those for which the head is specified in advance, and is held at this specified value through all time steps of the simulation. Inactive or no-flow cells are those for which no flow into or out of these cells are permitted, in any step of the simulation. The remaining cells, termed “variable-head” cells are characterized by heads which are unspecified and free to vary with time. An equation of the form of Equation (D.2) must be formulated for each variable-head cell, and the resulting system of equations must be solved simultaneously to estimate piezometric heads at each of these cells for each time step.
- Data requirements for the model may include transmissivity, hydraulic conductivity, specific yield, confined yield coefficient, vertical leakance, aquifer bottom elevation, and aquifer top elevation.
- The thickness of individual layers is never read explicitly by the program; rather this thickness is embedded in various hydraulic coefficients specified by the user. For example, in confined layers transmissivity, which is the product of hydraulic conductivity and layer thickness, is specified; and storage coefficient, the product of specific storage and layer thickness, is also used. For an unconfined layer,

aquifer bottom elevation and hydraulic conductivity are input for each cell. Saturated thickness is calculated by subtracting bottom elevation from piezometric head, and transmissivity is calculated by multiplying hydraulic conductivity with the saturated thickness. Thus, layer thickness can vary from cell to cell depending on bottom elevation.

- Since a backward difference form is used for the time derivative, a piezometric head distribution at the beginning of each time step is required to calculate the piezometric head distribution of that time step. Thus, for the first time step, "starting heads" need to be specified by the user.
- Four types of model layers can be defined (i) Type 0 for layer which is strictly confined; (ii) Type 1 for layer which is strictly unconfined; (iii) Type 2 for layer which is partially convertible between confined and unconfined. Transmissivity of each cell is constant throughout the simulation; (iv) Type 3 for layer which is fully convertible between confined and unconfined. During a flow simulation transmissivity of each cell varies with the saturated thickness of the aquifer (Chiang and Kinzelbach 2000).
- Transmissivity is required for layers of type 0 and 2. Transmissivity may be calculated from horizontal hydraulic conductivity and the elevations of the top and bottom of each layer by the program or may be manually specified by the user.
- Two values, one in the row direction, one in the column direction, are required for transmissivity and hydraulic conductivity in each cell. Only one value, one in the row direction, is read for each of these parameters. An anisotropy factor is

also specified for each layer by the user. The value of the parameter in column direction is calculated as the product of the value in row direction and the anisotropy factor.

- For flow simulations involving more than one layer, program requires the input of the vertical conductance term, known as vertical leakance, $Vcont$, between two layers. Vertical leakance may be manually specified by the user or may be calculated by the program with the following formula:

$$Vcont_{i,j,k+\frac{1}{2}} = \frac{2}{\frac{\triangle v_k}{(K_z)_{i,j,k}} + \frac{\triangle v_{k+1}}{(K_z)_{i,j,k+1}}}$$

where $\triangle v_k$ and $\triangle v_{k+1}$ are thicknesses of model layers k and $k+1$ respectively; $(K_z)_{i,j,k}$ and $(K_z)_{i,j,k+1}$ are vertical hydraulic conductivity values of layers k and $k+1$, respectively.

- An injection or a pumping well is represented by a cell. The user specifies an injection or a pumping rate for each cell. It is implicitly assumed that the well penetrates the full thickness of the cell.

APPENDIX E - GAMS (GENERAL ALGEBRAIC MODELING SYSTEM) SOFTWARE

GAMS (General Algebraic Modeling System) is a software product of GAMS Development Corporation (http://www.gams.com) which solves mathematical programs (Rardin 1999). It includes the capability to globally solve linear programs and integer linear programs, as well as to find local optima of nonlinear programs and integer nonlinear programs that have all nonlinearities in continuous variables (Rardin 1999).

GAMS is an algebraic modeling language. GAMS programs consists of one or more statements (sentences) that define data structures, initial values, data modifications, and symbolic relationships (equations) written in GAMS language (Rosenthal 2006). GAMS is best employed for medium and large sized models (more than 100 rows and/or columns) and can handle large problems (McCarl 2006). GAMS code is portable between computers. GAMS has been implemented on machines ranging from PCs to UNIX/LINUX workstations to CRAY super computers. Exactly the same code runs on all of the computer systems (McCarl 2006).

Various types of problems can be solved with GAMS. The type of the model must be specified in the program before it is intended to be solved. The type of problems that can be solved with GAMS include linear programming (LP), nonlinear programming (NLP), discontinuous nonlinear problem (DNLP), relaxed mixed integer programming (RMIP), mixed integer programming (MIP), mixed integer nonlinear programming (MINLP), constrained nonlinear system (CNS), relaxed mixed integer nonlinear programming (RMINLP) etc. Various solvers are available for each one of these problem types (Rosenthal 2006). Thus, GAMS itself does not solve the problem but it passes the problem to one of the available solvers. The available solvers include BARON, CONOPT, LINGO, LGO, MINOS, CPLEX, SNOP, PATH, XA, ZOOM, etc. (McCarl 2006).

GAMS is a two pass program. First the user creates a file with the extension GMS which contains GAMS instructions. Then the user submits the file to GAMS and in return GAMS executes those instructions causing calculations to be done, solvers to be used, and a solution file of the execution results to be created (McCarl 2006). GAMS offers a number of choices to solve a model and the user may switch solvers if they have appropriate licenses.

Solution process in GAMS can be summarized by the following steps (McCarl 2006): (i) The user has to specify the type of the problem, LP, NLP, MIP, etc.; (ii) GAMS checks the model type, and issues explanatory error messages if it discovers a model beyond the solution capabilities of the solver to be used; (iii) A solver is chosen which is either the default solver for that problem type, one specified on the GAMS command line, or solver chosen by an option statement; (iv) Necessary checks related with the program structure are conducted. These may include checks such as “all sets and parameters used in the equations are checked to insure they have had values assigned,” or “all equations in the model are checked to insure they have been defined;” (v) The model is translated into the representation required by the solver to be used; (vi) GAMS verifies that there are no errors such as inconsistent bounds, inconsistent equations, or unacceptable values in the problem; (vii) GAMS passes control to the solution subsystem and waits while the problem is solved; (viii) GAMS collects back information on the solution process from the solver and loads solution values back into the memory; and, (ix) a row by row and column by column listing of the solution is provided.

We used GAMS for two different problems in the second part of the thesis. First, the coupled simulation-optimization model is solved by GAMS. Optimum pumping rates of the potential wells are calculated by the coupled simulation-optimization model. Then, we used GAMS to solve the numerical groundwater flow model with fuzzy parameters. Groundwater model operator method of (Dou et al. 1995) is used to calculate piezometric head distribution within the model domain when transmissivities are characterized by fuzzy numbers. Both the coupled simulation-optimization model and the groundwater model operator method yield nonlinear optimization problems. Among the available solvers provided in GAMS, some are cable of solving nonlinear problems. For example, MINOS and CONOPT are solvers for large-scale nonlinear optimization problems (McCarl 2006). We used these two solvers to model the nonlinear problems we formulated at various stages of the groundwater resources management problem in the Savannah region. Brief explanations about these two solvers are provided below.

CONOPT is developed and maintained by A. Drud, ARKI Consulting and Development. CONOPT is a feasible path solver based on the generalized reduced gradient (GRG) method. CONOPT contains extensions to the GRG method. CONOPT can solve LP, RMIP, NLP, CNS, DNLP, and RMINLP (McCarl 2006).

MINOS is developed by B. Murtaugh and M. Saunders at Macquarie University and Stanford University. MINOS solves large-scale nonlinear optimization problems using a reduced gradient algorithm combined with a quasi-Newton algorithm. This involves a sequence of major iterations, each of which requires the solution of a linearly constrained subproblem. MINOS can solve LP, RMIP, NLP, DNLP, and RMINLP model types (McCarl 2006).

Mathematically a nonlinear programming (NLP) problem looks like (McCarl 2006):

$$\begin{array}{ll} \textit{Maximize or Minimize} & f(x) \\ \textit{subject to} & g(x) \ \breve{a} \ 0 \\ & L \leq x \leq U \end{array}$$

where x is a vector of variables that are continuous real numbers; $f(x)$ is the objective function, $g(x)$ represents the set of constraints, $\breve{a}$ is some mixture of $\leq$, $=$, and $\geq$ operators, and L and U are vectors of lower and upper bounds on the variables. Both $f(x)$ and $g(x)$ must be differentiable. Either the objective function or constraints contains nonlinear terms.

All GAMS can guarantee for a nonlinear problem is a local optimum (Rosenthal 2006). Formulating nonlinear problems requires that the modeler pays attention to some details that play no role when dealing with linear problems (Drud; Murtagh et al.). One of these issues is the starting points. While solving the nonlinear optimization problem resulting from the groundwater model operator method we used the piezometric head distribution obtained from the crisp numerical model as initial values and this helped GAMS to find solution for the fuzzy numerical groundwater flow model. Why starting points are important in nonlinear programming is explained below.

By default the initial value chosen for all variables is zero or the lower bound (McCarl 2006). Unfortunately, zero in many cases is a bad initial value for a nonlinear variable. For, example, an initial value of zero is especially bad if the variable appears in a product term since the initial derivative becomes zero, and it appears as if the function does not depend on the variable. Zero starting values can cause numerical difficulties when logarithms, exponentials, or divisions are involved. Also nonzero lower bound derived starting points may not be desirable as derivatives evaluated at small lower bounds may be very large and provide the algorithm with misleading information (McCarl 2006).

The specification of starting points involving good initial values for the individual variables is important in a NLP context for a number of reasons (McCarl 2006):

- Non-convex models may have multiple solutions and the solvers generally only try to find one local one. An initial point in the right neighborhood is more likely to return a desirable solution.
- Initial values that satisfy many of the constraints reduce the work involved in finding a first feasible solution.

- Initial values that are close to the optimal ones reduce the work required to find the optimal and therefore the solution time.
- The progress of the optimization algorithm is based on good directional information and therefore on good derivatives. The derivatives in a nonlinear model depend on the current point and an improved initial point can improve solver performance.

GAMS has been used in many research areas. A large number of applications in which GAMS is used are provided under “Contributed Documents” link provided in the GAMS Homepage (i.e., http://www.gams.com/). One of these applications which is of particular interest to us is “Basic Optimization Models for Water and Energy Management” (McKinney and Savitsky 2003). McKinney and Savitsky (2003) provides solution methodologies by GAMS for a collection of water and energy resources problems. A GAMS code for modeling two-dimensional flow by finite difference method is provided in this reference. The model is developed for the homogenous domain, thus a single transmissivity value is used.

APPENDIX F - OPTIMUM PUMPING RATES FROM POTENTIAL WELLS

Table F. 1 Optimum additional pumping rates from each potential well for UFA using $w_i = 1;\ i = 1,2,...,6$, $w_i = (2 - d_i / d_{\max})$, $w_i = (4 - d_i / d_{\max})$, and $w_i = (6 - d_i / d_{\max})$ and for LFA using $w_i = 1;\ i = 1,2,...,6$ and $w_i = (2 - d_i / d_{\max})$

Coord.		Pumping rate (MGal/d)					
X	y	UFA $w_i = 1, \forall i$	UFA $w_i = \left(2 - \frac{d_i}{d_{\max}}\right)$	UFA $w_i = \left(4 - \frac{d_i}{d_{\max}}\right)$	UFA $w_i = \left(6 - \frac{d_i}{d_{\max}}\right)$	LFA $w_i = 1, \forall i$	LFA $w_i = \left(2 - \frac{d_i}{d_{\max}}\right)$
10	66	0.666	0.608	0.285	0.205	0.646	0.591
20	66	0.559	0.483	0.258	0.191	0.548	0.472
30	66	0.385	0.343	0.22	0.171	0.388	0.341
40	66	0.146	0.19	0.173	0.145	0.168	0.198
50	66	0	0.022	0.118	0.113	0	0.047
60	66	0	0.042	0.122	0.115	0	0.065
70	66	0.272	0.257	0.193	0.156	0.239	0.234
25	61	0.387	0.347	0.221	0.171	0.393	0.347
35	61	0.069	0.144	0.159	0.137	0.099	0.158
45	61	0	0	0.076	0.09	0	0
55	61	0	0	0.027	0.061	0	0
65	61	0	0	0.075	0.088	0	0
75	61	0.577	0.405	0.244	0.186	0.22	0.223
10	56	0.639	0.569	0.278	0.201	0.622	0.553
20	56	0.463	0.404	0.237	0.18	0.46	0.399
30	56	0.165	0.201	0.177	0.147	0.181	0.207
50	56	0	0	0	0.029	0	0
60	56	0	0	0	0.011	0	0
70	56	0	0.088	0.133	0.121	0	0
80	56	0.716	0.467	0.266	0.199	0.272	0.247
15	51	0.553	0.478	0.256	0.19	0.542	0.467
25	51	0.322	0.3	0.208	0.164	0.327	0.3
35	51	0	7.46E-04	0.113	0.111	0	0.026
45	51	0	0	0	0.03	0	0
55	51	0	0	0	0	0	0
65	51	0	0	0	0	0	0
10	46	0.643	0.563	0.277	0.201	0.625	0.547
20	46	0.442	0.385	0.232	0.177	0.439	0.379
30	46	0.081	0.151	0.162	0.138	0.109	0.164
40	46	0	0	0.03	0.063	0	0
50	46	0	0	0	0	0	0
60	46	0	0	0	0	0	0
70	46	0	0	0	0	0	0
80	46	0.471	0.341	0.222	0.174	0	0
15	41	0.563	0.481	0.258	0.191	0.551	0.47

25	41	0.291	0.28	0.201	0.161	0.301	0.282
35	41	0	0	0.101	0.104	0	0
45	41	0	0	0	0.003	0	0
55	41	0	0	0	0	0	0
65	41	0	0	0	0	0	0
10	36	0.64	0.559	0.276	0.201	0.622	0.543
20	36	0.468	0.402	0.237	0.18	0.465	0.396
30	36	0.094	0.159	0.164	0.14	0.116	0.168
40	36	0	0	0.036	0.067	0	0
50	36	0	0	0	0	0	0
70	36	0	0	0	0	0	0
15	31	0.565	0.485	0.259	0.191	0.553	0.474
25	31	0.335	0.308	0.21	0.165	0.344	0.31
35	31	0	0.054	0.129	0.12	0	0.053
45	31	0	0	0	0.035	0	0
10	26	0.648	0.573	0.279	0.202	0.633	0.56
30	26	0.198	0.221	0.183	0.151	0.224	0.233
40	26	0	0	0.103	0.105	0	0
15	21	0.59	0.513	0.265	0.195	0.597	0.516
25	21	0.42	0.367	0.227	0.175	0.424	0.366
10	16	0.671	0.608	0.286	0.205	0.663	0.601
20	16	0.55	0.473	0.255	0.19	0.557	0.475
22	11	0.573	0.492	0.26	0.192	0.571	0.488
29	10	0.509	0.431	0.245	0.184	0.492	0.416
35	19	0.146	0.19	0.174	0.145	0.218	0.228
43	24	0	0	0.088	0.097	0	0
52	27	0	0	0	0	0	0
60	33	0	0	0	0	0	0
70	40	0	0	0	0	0	0
15	64	0.598	0.527	0.268	0.196	0.584	0.514
60	61	0	0	0.04	0.068	0	0
42	54	0	0	0.046	0.073	0	0
75	52	0.586	0.395	0.242	0.185	0	0
44	34	0	0	0	0.033	0	0
19	27	0.498	0.428	0.244	0.184	0.494	0.422
Total		16.449	14.735	9.838	8.103	14.687	13.48

Table F. 2 Optimum additional pumping rates from each potential well for LFA using $w_i = (4 - d_i / d_{\max})$ and $w_i = (6 - d_i / d_{\max})$, and for UFA+LFA using $w_i = 1;\ i = 1, 2, ..., 6$, $w_i = (2 - d_i / d_{\max})$, $w_i = (4 - d_i / d_{\max})$, and $w_i = (6 - d_i / d_{\max})$

Coord.		Pumping rate (MGal/d)					
x	y	LFA $w_i = \left(4 - \frac{d_i}{d_{\max}}\right)$	LFA $w_i = \left(6 - \frac{d_i}{d_{\max}}\right)$	UFA+LFA $w_i = 1, \forall i$	UFA+LFA $w_i = \left(2 - \frac{d_i}{d_{\max}}\right)$	UFA+LFA $w_i = \left(4 - \frac{d_i}{d_{\max}}\right)$	UFA+LFA $w_i = \left(6 - \frac{d_i}{d_{\max}}\right)$
10	66	0.272	0.194	0.557	0.523	0.234	0.163
20	66	0.247	0.181	0.421	0.374	0.202	0.147
30	66	0.212	0.162	0.199	0.201	0.158	0.124
40	66	0.169	0.139	0	0.008	0.103	0.095
50	66	0.119	0.11	0	0	0.038	0.06
60	66	0.123	0.112	0	0	0.044	0.062
70	66	0.179	0.144	0	0.078	0.118	0.103
25	61	0.214	0.163	0.205	0.207	0.16	0.125
35	61	0.157	0.132	0	0	0.086	0.086
45	61	0.083	0.089	0	0	0	0.033
55	61	0.036	0.062	0	0	0	0
65	61	0.077	0.085	0	0	0	0.029
75	61	0.175	0.142	0	0.066	0.113	0.1
10	56	0.265	0.19	0.524	0.477	0.225	0.158
20	56	0.228	0.171	0.3	0.276	0.178	0.135
30	56	0.172	0.14	0	0.015	0.106	0.097
50	56	0	0.03	0	0	0	0
60	56	0	0.011	0	0	0	0
70	56	0.05	0.069	0	0	0	0.009
80	56	0.183	0.147	0.038	0.104	0.124	0.106
15	51	0.246	0.18	0.413	0.368	0.2	0.146
25	51	0.2	0.156	0.114	0.144	0.142	0.116
35	51	0.114	0.107	0	0	0.031	0.056
45	51	0	0.032	0	0	0	0
55	51	0	0	0	0	0	0
65	51	0	0	0	0	0	0
10	46	0.265	0.19	0.529	0.472	0.225	0.158
20	46	0.223	0.168	0.269	0.252	0.171	0.131
30	46	0.159	0.133	0	0	0.088	0.087
40	46	0.036	0.062	0	0	0	0
50	46	0	0	0	0	0	0
60	46	0	0	0	0	0	0
70	46	0	0	0	0	0	0
80	46	0	0.025	0	0	0	0
15	41	0.247	0.181	0.426	0.374	0.202	0.147
25	41	0.195	0.153	0.077	0.12	0.136	0.112
35	41	0.105	0.102	0	0	0.019	0.049
45	41	0	0	0	0	0	0
55	41	0	0	0	0	0	0
65	41	0	0	0	0	0	0
10	36	0.264	0.19	0.524	0.468	0.224	0.158
20	36	0.228	0.171	0.306	0.277	0.178	0.135

30	36	0.16	0.133	0	0	0.09	0.088
40	36	0.034	0.061	0	0	0	0
50	36	0	0	0	0	0	0
70	36	0	0	0	0	0	0
15	31	0.248	0.181	0.429	0.378	0.203	0.148
25	31	0.203	0.157	0.138	0.159	0.146	0.118
35	31	0.123	0.112	0	0	0.042	0.062
45	31	0	0.019	0	0	0	0
10	26	0.267	0.191	0.54	0.488	0.228	0.16
30	26	0.18	0.145	0	0.054	0.116	0.102
40	26	0.089	0.093	0	0	0	0.038
15	21	0.258	0.187	0.49	0.433	0.216	0.154
25	21	0.219	0.166	0.249	0.235	0.167	0.129
10	16	0.276	0.196	0.582	0.539	0.238	0.165
20	16	0.248	0.182	0.434	0.381	0.204	0.148
22	11	0.251	0.183	0.454	0.398	0.208	0.15
29	10	0.233	0.174	0.344	0.304	0.185	0.138
35	19	0.179	0.144	0	0.05	0.115	0.101
43	24	0.069	0.081	0	0	0	0.024
52	27	0	0	0	0	0	0
60	33	0	0	0	0	0	0
70	40	0	0	0	0	0	0
15	64	0.256	0.186	0.471	0.426	0.213	0.153
60	61	0.043	0.066	0	0	0	0.004
42	54	0.054	0.073	0	0	0	0.013
75	52	0.026	0.055	0	0	0	0
44	34	0	0.016	0	0	0	0
19	27	0.234	0.174	0.346	0.31	0.186	0.139
10	66	-	-	0.557	0.523	0.234	0.163
20	66	-	-	0.421	0.374	0.202	0.147
30	66	-	-	0.2	0.201	0.158	0.124
40	66	-	-	0	0.01	0.103	0.095
50	66	-	-	0	0	0.037	0.059
60	66	-	-	0	0	0.043	0.062
70	66	-	-	0.056	0.111	0.128	0.109
25	61	-	-	0.203	0.205	0.159	0.125
35	61	-	-	0	0	0.086	0.086
45	61	-	-	0	0	0	0.032
55	61	-	-	0	0	0	0
65	61	-	-	0	0	0	0.031
75	61	-	-	0.444	0.31	0.19	0.142
10	56	-	-	0.524	0.477	0.225	0.158
20	56	-	-	0.3	0.276	0.178	0.135
30	56	-	-	0	0.02	0.107	0.097
50	56	-	-	0	0	0	0
60	56	-	-	0	0	0	0
70	56	-	-	0	0	0.057	0.069
80	56	-	-	0.622	0.393	0.216	0.157
15	51	-	-	0.413	0.368	0.2	0.146
25	51	-	-	0.119	0.147	0.143	0.116
35	51	-	-	0	0	0.031	0.056
45	51	-	-	0	0	0	0
55	51	-	-	0	0	0	0

65	51	-	-	0	0	0	0
10	46	-	-	0.529	0.473	0.225	0.158
20	46	-	-	0.272	0.253	0.172	0.131
30	46	-	-	0	0	0.089	0.087
40	46	-	-	0	0	0	0.002
50	46	-	-	0	0	0	0
60	46	-	-	0	0	0	0
70	46	-	-	0	0	0	0
80	46	-	-	0.309	0.232	0.165	0.129
15	41	-	-	0.427	0.374	0.202	0.147
25	41	-	-	0.08	0.122	0.136	0.113
35	41	-	-	0	0	0.016	0.048
45	41	-	-	0	0	0	0
55	41	-	-	0	0	0	0
65	41	-	-	0	0	0	0
10	36	-	-	0.524	0.468	0.224	0.158
20	36	-	-	0.306	0.277	0.178	0.135
30	36	-	-	0	0	0.092	0.089
40	36	-	-	0	0	0	0.006
50	36	-	-	0	0	0	0
70	36	-	-	0	0	0	0
15	31	-	-	0.429	0.378	0.203	0.148
25	31	-	-	0.135	0.158	0.146	0.118
35	31	-	-	0	0	0.051	0.067
45	31	-	-	0	0	0	0
10	26	-	-	0.535	0.484	0.227	0.159
30	26	-	-	0	0.048	0.115	0.101
40	26	-	-	0	0	0.019	0.049
15	21	-	-	0.461	0.41	0.21	0.151
25	21	-	-	0.245	0.233	0.166	0.129
10	16	-	-	0.564	0.524	0.234	0.163
20	16	-	-	0.41	0.363	0.199	0.146
22	11	-	-	0.439	0.387	0.205	0.149
29	10	-	-	0.358	0.313	0.188	0.14
35	19	-	-	0	0.009	0.103	0.095
43	24	-	-	0	0	0.002	0.04
52	27	-	-	0	0	0	0
60	33	-	-	0	0	0	0
70	40	-	-	0	0	0	0
15	64	-	-	0.471	0.425	0.213	0.153
60	61	-	-	0	0	0	0.008
42	54	-	-	0	0	0	0.013
75	52	-	-	0.456	0.302	0.188	0.142
44	34	-	-	0	0	0	0
19	27	-	-	0.344	0.308	0.186	0.139
Total		8.893	7.298	20.532	18.915	12.713	10.383

For UFA+LFA cases (i.e., the last four columns) a total of 140 potential wells are listed. The first 70 values (i.e., coordinates marked with shaded cells) represent to the pumping rates from the LFA and the last 70 represents pumping from the UFA.

APPENDIX G - FUZZY AGGREGATION

Individual satisfaction degrees can be aggregated into an overall performance value by using the fuzzy aggregation approach. The aggregation process can be realized by using various aggregation operators identified as conjunctive, disjunctive, or averaging operators. In this study we used “and”, “or”, and “ordered weight averaging (OWA)” as conjunctive, disjunctive, and averaging operators, respectively. Brief definitions of these aggregation operators are provided below.

Conjunctive Operator, “and”

Various approaches have been proposed for combining multiple objectives in a decision where there is some uncertainty. The goal is to select from the alternatives, $A_1,...,A_r$, the one which performs best with respect to the set of multiple objectives, $\widetilde{F}_k$, $k = 1,2,3,...,m$. Multiple objectives can be aggregated by a conjunctive operator which combines values as an “and” operator. This approach is proposed by Bellman and Zadeh (1970). The overall performance for alternative A_s, $D_s = D(A_s)$, can be calculated as follows:

$$\begin{aligned} &D = \widetilde{F}_1 \cap \widetilde{F}_2 \cap ... \cap \widetilde{F}_m \\ &D_s = D(A_s) = \min[\mu_{s,1}, \mu_{s,2}, ..., \mu_{s,m}] \quad s = 1,2,...,r \end{aligned} \tag{G.1}$$

where $\mu_{s,j}$ is the individual satisfaction of alternative A_s for fuzzy objective $\widetilde{F}_i$.

Bellman and Zadeh's formulation intrinsically assigns equal importance for each objective. However, multiple objectives of the decision problem may have differing degrees of importance. One approach to treat multiple objectives with different degrees of importance is proposed by Yager (1978). For this case, a non-negative number w is associated with each objective, indicating its power or importance in the decision, with higher numbers chosen for more important objectives:

$$D = \widetilde{F}_1^{\,w_1} \cap \widetilde{F}_2^{\,w_2} \cap \ldots \cap \widetilde{F}_m^{\,w_m} \tag{G.2}$$

where w_k, $k = 1, 2, 3, \ldots, m$ is the weights associated with each fuzzy objective $\widetilde{F}_k$, $k = 1, 2, 3, \ldots, m$.

The membership values range between zero and one. Thus, the membership grades in all objectives having little importance ($w < 1$) becomes larger, while those in objectives having more importance ($w > 1$) become smaller. Since the conjunctive operator (i.e., "and") used to aggregate the fuzzy objectives assigns the minimum individual membership grade as the overall performance, the important objective which has the smallest membership grade determines the overall satisfaction. The procedure for determining the power of importance is provided by Yager (1978). An application of Yager's approach can be found in Ravi and Reddy (1999). Selection of w_k is the manager's decision; here, we will assume that w_k, $k = 1, 2, 3, \ldots, m$ values are known.

Disjunctive Operator, "or"

Disjunctive operators combine values as an "or" operator, so that the result of the combination is high if some (at least one) values are high (Slowiński 1998). For the disjunctive operator fuzzy objectives are aggregated as follows:

$$\begin{aligned} &D = \widetilde{F_1} \cup \widetilde{F_2} \cup \ldots \cup \widetilde{F_m} \\ &D_s = D(A_s) = \max[\mu_{s,1}, \mu_{s,2}, \ldots, \mu_{s,m}] \quad s = 1, 2, \ldots, r \end{aligned} \tag{G.3}$$

Averaging Operator, "OWA"

Between conjunctive and disjunctive operators, there is room for a third category, namely averaging operators. They are located between minimum and maximum aggregators. Averaging operators have the property to be compensative, that is, low values can be compensated by high values, so that the results of combination will be medium (Slowiński 1998). Some common examples of averaging operators are mean operators, median and order statistics, ordered weight averaging operators, etc. Here, we choose OWA as an example of averaging operators. OWA operators are introduced by Yager (1988). These operators allow inclusion of behavioral properties into decision-making. A brief explanation of OWA operator is provided below.

An OWA operator of dimension m is a mapping that has an associated vector V with m members (Yager 1996a):

$$V = \begin{bmatrix} v_1 & v_2 & \dots & v_m \end{bmatrix}^T \quad \text{(G.4)}$$

such that $v_i \in [0,1]$ and $\sum_i v_i = 1$ where

$$f\left(\mu_{s,1}, \mu_{s,2}, \dots, \mu_{s,m}\right) = \sum_i v_i b_i \quad \text{(G.5)}$$

with b_i being the i^{th} largest of $\mu_{s,1}, \mu_{s,2}, \dots, \mu_{s,m}$. The aggregation operation is represented by f, and the individual satisfaction of alternative s, A_s, for fuzzy objective $\widetilde{F}_i$ is represented by $\mu_{s,i}$.

A fundamental aspect of this operation is the re-ordering step, in particular an aggregate $\mu_{s,i}$ is not associated with a particular weight v_i, but rather a weight is associated with a particular ordered position of the aggregate (Yager 1996a). That is, v_i is the weight associated with the i^{th} largest element. Different weighting vectors generate different OWA operators. Min (i.e., the conjunctive operator used in this study), max (i.e., the disjunctive operator used in this study), and simple average are special cases of OWA operator. Min, max, and simple average can be obtained by choosing the appropriate V vectors as follows:

Min: $v_m = 1$ and $v_i = 0$ for all other weights.

Max: $v_1 = 1$ and $v_i = 0$ for all other weights.

Simple average: $v_i = 1/m$ for all i

OWA operators provide an aggregation which lies in between the two extremes: aggregation with "and" and "or." The structure of OWA operators is suitable for combining the objectives under the guidance of a quantifier. For example, the requirement that "most" of the objectives be satisfied corresponds to one of the OWA operators. The process of determining the best strategy using a linguistic quantifier, $\widetilde{Q}$, is called quantifier guided aggregation. The linguistic quantifier $\widetilde{Q}$ can be represented as a fuzzy set $\widetilde{Q}$ of I, where for each $r \in I$, $Q(r)$ indicates the degree to which the proposition r satisfies the concept denoted by $\widetilde{Q}$. The form of decision function for this approach is:

"$\widetilde{Q}$ fuzzy objectives are satisfied by a good solution"

The decision-maker feels satisfaction of $\widetilde{Q}$ fuzzy objectives is necessary for a good solution. The procedure to evaluate this decision function is summarized as follows (Yager 1996b):

i. Use $Q(.)$ to generate a set of OWA weights $v_1, v_2, ..., v_m$

ii. For each alternative A_s calculate the overall satisfaction:

$$D_s(A_s) = f\left(\mu_{s,1}, \mu_{s,2}, ..., \mu_{s,m}\right) \tag{G.6}$$

where f is an OWA aggregation using the weights found in step i.

The procedure used to generate the weights from the quantifier depends on the type of the quantifier required by the decision-maker. For example, weights can be calculated using the following formula for cases in which the quantifier, $\widetilde{Q}$, is a Regular Increasing Monotone (RIM) quantifier like "all," "most" or "many" etc.:

$$v_i = Q\left(\frac{i}{m}\right) - Q\left(\frac{i-1}{m}\right) \quad for\ i = 1,2,\ldots,m \tag{G.7}$$

where m is the total number of fuzzy objectives. For more detailed information on various classes of quantifiers the reader may refer to Yager (1996a; 1996b).

It is also possible to use OWA aggregators with unequal objectives. Let's assume that each fuzzy objective has a value w_i indicating the importance of that objective, where $w_i \in [0,1]$. The larger the w_i, the more important is the fuzzy objective. Again considering $\widetilde{Q}$ being some RIM quantifier and assuming a decision function is in the form of "$\widetilde{Q}$ important criteria are satisfied by A_s", the procedure to evaluate the overall satisfaction of alternative A_s is summarized below:

i. Order the $\mu_{s,i}$ in descending order. Let b_j be the j^{th} largest of $\mu_{s,1}, \mu_{s,2}, \ldots, \mu_{s,m}$.

ii. Let u_j denote the importance associated with the objective that has the j^{th} largest satisfaction to A_s. Thus if $\mu_{s,2}$ is the largest of the $\mu_{s,1}, \mu_{s,2}, \ldots, \mu_{s,m}$, then $b_1 = \mu_{s,2}$ and $u_1 = w_2$.

iii. Calculate OWA weights using

$$v_j(A_s) = Q\left(\frac{\sum_{k=1}^{j} u_k}{T}\right) - Q\left(\frac{\sum_{k=1}^{j-1} u_k}{T}\right) \tag{G.8}$$

in the equation above $T = \sum_{k=1}^{m} u_k$, is the sum of importances.

iv. For each alternative, A_s, calculate the overall satisfaction

$$D_s(A_s) = f_v\left(\mu_{s,1}, \mu_{s,2}, \ldots, \mu_{s,m}\right) = \sum_{j=1}^{m} b_j \times v_j(A_s) \tag{G.9}$$

Various sets of fuzzy objectives are considered for evaluating the best management strategy for the Savannah region. Individual performances for each fuzzy objective is aggregated into a single overall performance value using "and," "or" and OWA as the aggregation operators. This analysis is provided in the following section for the hypothetical permit application case given earlier.

REFERENCES

Aguado, E., and Remson, I. (1974). "Groundwater hydraulics in aquifer management." *Journal of Hydraulics Division American Society of Civil Engineers*, 100(HY1), 103-118.

Akpan, U. O., Koko, T. S., Orisamolu, I. R., and Gallant, B. K. (2001a). "Practical fuzzy finite element analysis of structures." *Finite Element Analysis and Design*, 38, 93-111.

Akpan, U. O., Koko, T. S., Orisamolu, I. R., and Gallant, B. K. (2001b). "Fuzzy finite-element analysis of smart structures." *Smart Materials and Structures*, 10, 273-284.

Akter, T., and Simonovic, S. P. (2005). "Aggregation of fuzzy views of a large number of stakeholders for multi-objective flood management decision-making." *Journal of Environmental Management*, 77, 133-143.

Alho, J. M. (1997). "Scenarios, Uncertainty and Conditional Forecasts of the World Population." *Journal of Royal Statistical Society, Series A*, 160(1), 71-85.

Alho, J. M., and Spencer, B. D. (1985). "Uncertain population forecasting." *Journal of the American Statistical Association*, 80(390), 306-314.

Aly, A. H., and Peralta, R. C. (1999). "Optimal Design of Aquifer Cleanup Systems Under Uncertainty Using a Neural Network and a Genetic Algorithm." *Water Resources Research*, 35(8), 2523-2532.

Aral, M. M. (1989). "Waste Stabilization in Multilayer Aquifers by Optimal Hydraulic Control." *Groundwater*, 27(4), 517-523.

Barbeau, B. (2000). "Evaluating the Risk of Infection from the Presence of Giardia and Cryptosporidium in Drinking Water." *Quantitative Microbiology*, 2, 37-54.

Bárdossy, A. (2003). "Automatic fuzzy-rule assessment and its application to the modelling of nitrogen leaching for large regions." *Soft Computing*, 7, 370-385.

Bárdossy, A., Bogardi, I., and Duchstein, L. (1991). "Fuzzy set and probabilistic techniques for health-risk analysis." *Applied Mathematics and Computation*, 45, 241-268.

Bárdossy, A., Bogardi, I., and Duckstein, L. (1990b). "Fuzzy Regression in Hydrology." *Water Resources Research*, 26(7), 1497-1508.

Bárdossy, A., and Disse, M. (1993). "Fuzzy Rule-Based Models for Infiltration." *Water Resources Research*, 29(2), 373-382.

Baudrit, C., and Dubois, D. "Comparing methods for joint objective and subjective uncertainty propagation with an example in a risk assessment." *4th International Symposium on Imprecise Probabilities and Their Applications (2005)*, Pittsburg, Pennsylvania.

Baudrit, C., Dubois, D., and Fargier, H. "Practical Representation of Incomplete Probabilistic Information." *2nd International Conference on Soft Methods in Probability and Statistics*, Oviedo, 2-4 September 2004.

Bauer, P., Held, R. J., Zimmermann, S., Linn, F., and Kinzelbach, W. (2006). "Coupled flow and salinity transport modelling in semi-arid environments: The Shashe River Valley, Botswana." *Journal of Hydrology*, 316, 163-183.

Bear, J. (1972). *Dynamics of fluids in porous media*, Dover Publications, Inc., New York.

Bellman, R. E., and Zadeh, L. A. (1970). "Decision-making in a fuzzy environment." *Management Science*, 17(4), B-141 - B-164.

Bender, M. J., and Simonovic, S. P. (2000). "A fuzzy compromise approach to water resources systems planning under uncertainty." *Fuzzy Sets and Systems*, 115, 35-44.

Bennett, D. H., Kastenberg, W. E., and McKone, T. E. (1999). "A multimedia, multi pathway risk assessment of atrazine: the impact of age differentiated exposure including joint uncertainty and variability." *Reliability Engineering and Systems Safety*, 63, 185-198.

Bezdek, J. C., Dubois, D., and Prade, H. (1999). "Fuzzy sets in approximate reasoning and information systems." Kluwer Academics, Boston.

Black, M. (1937). "Vagueness: an exercise in logical analysis." *Philosophy of Science*, 4(4), 427-455.

Bodjanova, S. (1997). "Approximation of fuzzy concepts in decision making." *Fuzzy Sets and Systems*, 85, 23-29.

Bogardi, I., Bardossy, A., and Duckstein, L. "Risk management for groundwater contamination. Fuzzy set approach." *Optimizing the Resources for Water Management - Proceedings of the ASCE 17th Annual National Conference*, Forth Worth, TX, USA.

Bogen, K. T., and Spear, R. C. (1987). "Integrating uncertainty and individual variability in environmental risk assessment." *Risk Analysis*, 7, 427-436.

Bonano, E. J., Apostolakis, G. E., Salter, P. F., Ghassemi, A., and Jennings, S. (2000). "Application of risk assessment and decision analysis to the evaluation, ranking and selection of environmental remediation alternatives." *Journal of Hazardous Materials*, 71, 35-57.

Brooke, A., Kendrick, D., Meeraus, A., Raman, R., and Rosenthal, T. b. R. E. (1998). "GAMS - A User's Guide." GAMS Development Corporation. http://www.gams.com/docs/gams/GAMSUsersGuide.pdf (January 2005).

Buckley, J. J. (1990). "Stochastic versus Possibilistic Programming." *Fuzzy Sets and Systems*, 34, 173-177.

Bush, P. W., and Johnston, R. H. (1988). "Ground-water hydraulics, regional flow, and ground-water development of the Floridan aquifer system in Florida, and in parts of Georgia, South Carolina, and Alabama: U.S." *Geological Survey Professional Paper 1403-C*, 80 p.

Calvo, T., Mesiar, R., and Yager, R. R. (2004). "Quantitative Weights and Aggregation." *IEEE Transactions on Fuzzy Systems*, 12(1), 62-69.

Carlsson, C., and Fullér, R. (1996). "Fuzzy multiple criteria decision making: Recent developments." *Fuzzy Sets and Systems*, 78, 139-153.

Carlsson, C., and Fullér, R. (2001). "On possibilistic mean value and variance for fuzzy numbers." *Fuzzy Sets and Systems*, 122, 315-326.

Carlsson, C., and Fullér, R. (2002). *Fuzzy reasoning in decision making and optimization*, New York Physica-Verlag Heidelberg.

Chan, T. P., and Govindaraju, R. S. (2001). "Interval Computing Method for Analyzing Field-Scale Solute Transport." *Journal of Hydrologic Engineering*, 6(6), 480-489.

Chang, S. K. "Fuzzy Programs - Theory and Application." *Polytechnic Institute of Brooklyn Conference on Computers and Automata*, Brooklyn, New York.

Chen, Z., Huang, G. H., and Chakma, A. (1998). "Integrated Environmental Risk Assessment for Petroleum-Contaminated Sites - A North American Case Study." *Water Science and Technology*, 38(4-5), 131-138.

Chen, Z., Huang, G. H., and Chakma, A. (2003). "Hybrid Fuzzy-Stochastic Modeling Approach for Assessing Environmental Risks at Contaminated Groundwater Systems." *Journal of Environmental Engineering*, 129(1), 79-88.

Chiang, W.-H., and Kinzelbach, W. (2000). *3D-Groundwater Modeling with PMWIN*, Springer, New York.

Christakos, G. (2004). "A sociological approach to the state of stochastic hydrogeology." *Stochastic Environmental Research and Risk Assessment*, 18, 274-277.

Clarke, J. S., and Krause, R. E. (2000). "Design, Revision, and Application of Ground-Water Flow Models for Simulation of Selected Water-Management Scenarios in the Coastal Area of Georgia and Adjacent Parts of South Carolina and Florida." U.S. Department of the Interior, U.S. Geological Survey, prepared in cooperation with Georgia Department of Natural Resources, Environmental Protection Division and Georgia Geologic Survey, Water-Resources Investigations Report 00-4084, Atlanta.

Clarke, J. S., and Krause, R. E. "Use of Ground-Water Flow Models for Simulation of Water-Management Scenarios for Coastal Georgia and Adjacent Parts of South Carolina." *2001 Georgia Water Resources Conference*, The University of Georgia, Athens, Georgia.

Clarke, J. S., Leeth, D. C., Taylor-Harris, D., Painter, J. A., and L., L. (2004). "Summary of hydraulic properties of the Floridan aquifer system in coastal Georgia and adjacent parts of South Carolina and Florida." U.S. Geological Survey, Scientific Investigation Report.

Clewell, H. J., Gearhart, J. M., Gentry, P. R., Covington, T. R., VanLandingham, C. B., Crump, K. S., and Shipp, A. M. (1999). "Evaluation of the Uncertainty in an Oral Reference Dose for Methylmercury Due to Interindividual Variability in Pharmacokinetics." *Risk Analysis*, 19(4), 547-558.

Cohrssen, J. J., and Covello, V. T. (1989). *Risk Analysis: A Guide to Principles and Methods for Analyzing Health and Environmental Risk*, The National Technical Information Service.

Colarullo, S. J., Heidari, M., and III, T. M. (1984). "Identification of an Optimal Groundwater Management Strategy in a Contaminated Aquifer." *Water Resources Bulletin*, 20(5), 747-760.

Côme, B., Guyonnet, D., and Magnouni, S. (1997). "Apports potentiels de la théori des possibilitiés á l'évaluation des risques environnementaux." *Déchets-Sci. et Techniques*, 7, 40-46.

Cooper, J. A., Ferson, S., and Ginzburg, L. R. (1996). "Hybrid processing of stochastic and subjective uncertainty data." *Risk Analysis*, 16, 785-791.

Coppola Jr, E. A., Duckstein, L., and Davis, D. (2002). "Fuzzy Rule-based Methodology for Estimating Monthly Groundwater Recharge in a Temperate Watershed." *Journal of Hydrologic Engineering*, 7(4), 326-335.

Critto, A., Nadal, N., Carlon, C., and Marcomini, A. (2005). "A Method for Risk Assessment for Three Contaminated Sites in Northern Italy." *Annali di Chimica*, 95(11-12), 833-844.

Cullen, A. C., and Frey, H. C. (1999). *Probabilistic techniques in exposure assessment: a handbook for dealing with variability and uncertainty in models and inputs*, Plenum Press, New York.

Dagan, G. (1982). "Analysis of flow through heterogenous random aquifer." *Water Resources Research*, 18(5), 1571-1585.

Dagan, G. (1986). "Statistical Theory of Groundwater Flow and Transport: Pore to Laboratory, Laboratory to Formation, and Formation to Regional Scale." *Water Resources Research*, 22(9), 120S-134S.

Dagan, G. (2004). "On application of stochastic modeling of groundwater flow and transport." *Stochastic Environmental Research and Risk Assessment*, 18, 266-267.

Daniels, J. I., Bogen, K. T., and Hall, L. C. (2000). "Analysis of Uncertainty and Variability in Exposure to Characterize Risk: Case study Involving Trichloroethylene Groundwater Contamination at Beale Air Force Base in California." *Water, Air, and Soil Pollution*, 123, 273-298.

Das, A., and Datta, B. (1999a). "Development of Management Models for Sustainable Use of Coastal Aquifers." *Journal of Irrigation and Drainage Engineering*, 125(3), 112-121.

Das, A., and Datta, B. (1999b). "Development of Multiobjective Management Models for Coastal Aquifers." *Journal of Water Resources Planning and Management*, 125(2), 76-87.

Das, A., and Datta, B. (2001). "Application of Optimization Techniques in Groundwater Quantity and Quality Management." *Sadhana*, 26(4), 293-316.

Daskin, W. R., and Gorelick, S. M. (1985). "A Policy Evaluation Tool: Management of a Multiaquifer System Using Controlled Stream Recharge." *Water Resources Research*, 21(11), 1731-1747.

de Marsily, G. (1986). *Quantitative Hydrogeology*, Academic Press.

Delgado, M., Vila, M. A., and Voxman, W. (1998b). "A fuzziness measure for fuzzy numbers: Applications." *Fuzzy Sets and Systems*, 94, 205-216.

Deninger, R. A. (1970). "Systems analysis of water supply systems." *Water Resources Bulletin*, 6(4), 573-579.

Despic, O., and Simonovic, S. P. (2000). "Aggregation operators for soft decision making." *Fuzzy Sets and Systems*, 115(1), 11-33.

Don, N. C., Araki, H., Yamanishi, H., and Koga, K. (2005). "Simulation of groundwater flow and environmental effects resulting from pumping." *Environmental Geology*, 47, 361-374.

Dou, C., Woldt, W., and Bogardi, I. (1999). "Fuzzy rule-based approach to describe solute transport in the unsaturated zone." *Journal of Hydrology*, 220, 74-85.

Dou, C., Woldt, W., Bogardi, I., and Dahab, M. (1995). "Steady state groundwater flow simulation with imprecise parameters." *Water Resources Research*, 31(11), 2709-2719.

Dou, C., Woldt, W., Bogardi, I., and Dahab, M. (1997b). "Numerical solute transport simulation using fuzzy set approach." *Contaminant Hydrology*, 27, 107-126.

Dou, C., Woldt, W., Dahab, M., and Bogardi, I. (1997a). "Transient Ground-Water Flow Simulation Using a Fuzzy Set Approach." *Ground Water*, 35(2), 205-215.

Drud, A. "CONOPT." ARKI Consulting and Development A/S. Bagsvaerd, Denmark. http://www.gams.com/solvers/conopt.pdf (January 2005).

Dubois, D., Godo, L., and Prade, H. (1999). "On the possibilistic decision model: from decision under uncertainty to case-based decision." *International Journal of Uncertainty Fuzziness Knowledge-Based Systems*, 7(6), 631-670.

Dubois, D., Nguyen, H. T., and Prade, H. (2000). "Possibility theory, probability and fuzzy sets: misunderstandings, bridges and gaps." Fundamentals of Fuzzy Sets, D. Dubois and H. Prade, eds., Kluwer, Boston, Mass, 343-438.

Dubois, D., and Parade, H. (1992). "When upper probabilities are possibility measures." *Fuzzy Sets and Systems*, 49, 65-74.

Dubois, D., and Parade, H. "Possibility Theory and its Applications: A Retrospective and Prospective View." *Proceedings of the 12th IEEE International Conference on Fuzzy Systems*, St. Louis, Missouri, USA, 3-11.

Dubois, D., and Prade, H. (1980). *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, Inc., New York, New York.

Dubois, D., and Prade, H. (1988). *Possibility Theory. An Approach to Computerized Processing of Uncertainty*, Plenum Press, New York.

Dubois, D., and Prade, H. "Fuzzy sets and probability: Misunderstandings, bridges and gaps." *Second IEEE International Conference on Fuzzy Systems*, 1059-1068.

Dubois, D., and Prade, H. (1994). "Fuzzy Sets - A Convenient Fiction for Modeling Vagueness and Possibility." *IEEE Transactions on Fuzzy Systems*, 2(1), 16-21.

Dubois, D., Prade, H., and Sessa, S. (2001). "Recent Literature." *Fuzzy Sets and Systems*, 119, 355-363.

Dubois, D., Prade, H., and Yager, R. R. (1997). "Fuzzy information engineering: a guided tour of applications." John Wile & Sons, Inc., New York.

Ekel, P., Pedrycz, W., and Schinzinger, R. (1998). "A general approach to solving a wide class of fuzzy optimization problems." *Fuzzy Sets and Systems*, 97, 49-66.

Ekel, P. Y. (2001). "Methods of decision making in fuzzy environment and their applications." *Nonlinear Analysis*, 47, 979-990.

Ekel, P. Y. (2002). "Fuzzy Sets and Models of Decision Making." *Computers and Mathematics with Applications*, 44, 863-875.

Emch, P. G., and Yeh, W. W.-G. (1998). "Management Model for Conjunctive Use of Coastal Surface Water and Ground Water." *Journal of Water Resources Planning and Management*, 124(3), 129-139.

EPD. (1997). "Interim Strategy for Managing Saltwater Intrusion in the Upper Floridan Aquifer of Southeast Georgia." http://www.dnr.state.ga.us/dnr/environ/techguide_files/wrb/interim.htm (January 2005).

EPD. (2001). "Supplement to the Interim Strategy for Managing Saltwater Intrusion in the Upper Floridan Aquifer of Southeast Georgia." http://www.dnr.state.ga.us/dnr/environ/techguide_files/wrb/interim_supp.pdf (January 2005).

EPD. (2003). "Interim Strategy for Permitting Lower Floridan Withdrawals in Coastal Georgia." http://www.ganet.org/dnr/envrion (January 2005).

EPD. (2004). "Clarification to Interim Strategy for Permitting Lower Floridan Withdrawals In Coastal Georgia." http://www.ganet.org/dnr/envrion (January 2005).

European Commission. (1995). "Unbiased guess, a concept to cope with fuzzy and random parameters?" *EUR 16199 EN*, Compiled by S. Wonneberger, S. Kistinger, A. Deckert.

Ferson, S., and Ginzburg, L. "Hybrid Arithmetic." *Proceedings of the 1995 Joint ISUMA Third International Symposium on Uncertainty Modeling and Analysis and NAFIPS Annual Conference on the North American Fuzzy Information Processing Society*, California.

Ferson, S., Ginzburg, L., and Akcakaya, R. (To appear). "Whereof one cannot speak: when input distributions are unknown." *Risk Analysis*

Ferson, S., Ginzburg, L., Kreinovich, V., Myers, D. M., and Sentz, K. (2003). "Consturction of Probability Boxes and Dempster-Shafer Structures." Sandia National Laboratories, Technical Report SANDD2002-4015.

Ferson, S., and Ginzburg, L. R. (1996). "Different methods are needed to propagate ignorance and variability." *Reliability Engineering and System Safety*, 54, 133-144.

Ferson, S., Ginzburg, L. R., Kreinovich, V., Nguyen, H. T., and Starks, S. A. "Uncertainty in risk analysis: towards a general second-order approach combining interval probabilities, and fuzzy techniques." *Proceedings of the 2002 IEEE International Conference on Fuzzy Systems*, Honolulu, HI, USA, 1342-1347.

Ferson, S., and Kreinovich, V. (2001). "Representation, Elicitation, and Aggregation of Uncertainty in Risk Analysis - From Traditional Probabilistic Techniques to More General, More Realistic Approaches: A Survey."

Feyen, L., and Gorelick, S. M. (2004). "Reliable groundwater management in hydroecologically sensitive areas." *Water Resources Research*, 40(W07408), 1-14.

Filev, D. P., and Yager, R. R. "Learning OWA operator weights from data." *Third IEEE Conference on Fuzzy Systems*, 468-473.

Finley, B., Lau, V., and Paustenbach, D. (1992). "Using an uncertainty analysis of direct and indirect exposure to contaminated groundwater to evaluate EPA's MCLs and health-based cleanup goals." *Journal of Hazardous Materials*, 32, 263-274.

Finney, B. A., Samsuhadi, and Willis, R. (1992). "Quasi-Three-Dimensional Optimization Model of Jakarta Basin." *Journal of Water Resources Planning and Management*, 118(1), 18-31.

Fortemps, P., and Roubens, M. (1996). "Ranking and defuzzification methods based on area compensation." *Fuzzy Sets and Systems*, 82, 319-330.

Freeze, R. A. (2004). "The role of stochastic hydrogeological modeling in real-world engineering applications." *Stochastic Environmental Research and Risk Assessment*, 18, 286-289.

Frind, E. O. (2003). *Fundamentals of Groundwater Flow Modelling*.

Garza, R., and Krause, R. E. (1996). "Water-Supply Potential of Major Streams and the Upper Floridan Aquifer in the Vicinity of Savannah, Georgia." *U.S. Geological Survey Water-Supply Paper 2411*, 38 p.

Gelhar, L. W. (1986). "Stochastic Subsurface Hydrology From Theory to Applications." *Water Resources Research*, 22(9), 135S-145S.

Gelhar, L. W., and Axness, C. L. (1983). "Three dimensional stochastic analysis of macrodispersion in aquifers." *Water Resources Research*, 19(1), 161-180.

Geng, L., Chen, Z., Chan, C. W., and Huang, G. H. (2001). "An intelligent decision support system for management of petroleum contaminated sites." *Expert Systems with Applications*, 20, 251-260.

Georgakakos, A. P., and Vlatsa, D. A. (1991). "Stochastic Control of Groundwater Systems." *Water Resources Research*, 27(8), 2077-2090.

Georgia Department of Natural Resources. (2002). "Proposed Permit Protocol: Wells withdrawing from the Lower Floridan Aquifer in the Coastal Area of Georgia."

Ghosh, S., and Mujumdar, P. P. (2006). "Risk minimization in water quality control problems of a river system." *Advances in Water Resources*, 29, 458-470.

Glickman, T. S., and Gough, M. (1990). "Reading in Risk." Resources for the Future, Washington, D.C.

Glorennec, P. (2006). "Analysis and reduction of the uncertainty of the assessment of children's lead exposure around an old mine." *Environmental Research*, 100(2), 150-158.

Gorelick, S. M., and Remson, I. (1982). "Optimal Dynamic Management of Groundwater Pollution Sources." *Water Resources Research*, 18(1), 71-76.

Gorelick, S. M., Voss, C. I., Gill, P. E., Murray, W., Saunders, M. A., and Wright, M. H. (1984). "Aquifer Reclamation Design: The Use of Contaminant Transport Simulation Combined With Nonlinear Programming." *Water Resources Research*, 20(4), 415-427.

Grin, T. R. (2004). "On the applicationo f stochastic approaches in hydrogeology." *Stochastic Environmental Research and Risk Assessment*, 18, 282-284.

Guo, P., and Tanaka, H. "Fuzzy Decisions in Possibility Programming Problems." *Proceedings of the 1996 Asian Fuzzy Systems Symposium*, 278-283.

Guyonnet, D., Bourgine, B., Dobois, D., Fargier, H., Côme, B., and Chilés, J.-P. (2003). "Hybrid Approach for Addressing Uncertainty in Risk Assessment." *Journal of Environmental Engineering*, 129(1), 68-78.

Guyonnet, D., Côme, B., Perrochet, P., and Parriaux, A. (1999). "Comparing Two Methods for Addressing Uncertainty in Risk Assessment." *Journal of Environmental Engineering*, 125(7), 660-666.

Guyonnet, D., Ménard, C., Baudrit, C., and Dubois, D. (2005). "-HyRisk - Hybrid Treatment of Uncertainties in Risk Assessment." BRGM Report RP 53713: 40.

H. A. Loáiciga, R. B. L. (2000). "Closed-form Solution for Coastal Aquifer Management." *Journal of Water Resources Planning and Management - ASCE*, 126(1), 30-35.

Hallaji, K., and Yazicigil, H. (1996). "Optimal Management of Coastal Aquifer in Southern Turkey." *Journal of Water Resources Planning and Management*, 122(4), 233-244.

Hamed, M. M. (1997). "First-Order Reliability Analysis of Public Health Risk." *Risk Analysis*, 17(2), 177-185.

Harbough, A. W., Banta, E. R., Hill, M. C., and McDonald, M. G. (2000). "MODFLOW-2000, The U.S. Geological Survey Modular Ground-Water Model - User Guide to Modularization Concepts and the Ground-Water Flow Process." U.S. Geological Survey. Open-File Report 00-92.

Harter, T. (1994). "Unconditional and Conditional Flow and Transport in Heterogeneous Variably Saturated Porous Media," University of Arizona, Tuscon, Arizona.

Hattis, D., and Kennedy, D. (1986). "Assessing Risks from Health Hazards: An Imprefect Science." *Technology Review*, 89(4), 60-71.

Hayes, L. R. (1979). "The ground-water resources of Beaufort, Colleton, Hampton, and Jasper Counties, South Carolina." 91 p.

Heidari, M. (1982). "Application of Linear System's Theory and Linear Programming to Ground Water Management in Kansas." *Water Resources Bulletin*, 18(6), 1003-1012.

Helton, J. C. (2004). "Alternative representation of epistemic uncertainty." *Reliability Engineering and System Safety*, 85, 1-10.

Holmblad, L. P., and Østergaard, J. J. (1982). "Control of a cement kiln by fuzzy logic." Fuzzy Information and Decision Processes, M. M. Gupta and E. Sanchez, eds., North-Holland, Amsterdam, 398-409.

Huang, G. H., Chen, Z., Tontiwachwuthikul, P., and Chakma, A. (1999). "Environmental Risk Assessment for Underground Storage Tanks Through an Interval Parameter Fuzzy Relation Analysis Approach." *Energy Sources*, 21, 75-96.

Huyakorn, P. S., and Pinder, G. F. (1983). *Computational Methods in Subsurface Flow*, Academic Press, Orlando, Fla.

Inuiguchi, M., Greco, S., Slowinski, R., and Tanino, T. (2003). "Possibility and necessity measure specification using modifiers for decision making under fuzziness." *Fuzzy Sets and Systems*, 137, 151-175.

Inuiguchi, M., Ichihashi, H., and Tanaka, H. (1987). "Decision Procedures Based on Modal Concept and Its Application to Fuzzy Multiobjective Linear-Programming." *Journal of Operations Research Society of Japan*, 30(4), 471-491.

Inuiguchi, M., Ichihashi, H., and Tanaka, H. (1989). "Possibilistic Linear Programming with Measurable Multiattribute Value Functions." *ORSA Journal on Computing*, 1(3), 146-158.

Inuiguchi, M., and Tanino, T. (2002). "Possibilistic Linear Programming with Fuzzy If-Then Rule Coefficients." *Fuzzy Optimizaiton and Decision Making*, 1, 65-91.

Jain, R. (1976). "Decisionmaking in Presence of Fuzzy Variables." *IEEE Transactions on Systems, Man, and Cybernetics*, CYB 6(10), 698-703.

Ji, J.-H., and Chang, N.-B. (2005). "Risk assessment for optimal freshwater inflow in response to sustainability indicators in semi-arid coastal bay." *Stochastic Environmental Research and Risk Assessment*, 19(111-124).

Jumarie, G. (1995). "Possibility, probability and relative information. A unified approach via geometric programming." *Kybernetes*, 24(1), 18-33.

Kacprzyk, J., and Fedrizzi, M. (1990). "Multiperson Decision Making Models Using Fuzzy Sets and Possibility Theory." Theory and Decision Library, Series B: Mathematical and Statistical Methods, W. Leinfellner and G. Eberlein, eds., Kluwer Academic Publishers, Dordrecht, the Netherlands.

Kacprzyk, J., and Nurmi, H. (1998). "Group Decision Making Under Fuzziness." Fuzzy Sets in Decision Analysis. Operations Research and Statistics, R. Slowinski, ed., Kluwer Academic Publishers. Dordetch.

Kacprzyk, J., and Orlovski, S. A. (1987). "Optimization models using fuzzy sets and possibility theory." Theory and Decision Library, Series B: Mathematical and Statistical Methods, H. J. Skala, ed., Kluwer Academic Publishers, Dordrecht, The Netherlands.

Kandel, A. (1986). *Fuzzy mathematical techniques with applications*, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts.

Kangas, A. S., and Kangas, J. (2004). "Probability, possibility and evidence: approaches to consider risk and uncertainty in forestry decision analysis." *Forest Policy and Economics*, 6, 169-188.

Kato, Y., Izuka, T., Ohtsuki, R., and Yamaguchi, S. "A proposal for a new fuzzy probability distribution function." *1999 IEEE International Fuzzy Systems Conference*, Seoul, Korea. Volume III, 1334-1339, 1334-1339.

Kaufmann, A., and Gupta, M. M. (1985). *Introduction to Fuzzy Arithmetic. Theory and Applications*, Van Nostrand Reinhold Company Inc., New York.

Kaufmann, A., and Gupta, M. M. (1988). *Fuzzy mathematical modeling in engineering and management science*, Elsevier Science Pub. Co., Amsterdam.

Kaunas, J. R., and Haimes, Y. Y. (1985). "Risk Management of Groundwater Contamination in a Multiobjective Framework." *Water Resources Research*, 21(11), 1721-1730.

Kelly, E. J., and Campbell, K. (2000). "Separating Variability and Uncertainty in Environmental Risk Assessment - Making Choices." *Human and Ecological Risk Assessment*, 6(1), 1-13.

Kentel, E., and Aral, M. M. (2004). "Probabilistic-Fuzzy Health Risk Modeling." *Stochastic Environmental Research and Risk Assessment*, 18, 324-338.

Kentel, E., and Aral, M. M. (2005). "2D Monte Carlo versus 2D Fuzzy Monte Carlo Health Risk Assessment." *Stochastic Environmental Research and Risk Assessment*, 19, 86-96.

Kentel, E., Gill, H., and Aral, M. M. (2005). "Evaluation of Groundwater Resources Potential of Savannah Georgia Region." Multimedia Environmental Simulation Laboratory, School of Civil and Environmental Engineering, Georgia Institute of Technology.

Khadam, I. M., and Kaluarachchi, J. J. (2003). "Multi-criteria decision analysis with probabilistic risk assessment for management of contaminated ground water." *Environmental Impact Assessment Review*, 23, 683-721.

Kikuchi, S., and Pursula, M. (1998). "Treatment of Uncertainty in Study of Transportation: Fuzzy Set Theory and Evidence Theory." *Journal of Transportation Engineering*, 124(1), 1-8.

Klir, G. J. (1995). "Principles of uncertainty: What are they? Why do we need them?" *Fuzzy Sets and Systems*, 74, 15-31.

Klir, G. J., and Wierman, M. J. (1999). *Uncertainty-Based Information: Elements of Generalized Information Theory*, Physica-Verl., Heidelberg.

Klir, G. J., and Yuan, B. (1995). *Fuzzy sets and fuzzy logic: theory and applications*, Prentice-Hall Inc., Upper Saddle River, NJ.

Kolluru, R., Bartell, S., Pitblado, R., and Stricoff, S. (1996). "Risk Assessment and Management Handbook for Environmental, Health, and Safety Professionals." McGraw-Hill, Inc., New York.

Kooi, H., and Groen, J. (2003). "Geological processes and the management of groundwater resources in coastal areas." *Netherlands Journal of Geosciences-Geologie En Mijnbouw*, 82(1), 31-40.

Köylüoglu, H. U., Cakmak, A. S., and Nielsen, S. R. K. (1995). "Interval algebra to deal with pattern loading and structural uncertainty." *Journal of Engineering Mechanics, ASCE*, 121(11), 1149-1157.

Krause, R. E., and Randolph, R. B. (1989). "Hydrology of the Floridan Aquifer System in Southeast Georgia and Adjacent Parts of Florida and South Carolina." *U.S. Geological Survey Professional Paper 1403-D*, 65 p.

Kreinovich, V., Langrand, C., and Nguyen, H. T. "Combining Fuzzy and Probabilistic Knowledge Using Belief Functions." *Proceedings of 2nd Vietnam-Japan Bilateral Symposium on Fuzzy Systems and Applications, VJFUZZY'2001*, Hanoi, Vietnam, 191-198.

Kwanyuen, B., and Fontane, D. G. (1998). "Heuristic Branch-and-Bound Method for Ground Water Development Planning." *Journal of Water Resources Planning and Management*, 124(3), 140-148.

Labieniec, P. A., Dzombak, D. A., and Siegrist, R. L. (1996). "SoilRisk: Risk Assessment Model for Contaminated Soil." *Journal of Environmental Engineering*, 122(5), 388-398.

Labieniec, P. A., Dzombak, D. A., and Siegrist, R. L. (1997). "Evaluation of Uncertainty in a Site-Specific Risk Assessment." *Journal of Environmental Engineering*, 123(3), 234-243.

Lai, Y.-J., and Hwang, C.-L. (1992). *Fuzzy Mathematical Programming*, Springer-Verlag, Berlin Heidelberg.

Lai, Y.-J., and Hwang, C.-L. (1994). *Fuzzy Multiple Objective Decision Making*, Springer-Verlag, Berlin Heidelberg.

Lai, Y.-J., Liu, T.-Y., and Hwang, C.-L. (1994). "TOPSIS for MODM." *European Journal of Operational Research*, 76(3), 486-500.

Lee, A. S., and Aronofsky, J. S. (1958). "A linear programming model for scheduling crude oil production." *JPT Journal of Petroleum Technology*, 213, 51-54.

Lee, J., Liu, K. F. R., and Chiang, W. (2000). "A possibilistic-logic-based approach to integrating imprecise and uncertain information." *Fuzzy Sets and Systems*, 113, 309-322.

Lee, Y. W., Dahab, M. F., and Bogardi, I. (1994). "Fuzzy decision making in groundwater nitrate risk management." *Water Resources Bulletin*, 30(1), 135-148.

Lee, Y. W., Dahab, M. F., and Bogardi, I. (1995). "Nitrate-Risk Assessment Using Fuzzy-Set Approach." *Journal of Environmental Engineering*, 121(3), 245-255.

Leeth, D. C., Clarke, J. S., Craigg, S. D., and Wipperfurth, C. J. (2003). "Ground-Water Conditions and Studies in Georgia, 2001." U.S. Geological Survey, Atlanta, GA.

Lindner, W., Lindner, K., and Karadi, G. (1988). "Optimal Groundwater Management in Two-Aquifer Systems." *Water Resources Bulletin*, 24(1), 27-33.

Liu, C. M., Yu, J. J., and Kendy, E. (2001). "Groundwater exploitation and its impact on the environment in the North China Plain." *Water International*, 26(2), 265-272.

Liu, L., Cheng, S. Y., and Guo, H. C. (2004a). "A Simulation-Assessment Modeling Approach for Analyzing Environmental Risks of Groundwater Contamination at Waste Landfill Sites." *Human and Ecological Risk Assessment*, 10, 373-388.

Liu, L., Cheng, S. Y., and Guo, H. C. (2004b). "A Simulation-Assessment Modeling Approach for Analyzing Environmental Risks of Groundwater Contamination at Waste Landfill Sites." *Human and Ecological Risk Assessment*, 10, 373-388.

Liu, X. (2004b). "On the Methods of Decision Making under Uncertainty with Probability Information." *International Journal of Intelligent Systems*, 19, 1217-1238.

Louvar, J. F., and Louvar, B. D. (1998). *Health and environmental risk analysis: fundamentals with applications*, Prentice Hall PTR, Upper Saddle River, NJ.

Lutz, W., Sanderson, W., and Scherbov, S. (2001). "The end of world population growth." *Nature*, 412, 543-545.

Ma, H.-W. (2000). "The incorporation of stochasticity in risk analysis and management: a case study." *Stochastic Environmental Research and Risk Assessment*, 14, 195-206.

Ma, H.-W. (2002). "Stochastic multimedia risk assessment for a site with contaminated groundwater." *Stochastic Environmental Research and Risk Assessment*, 16, 464-478.

Ma, H.-W., Wu, K.-Y., and Ton, C.-D. (2002). "Setting information priorities for remediation decisions at a contaminated groundwater site." *Chemosphere*, 46, 45-81.

Maddock III, T. (1972). "Algebraic Technological Function from a Simulation Model." *Water Resources Research*, 8(1), 129-134.

Maddock III, T. (1974). "The operation of stream-aquifer system under stochastic demand." *Water Resources Research*, 10(1), 1-10.

Maddock III, T., and Lacher, L. J. (1991). "Drawdown, Velocity, Storage, and Capture response functions for multi-aquifer systems." *Water Resources Research*, 27(11), 2885-2895.

Mamdani, E. H., and Assilian, S. (1975). "An experiment in linguistic synthesis with a fuzzy logic controller." *International Journal of Man-machine Studies*, 7(1), 1-13.

Mantoglou, A. (2003). "Pumping management of coastal aquifers using analytical models of saltwater intrusion." *Water Resources Research*, 39(12), 5-1 - 5-12.

Mantoglou, A., Papantoniou, M., and Giannoulopoulos, P. (2004). "Management of coastal aquifers based on nonlinear optimization and evolutionary algorithms." *Journal of Hydrology*, 297, 209-228.

Maqsood, M., Huang, G. H., and Yeomans, J. S. (2005). "An interval-parameter fuzzy two-stage stochastic program for water resources management under uncertainty." *European Journal of Operational Research*, 167(1), 208-225.

Martin, E. H., Clarke, J. S., and McCallum, B. E. (2005). "U.S. Geological Survey Science Plan for Georgia, 2005." U.S. Geological Survey Open-File Report 2005-1074, 35 p.

Maxwell, R. M., and Kastenberg, W. E. (1999). "Stochastic environmental risk analysis: an integrated methodology for predicting cancer risk from contaminated groundwater." *Stochastic Environmental Research and Risk Assessment*, 13, 27-47.

Maxwell, R. M., Pelmulder, S. D., Tompson, A. F. B., and Kastenberg, W. E. (1998). "On the development of a new methodology for groundwater-driven health risk assessment." *Water Resources Research*, 34(4), 833-847.

McCarl, B. A. (2006). "GAMS User Guide: 2006." http://www.gams.com/dd/docs/bigdocs/gams2002/mccarlgamsuserguide.pdf (January 2005).

McDonald, M. G., and Harbaugh, A. W. (1988). "A Modular Three-Dimensional Finite-Difference Ground-Water Flow Model." U.S. Geological Survey, Techniques of Water-Resources Investigations, Book 6, Chapter A1, Washington.

McKinney, D. C., and Savitsky, A. G. (2003). "Basic Optimization Models for Water and Energy Management."

McKone, T. E., and Bogen, K. T. (1991). "Predicting the uncertainties in risk assessment." *Environmental Science and Technology*, 25(10), 674-681.

McKone, T. E., and Deshpande, A. W. (2005). "Can Fuzzy Logic Bring Complex Environmental Problems into Focus?" *Environmental Science and Technology*, 39(2), 42A-47A.

McPhee, J., and Yeh, W. W.-G. (2004). "Multiobjective Optimization for Sustainable Groundwater Management in Semiarid Regions." *Journal of Water Resources Planning and Management*, 130(6), 490-497.

Meghdadi, A. H., and Akbarzadeh-T., M.-R. "Uncertainty Modeling through Probabilistic Fuzzy Systems." *Proceedings of the Fourth International Symposium on Uncertainty Modeling and Analysis (ISUMA'03)*, College Park, Maryland.

Melloul, A. J., and Collin, M. L. (2000). "Sustainable groundwater management of the stressed Coastal aquifer in the Gaza region." *Hydrological Sciences Journal - Journal Des Sciences Hydrologiques*, 45(1), 147-159.

Miller, J. S. (1986). "Hydrogeologic Framework of the Floridan System in Florida and in Parts of Georgia, Alabama, and South Carolina." *USGS Professional Paper 1403-B*.

Moens, D., and Vandepitte, D. (2005). "A survey of non-probabilistic uncertainty treatment in finite element analysis." *Computer methods in applied mechanics and engineering*, 194, 1527-1555.

Mohamed, A. M. O., and Côté, K. (1999). "Decision analysis of polluted sites - a fuzzy set approach." *Waste Management*, 19, 519-533.

Molz, F. (2004). "A rational role for stochastic concepts in subsurface hydrology: a personal perspective." *Stochastic Environmental Research and Risk Assessment*, 18, 278-279.

Moore, R. E. (1979). *Methods and Applications of Interval Analysis*, Siam, Philadelphia.

Muhanna, R. L., and Mullen, R. L. "Development of interval based methods for fuzziness in continuum mechanics." *ISUMA-NAFIPS'95. 145-150*, 145-150.

Muhanna, R. L., and Mullen, R. L. (1999). "Formulation of fuzzy finite element methods for mechanics problems." *Computer-aided Civil and Infrastructure Engineering*, 14, 107-117.

Muhanna, R. L., and Mullen, R. L. (2001). "Uncertainty in Mechanics Problems - Interval-Based Approach." *Journal of Engineering Mechanics, ASCE*, 127(6), 557-566.

Mujumdar, P. P., and Sasikumar, K. (2002). "A fuzzy risk approach for seasonal water quality management of a river system." *Water Resources Research*, 38(1), 5-1 - 5-9.

Murata, T., Ishibuchi, H., and Tanaka, H. (1996). "Multi-objective Genetic Algorithm and Its Applications to Flowshop Scheduling." *Computers and Industrial Engineering*, 30(4), 957-968.

Murtagh, B. A., Saunders, M. A., Murray, W., Gill, P. E., and Raman, R. "MINOS."

Mylopoulos, Y. A., Theodosiou, N., and Mylopoulos, N. A. (1999). "A Stochastic Optimization Approach in the Design of an Aquifer Remediation Under Hydrogeological Uncertainty." *Water Resources Management*, 13, 335-351.

National Research Council. (1983). "Risk assessment in the federal government: Managing the process." National Academies Press, Washington, DC.

National Research Council. (1994). "Science and judgment in risk assessment." National Academies Press, Washington, DC.

Nau, R. F. (2002). "The aggregation of imprecise probabilities." *Journal of Statistical Planning and Inference*, 105, 265-282.

Ndambuki, J. M., Otieno, F. A. O., Stroet, C. B. M., Terlaky, T., and Verling, E. J. M. (2003). "An efficient optimization method in groundwater resources management." *Water SA*, 29(4), 359-363.

Ndambuki, J. M., Otieno, F. A. O., Stroet, C. B. M., and Veling, E. J. M. (2000). "Groundwater Management Under Uncertainty: A Multi-objective Approach." *Water SA*, 26(1), 35-42.

Neuman, S. P. (2004). "Stochastic groundwater models in practice." *Stochastic Environmental Research and Risk Assessment*, 18, 268-270.

Neuman, S. P., and Yakowitze, S. (1979). "A statistical approach to the inverse problem of aquifer hydrology." *Water Resources Research*, 15(4), 845-860.

Nguyen, H. T. (1997). "Fuzzy sets and probability." *Fuzzy Sets and Systems*, 90, 129-132.

Nguyen, H. T., and Sugeno, M. (1998). "Fuzzy systems: modeling and control." Kluwer Academic Publishers, Boston.

Nguyen, H. T., and Walker, E. A. (1997). *A first course in fuzzy logic*, CRC Press, Boca Raton.

Oberkampf, W. L., Helton, J. C., Joslyn, C. A., Wojtkiewicz, S. F., and Ferson, S. (2004). "Challenge problem: uncertainty in system response given uncertain parameters." *Reliability Engineering and System Safety*, 85, 11-19.

Orr, S. (1993). "Stochastic Approach to Steady State Flow in Nonuniform Geologic Media," University of Arizona, Tuscon, Arizona.

Ozbek, M. M., and Pinder, G. F. "Fuzzy logic approach to health risk based design of groundwater remediation." *Proceedings of the 1998 12th International Conference on Computational Methods in Water Resources, CMWR, XII'98. Part 1*, Crete, Greece.

Padet, C., Ramer, A., and Yager, R. (1995). "A note on multi-objective information measures." *Advances in Intelligent Computing - IPMU'94 Lecture Notes in Computer Science*, 945, 70-76.

Papadopoulou, M. P. (2002). "Enhanced Methodology for the Solution of Groundwater Management Problems," The Faculty of the Graduate College of The University of Vermont.

Payne, D. F., Rumman, M. A., and Clarke, J. S. (2005). "Simulation of ground-water flow in coastal Georgia and adjacent parts of South Carolina and Florida - Predevelopment, 1980, and 2000." U.S. Geological Survey, Scientific Investigations Report 2005-5089, 91 p.

Pedrycz, W., and Gomide, F. (1998). *An introduction to fuzzy sets: analysis and design*, The MIT Press, Cambridge, Massachusetts.

Provost, A. M., Payne, D. F., and Voss, C. I. (2005). "Simulation of saltwater movement in the Upper Floridan aquifer in the Savannah, Georgia-Hilton Head Island, South Carolina, area, predevelopment-2004, and projected movement for 2000 pumping conditions." *Scientific Investigation Report 2006-5058*.

Rajkumar, T., and Guesgen, H. W. "Fuzzy Exposure Model." *Proceedings of the 1996 IEEE International Conference on Systems, Man and Cybernetics. Part 2*, Beijing, China.

Ramik, J., and Vlach, M. (2002). "Fuzzy Mathematical Programming: A Unified Approach Based on Fuzzy Relations." *Fuzzy Optimization and Decision Making*, 1, 335-346.

Randolph, R. B., and Krause, R. E. (1984). "Analysis of the effects of proposed pumping from the principal artesian aquifer, Savannah, Georgia area." 26.

Randolph, R. B., and Krause, R. E. (1990). "Analysis of the effect of hypothetical changes in ground-water withdrawal from the Floridan aquifer system in the area of Glynn County, Georgia." U.S. Geological Survey Water-Resources Investigations Report 90-4027.

Randolph, R. B., Pernik, M., and Garza, R. (1991). "Water-supply potential of the Floridan aquifer system in coastal Georgia - a digital model approach." *Georgia Geological Survey Bulletin*, 116, 30.

Rao, S. S., and Berke, L. (1997). "Analysis of uncertain structural systems using interval analysis." *AIAA Journal*, 35(4), 727-735.

Rao, S. S., and Chen, L. (1998). "Numerical solution of fuzzy linear equations in engineering analysis." *International Journal for Numerical Methods in Engineering*, 43(391-408).

Rao, S. S., and Sawyer, P. (1995). "Fuzzy finite element approach for analysis of imprecisely defined systems." *AIAA Journal*, 33(12), 2364-2370.

Rao, S. V. N., Bhallamudi, S. M., Thandaveswara, B. S., and Mishra, G. C. (2004). "Conjunctive Use of Surface and Groundwater for Coastal and Deltaic Systems." *Journal of Water Resources Planning and Management - ASCE*, 130(3), 255-267.

Rao, S. V. N., Thandaveswara, B. S., Bhallamudi, S. M., and Srinivasulu, V. (2003). "Optimal groundwater management in deltaic regions using simulated annealing and neural networks." *Water Resources Management*, 17, 409-428.

Rardin, R. L. (1999). "Notes on GAMS for Optimization." http://gilbreth.ecn.purdue.edu/~rardin/gams/notes.html (January 2005).

Ravi, V., and Reddy, P. J. (1999). "Ranking of Indian coals via fuzzy multi attribute decision making." *Fuzzy Sets and Systems*, 103, 369-377.

Remson, I., Hornberger, G. M., and Molz, F. J. (1971). *Numerical Methods in Subsurface Hydrology with an introduction to the finite element method*, Wiley-Interscience, a division of John Wiley & Sons, Inc., New York.

Ribeiro, R. A. (1996). "Fuzzy multiple attribute decision making: A review and new preference elicitation techniques." *Fuzzy Sets and Systems*, 78, 155-181.

Rosenthal, R. E. (2006). "GAMS - A User's Guide." GAMS Development Corporation, Washington, DC, USA. http://www.gams.com/docs/gams/GAMSUsersGuide.pdf (January 2005).

Ross, T. J. (1995). *Fuzzy logic with engineering applications*, McGraw-Hill, Inc., New York.

Roubens, M. (1997). "Fuzzy sets and decision analysis." *Fuzzy Sets and Systems*, 90, 199-206.

Rubin, Y. (2004). "Stochastic hydrogeology - challenges and misconceptions." *Stochastic Environmental Research and Risk Assessment*, 18, 280-281.

Sakawa, M., Inuiguchi, M., Kato, K., and Ikeda, T. (1993). "A fuzzy satisficing method for multiobjective linear optimal control problems." *Fuzzy Sets and Systems*, 78, 223-229.

Sakawa, M., Inuiguchi, M., Kato, K., and Ikeda, T. (1999). "An interactive fuzzy satisficing mehtod for multiobjective optimal control problems in linear distributed-parameter systems." *Fuzzy Sets and Systems*, 102, 237-246.

Sanderson, W., Scherbov, S., O'Neill, B. C., and Lutz, W. (2003). "Conditional Probabilistic Population Forecasting." *IR-03-052*, Laxenburg, Austria: International Institute for Applied Systems Analysis. http://www.iiasa.ac.at/Publications/Documents/IR-03-052.pdf (May 2005).

Sandri, S. A., Dubois, D., and Kalfsbeek, H. W. (1995). "Elicitation, Assessment, and Pooling of Expert Judgments Using Possibility Theory." *IEEE Transactions on Fuzzy Systems*, 3(3), 131-335.

Sasikumar, K., and Mujumdar, P. P. (2000). "Application of fuzzy probability in water quality management of a river system." *International Journal of Systems Science*, 31(5), 575-591.

Sawyer, C. S., and Lin, Y.-F. (1998). "Mixed-Integer Chance-Constrained Models for Ground-Water Remediation." *Journal of Water Resources Planning and Management*, 124(5), 285-294.

Schuhmacher, M., Meneses, M., Xifró, A., and Domingo, J. L. (2001). "The use of Monte-Carlo simulation techniques for risk assessment: study of a municipal waste incinerator." *Chemosphere*, 43(4-7), 787-799.

Schulz, K., and Huwe, B. (1997). "Water flow modeling in the unsaturated zone with imprecise parameters using a fuzzy approach." *Journal of Hydrology*, 201, 211-229.

Shafer, G. (1976). *A Mathematical Theory of Evidence*, Princeton University Press, Princeton and London.

Shafike, N. G. (1994). "Groundwater flow simulations and management under imprecise parameters," The University of Arizona, Arizona.

Shamir, U., Bear, J., and Gamliel, A. (1984). "Optimal Annual Operation of a Coastal Aquifer." *Water Resources Research*, 20(4), 435-444.

Shen, W.-b., Dong, D.-m., and Yu, M.-q. (2004). "Optimal Groundwater Management Model in a Two-Aquifer System." *Practice Periodical of Hazardous, Toxic, and Radioactive Waste Management*, 8(2), 119-129.

Slovinski, R. (1998). "Fuzzy sets in decision analysis, operations research, and statistics." The Handbook of Fuzzy Sets Series, D. Dubois and H. Prade, eds., Kluwer Academic Publishers, Massachusetts.

Słowiński, R. (1998). "Fuzzy sets in decision analysis, operations research, and statistics." The Handbook of Fuzzy Sets Series, D. Dubois and H. Prade, eds., Kluwer Academic Publishers, Massachusetts.

Smith, B. S. (1988). "Ground-water flow and saltwater encroachment in the Upper Floridan aquifer, Beaufort and Jasper Counties, South Carolina." 61 p.

Smith, R. L. (1994). "Technical Guidance Manual - Use of Monte Carlo Simulation in Risk Assessment." *EPA903-F-94-001*, U.S. EPA, Region III, Hazardous Waste Management Division, Office of Superfund Programs.

Stanciulescu, C., Fortemps, P., Installé, M., and Wertz, V. (2003). "Multiobjective fuzzy linear programming problems with fuzzy decision variables." *European Journal of Operational Research*, 149, 654-675.

Sudicky, E. (2004). "On certain stochastic hydrology issues." *Stochastic Environmental Research and Risk Assessment*, 18, 285.

Sugeno, M., Murofushi, T., Mori, T., Tatematsu, T., and Tanaka, J. (1989). "Fuzzy algorithmic control of a model car by oral instructions." *Fuzzy Sets and Systems*, 32, 207-219.

Sun, N. Z., and Yeh, W. W. G. (1992). "A Stochastic Inverse Solution for Transient Groundwater-Flow - Parameter-Identification and Reliability-Analysis." *Water Resources Research*, 28(12), 3269-3280.

Tanaka, H., Okuda, T., and Asai, K. (1974). "On Fuzzy-Mathematical Programming." *Journal of Cybernetics*, 3(4), 37-46.

Tanaka, H., Okuda, T., and Asai, K. (1975). "A formulation of fuzzy decision problems and its application." *Kybernetes*, 5, 25-30.

Theodossiou, N. P. (2004). "Application of Non-Linear Simulation and Optimization Models in Groundwater Aquifer Management." *Water Resources Management*, 18, 125-141.

Tilmant, A., Vanclooster, M., Duckstein, L., and Persoons, E. (2002). "Comparison of Fuzzy and Nonfuzzy Optimal Reservoir Operating Policies." *Journal of Water Resources Planning and Management*, 128(6), 390-398.

Tonon, F., Bernardini, A., and Elishakoff, I. (2001). "Hybrid analysis of uncertainty: probability, fuzziness and antioptimization." *Chaos Solitons and Fractals*, 12, 1403-1414.

U.S. EPA. (1989). "Risk Assessment Guidance for Superfund, Volume I, Human Health Evaluation Manual (Part A)." *EPA/540/1-89/002*, Office of emergency and remedial response, Washington, DC.

U.S. EPA. (1997). "Guiding Principles for Monte Carlo Analysis." *EPA/630/R-97/001*.

U.S. EPA. (1998). "Human Health Risk Assessment Protocol for Hazardous Waste." *EPA530-D-98-001A*.

U.S. EPA. (2001). "Risk Assessment Guidance for Superfund (RAGS), Volume III - Part A, Process for conducting probabilistic risk assessment." *EPA 540-R-02-002*, Office of emergency and remedial response, Washington, DC.

U.S. EPA. (2003). "Human Health Research Strategy." Office of Research and Development, U.S. EPA. EPA/600/R-02/050, Washington, DC.

U.S. EPA. (2004). "An Examination of EPA Risk Assessment Principles and Practices." Staff paper prepared for U.S. Environmental Protection Agency by members of the Risk Assessment Task Force Office of the Science Advisor. EPA/100/B-04/001.

U.S. EPA. (2005). "Guidelines for Carcinogen Risk." U.S. EPA, Washington, DC. EPA/630/P-03/0018.

U.S. Geological Survey. (2000). "Project Proposal: Evaluation of Ground-Water Flow, Saltwater Contamination, and Alternative Water Sources in the Coastal Area of Georgia. Georgia Coastal Sound Science Initiative."

Valberg, P. A., Drivas, P. J., McCarthy, S., and Watson, A. Y. (1996). "Evaluating the health impacts of incinerator emissions." *Journal of Hazardous Materials*, 47, 205-227.

Vamvakeridou-Lyroudia, L. S., Walters, G. A., and Savic, D. A. (2005). "Fuzzy multiobjective optimization of water distribution networks." *Journal of Water Resources Planning and Management - ASCE*, 131(6), 467-476.

Vose, D. (1996). *Quantitative risk analysis - A guide to Monte Carlo simulation modeling*, Wiley, New York.

Wagner, B. J., and Gorelick, S. M. (1989). "Reliable Aquifer Remediation in the Presence of Spatially Variable Hydraulic Conductivity: From Data to Design." *Water Resources Research*, 25(10), 2211-2225.

Wagner, J. M., Shamir, U., and Nemati, H. R. (1992). "Groundwater Quality Management under Uncertainty: Stochastic Programming Approaches and the Value of Information." *Water Resources Research*, 28(5), 1233-1246.

Wang, H.-F., and Huang, Z.-H. (2002). "Top-Down Fuzzy Decision Making with Partial Preference Information." *Fuzzy Optimization and Decision Making*, 1, 161-176.

Wang, J. R. (2001). "Ranking engineering design concepts using a fuzzy outranking preference method." *Fuzzy Sets and Systems*, 119, 161-170.

Wang, L.-X. (1997). *A course in fuzzy systems and control*, Prentice Hall PTR, Upper Saddle River, NJ.

Wang, P. P. (1982). "Advances in fuzzy sets, possibility theory, and applications." Plenum Press, New York.

Wang, T. A., and McTernan, W. F. (2002). "The development and application of a multilevel decision analysis model for the remediation of contaminated groundwater under uncertainty." *Journal of Environmental Management*, 64, 221-235.

Watson, S. R., Weiss, J. J., and Donnell, M. L. (1979). "Fuzzy Decision Analysis." *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-9(1), 1-9.

Willis, R., and Finney, B. A. (1988). "Planning Model for Optimal Control of Saltwater Intrusion." *Journal of Water Resources Planning and Management*, 114(2), 163-178.

Winter, C. L. (2004). "Stochastic hydrology: practical alternatives exist." *Stochastic Environmental Research and Risk Assessment*, 18, 271-273.

Woldt, W., Dahab, M., Bogardi, I., and Dou, C. (1996). "Management of Diffuse Pollution in Groundwater Under Imprecise Conditions Using Fuzzy Models." *Water Science and Technology*, 33(4-5), 249-257.

Wong, H. S., and Yeh, W. W.-G. (2002). "Uncertainty Analysis in Contaminated Aquifer Management." *Journal of Water Resources Planning and Management*, 128(1), 33-45.

Xia, Z. (2001). "Fuzzy probability system: fuzzy-probability space (1)." *Fuzzy Sets and Systems*, 120, 469-486.

Yager, R. (1977). "Multiple Objective Decision-Making Using Fuzzy Sets." *International Journal of Man-Machine Studies*, 9(4), 375-382.

Yager, R. (1984). "Multiple-Objective Decision Functions and Linguistically Quantified Statements." *International Journal of Man-Machine Studies*, 21(5), 389-400.

Yager, R. R. (1978). "Fuzzy Decision Making Including Unequal Objectives." *Fuzzy Sets and Systems*, 1, 87-95.

Yager, R. R. (1982). "Fuzzy set and possibility theory." Pergamon Press Inc., New York.

Yager, R. R. (1988). "On ordered weighted averaging aggregation operations in multicriteria decision making." *IEEE Transactions on Systems, Man, and Cybernetics*, 18(1), 183-190.

Yager, R. R. (1993). "Families of OWA operators." *Fuzzy Sets and Systems*, 59, 125-148.

Yager, R. R. (1995). "An Approach to Ordinal Decision Making." *International Journal of Approximate Reasoning*, 12, 237-261.

Yager, R. R. (1996a). "Quantifier Guided Aggregation Using OWA Operators." *International Journal of Intelligent Systems*, 11, 49-73.

Yager, R. R. (1996b). "Fuzzy logic in the formulation of decision functions from linguistic specifications." *Kybernetes*, 25(4), 119-130.

Yager, R. R. (1998a). "On Ordered Weighted Averaging Aggregation Operators in Multicriteria Decisionmaking." *IEEE Transactions on Systems, Man, and Cybernetics*, 18(1), 183-190.

Yager, R. R. (1998b). "Including importance in OWA aggregations using fuzzy systems models." *IEEE Transactions on Fuzzy Systems*, 6(2), 286-294.

Yager, R. R. (1999b). "Including decision attitude in probabilistic decision making." *International Journal of Approximate Reasoning*, 21, 1-21.

Yager, R. R. (2000). "Fuzzy modeling for intelligent decision making under uncertainty." *IEEE Transactions Systems, Man, Cybernetics*, 30, 60-70.

Yager, R. R. (2002). "On the valuation of alternatives for decision-making under uncertainty." *International Journal of Intelligent Systems*, 17, 687-707.

Yager, R. R. (2004a). "Uncertainty modeling and decision support." *Reliability Engineering and Systems Safety*, 85, 341-354.

Yager, R. R. (2004b). "OWA Aggregation Over a Continuous Interval Argument With Applications to Decision Making." *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 34(5), 1952-1963.

Yager, R. R. (2004c). "Modeling Prioritized Multicriteria Decision Making." *IEEE Transactions on Systems, Man, and Cybernetics - Part B: Cybernetics*, 34(6), 2396-2404.

Yager, R. R., and Filev, D. P. (1994). *Essentials of fuzzy modeling and control*, John Wiley & Sons, Inc., New York.

Yager, R. R., and Filev, D. P. (1999). "Induced ordered weighted averaging operators." *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 29(2), 141-150.

Yasunobu, S., and Miyamoto, S. (1985). "Automatic train operation by predictive fuzzy control." Industrial Application of Fuzzy Control, M. Sugeno, ed., Amsterdam: North-Holland, 1-18.

Yazicigil, H., Al-Layla, R. I., and Jong, R. L. d. (1987). "Optimal Management of a Regional Aquifer in Eastern Saudi Arabia." *Water Resources Bulletin*, 23(3), 423-434.

Yazicigil, H., and Rasheeduddin, M. (1987). "Optimization Model for Groundwater Management in Multi-Aquifer Systems." *Journal of Water Resources Planning and Management*, 113(2), 257-273.

Yin, Y. Y., Huang, G. H., and Hipel, K. W. (1999). "Fuzzy Relation Analysis for Multicriteria Water Resources Management." *Journal of Water Resources Planning and Management*, 125(1), 41-47.

Zadeh, L. A. (1964). "Fuzzy Sets." *No. 64-44*, University of California, Berkley.

Zadeh, L. A. (1965). "Fuzzy Sets." *Information and Control*, 8(3), 338-353.

Zadeh, L. A. (1968a). "Fuzzy Algorithms." *Information and Control*, 12(2), 94-102.

Zadeh, L. A. (1968b). "Probability Measures of Fuzzy Events." *Journal of Mathematical Analysis and Applications*, 23, 421-427.

Zadeh, L. A. (1971). "Toward a Theory of Fuzzy Systems." Aspects of Networks and Systems Theory, R. E. Kalman and R. N. D. Claris, eds., Oxford University Press Inc., 469-490.

Zadeh, L. A. (1972a). "A Rationale for Fuzzy Control." *Journal of Dynamical Systems, Measurement, and Control (Transactions of the ASME)*, 94(1), 3-4.

Zadeh, L. A. (1973). "Outline of a New Approach to the Analysis of Complex Systems and Decision Process." *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-3(1), 28-44.

Zadeh, L. A. (1974). "On the analysis of large-scale systems." Systems Approaches and Environmental Problems, H. Gottinger, ed., Vandenhoeck and Ruprecht, Gottingen, 23-37.

Zadeh, L. A. (1978). "Fuzzy sets as a basis for a theory of possibility." *Fuzzy Sets and Systems*, 1(1), 3-28.

Zadeh, L. A. (1979). "Fuzzy Sets." Operations Research Support Methodology, A. G. Holzman, ed., Marcel Dekker Inc., New York, 569-606.

Zadeh, L. A. (1981a). "Fuzzy Systems Theory: A Framework for the Analysis of Humanistic Systems." Systems Methodology in Social Science Research, R. Cavallo, ed., Kluwer, Boston, 25-41.

Zadeh, L. A. (1981b). "Possibility theory and soft data analysis." Mathematical Frontiers of the Social and Policy Sciences, L. Cobb and R. M. Thrall, eds., Westview Press, Boulder, Colorado, 69-129.

Zadeh, L. A. (1994). "The role of fuzzy logic in modeling, identification and control." *Modeling Identification and Control*, 15(3), 191-203.

Zadeh, L. A. (1995). "Discussion: Probability Theory and Fuzzy Logic Are Complementary Rather Than Competitive." *Technometrics*, 37(3), 271-276.

Zhang, D. (2002). *Stochastic Methods for Flow in Porous Media: Coping with Uncertainties*, Academic Press, San Diego, CA.

Zhang, Y.-K., and Zhang, D. (2004). "Forum: The state of stochastic hydrology." *Stochastic Environmental Research and Risk Assessment*, 18, 265.

Zhou, X., Chen, M., and Liang, C. (2003). "Optimal Schemes of Groundwater Exploitation for Prevention of Seawater Intrusion in the Leizhou Peninsula in Southern China." *Environmental Geology*, 43, 978-985.

Zimmermann, H. J. (1985). *Fuzzy Set Theory and Its Applications*, Kluwer-Nijhoff Publishing, Boston.